

Chapter Eight. Stratification of the Universe of Objects.

§ 37. Adequacy and the stratified ontology.

The introduction of S/A quantifiers in place of (or in tandem with) the more familiar \exists and \forall , may be held to represent merely a harmless 'horizontal' extension of the discrete ontology, as if we were to add a single object or finite group of objects at the periphery, so to speak, of our object-domain, and then redetermine all our laws and statements in order to take account of these additions as appropriate. Such a view would receive support from the fact that the introduction of S and A does not seem to demand any structural modification of the syntactic machinery of \mathcal{L}^1 .¹⁶⁷ It would nevertheless be a mistaken view, for the arguments which justify the introduction of S and A are such as to demand a radical overhaul of the conception of the universe which is built into the discrete ontology and reflected in the machinery of \mathcal{L}^1 . This overhaul, which is required equally by $\mathcal{O}^2, \mathcal{O}^3$, etc., and correspondingly by any language in the sequence $\mathcal{L}^2, \mathcal{L}^3$, etc., turns on the nature of the universe of objects. This is presented in the discrete ontology as a homogeneous, unstratified domain, something which is revealed as massively inadequate to the multi-level universe of objects which is given in experience. Indeed, even on the level of microphysical particles the unstratified model of discrete, autonomous objects seems to receive little support from the data obtained.¹⁶⁸ And when we extend our purview beyond the field of microphysical particles to encompass, for example, mathematical objects, persons, works of art, political organisations (parish councils, cities, nation states), it becomes clear that any adequate ontology, with its corresponding universal characteristic, must allow for a radical non-uniformity in the domain of objects countenanced. For there is an essential stratification in this domain, objects on lower strata combining together in different ways to 'found' or constitute objects within higher strata. The 'discreteness' of the discrete ontology is accep-

table only within particular strata, thus the stratified ontology may be characterised as a relatively discrete ontology, since the discrete or 'atomic' character of individual objects is something which occurs only relative to each particular stratum: what is atomic (structurally simple in the sense of non-decomposable) on one stratum may reveal itself as complex on a different, lower stratum.

The stratified conception of the universe, according to which the minds, tribes, gods, myths, wars, which constitute the higher strata are recognised as ontologically dependent on but not reducible to the objects of lower strata was advanced in the 1930's by Nicolai Hartmann (see especially his 1940, and also Kuhn, 1950/51.) It found an echo in the doctrines of manifold realities and of multiple realities advanced, respectively, by Chwistek and Schutz. (See Chwistek, 1948 and Pasenkiewicz, 1964; and Schutz, 1945). And it has been recently revived, in the context of the philosophy of logic and mathematics (though without reference to any of these thinkers) by Hao Wang, (1974). Wang argues that we should pursue such sciences as

biology and psychology ... not for the practical reason that we do not know enough of the physics and chemistry of life and mind, but for the deeper reason that higher forms of being come into existence only because they are sufficiently stable and even self-contained. (Op. cit. p.4).

The policy of 'reductionism' which motivates acceptance of an ontology such as \mathcal{O}^1 , a policy which can be characterised as an exclusive concern with the results of the earlier sciences in the dependence-hierarchy: physics, chemistry, biology, physiology, sociology, (musicology, etc.), is inadequate in virtue of the fact that the objects of the 'later' sciences are not wholly reducible to the objects of the earlier: major novels, for example, seem to survive translation into other languages in a way which suggests a limited independence of the novel as such from any given linguistic structure. And in mathematical logic, a configuration such as the Lewis S_4 modal structure consistently reappears in very diverse fields of logical inquiry. Wang himself cites the example of higher-level machine languages which are indep-

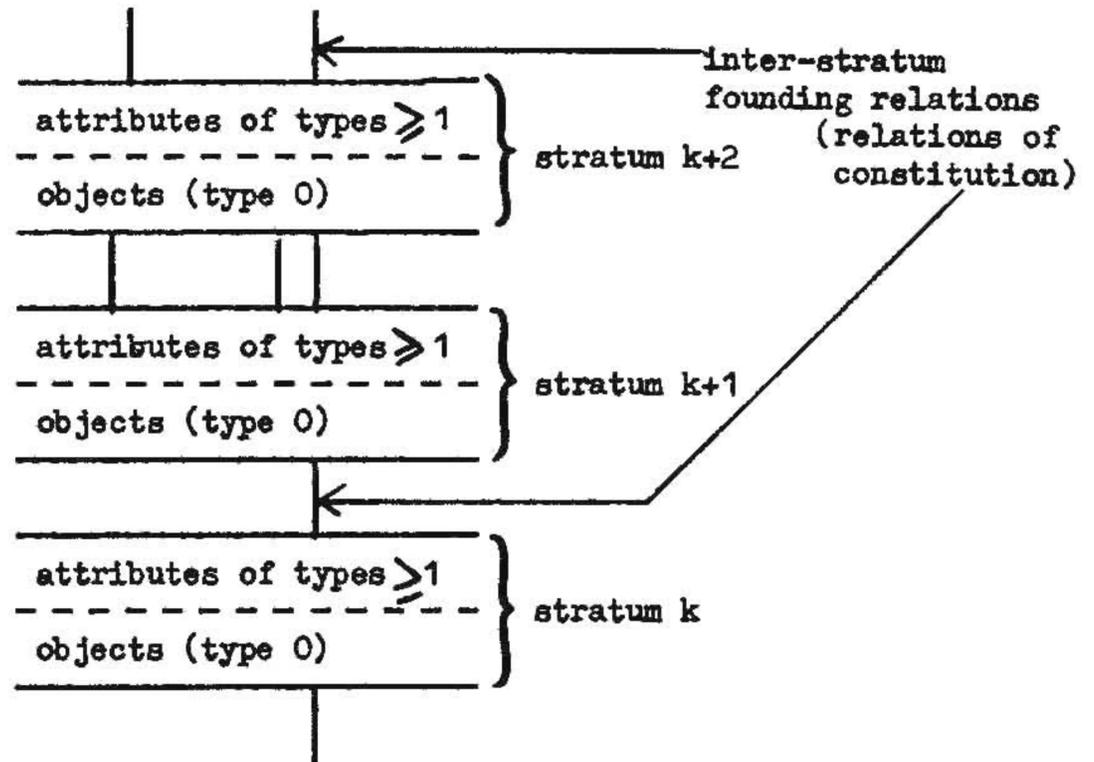
endent of the particular machine on which they are realised. He points out that there is a stabilisation on various levels of being which

enables us to disregard the details as to how [entities on higher strata] are made up from simpler elements. In particular the human mind has its own sphere of operation and can be investigated fruitfully independently of the physical and biological details of the functioning of the brain. (Loc.cit.)

The policy of reductionism can be seen to have a deleterious effect first of all on the practice of the natural sciences themselves, and of mathematics: particularly clear illustrations of this effect in the field of foundational mathematics have been provided by Gödel (see Wang, Op.cit., pp.8-13). But it has ill-effects also for philosophy: in ontology, in particular, the policy of reductionism, which has led to the acceptance of positions of the type of \mathcal{O}^1 , has acquired a position of orthodoxy hand in hand with the assumption that ontology is conceptually a very simple subject, that the demands of formal simplicity and of intellectual elegance may override the requirement of adequacy to die Sachen selbst. This is seen, for example, in the attempted reductions of ontology to set theory, or in Frege's reduction of ontology to his function/object theory.

The stratification into distinct domains of objects at distinct levels should not be confused with the 'type theoretic' differentiation into objects (of type 0) and attributes of various types, i.e.: attributes of objects (having type 1), attributes of attributes of objects (having type 2), and attributes in general of attributes of type n (having type n+1). (Problems of mixed types will not concern us here). In fact both the stratificational dimension and the dimension of (ontological) types are indispensable to the complete ontological fabric. Also indispensable is the dimension of temporality: events, processes, etc. are not included in the schemata here considered, and nor is any recognition of the distinction in mode of being between objects of the past and objects of the present. Exhaustive analyses of the issues raised by temporality would have to be included in any full development of the theory here advanced. (See StEW, I, Ch.v and § 33). We may now represent, in a highly schematic way, one particular section through the

universe of objects as it is now beginning to appear through the theory here advanced:



It is neither asserted nor denied that there is some lowest stratum of absolutely simple objects; in general however it will be acceptable to assume that the universe is stratificationally 'open ended' from below. This can be assumed also with regard to the upper limits of the structure under consideration, for whilst, at any given stage there will be a highest stratum at that stage: the 'highest' —in a purely stratificational sense — product of our cultural ingenuity, no limit is set to the potential of that cultural ingenuity to constitute objects having an order of complexity hitherto un contemplated. The considerations involved may best be indicated by means of examples, which will serve to illustrate also the different types of founding relations which may exist between the members of a given stratum k and those members of lower strata which together co-constitute the stratum k. Such relations will include relations between sub-atomic particles and the atoms and molecules which they co-constitute; relations between molecules and the material bodies (including living beings) which they co-constitute; relations between planetary and stellar bodies and the galaxies which they co-constitute; between actual copies of literary or musical works and the works themselves; between individual persons and the organisations, political unities, tribes, etc. which they co-constitute; between real numbers and real intervals on the mathematical continuum; and so on.

As these examples will make clear, the diversity of the objects in the universe is reflected in a diversity of founding or constitution relations between objects of different strata. The 'ontology' or 'descriptive mathematics' of such founding relations has received little attention either from mathematicians and logicians or from philosophers. What has received exhaustive study is the set or aggregate founding relation ' \in ', and set-theoretical work has provided some useful insights into the nature of a general theory of founding relations and of higher order objects, should such a theory ever come to be developed. The interest of the relation ' \in ' and of the higher-order totalities which are founded through this relation lies in the fact that every stratum of objects generates a derivative higher-level stratum (or, in some cases, a series of such strata) correlated with it, in virtue of the fact that any (homogeneous) totality of objects can be understood as forming a

collection into a whole [Zusammenfassung zu einem Ganzen] of definite and separate objects of our intuition or our thought.
(Cantor, 1895/97, p.282).

Despite this universality of ' \in ' however, the object-wholes which are constituted by it are of little intrinsic interest from the point of view of ontology, when set against examples of higher-order objects such as those which we have instanced above, and such as the examples of higher-order abstract objects discussed in earlier chapters. What, then, can be held to account for the virtual exclusive devotion of mathematicians to ' \in ' at the expense of other founding-relations? Four separate considerations can be distinguished as involved in this matter:

(1) The universal applicability of set theory to all object-strata has been confused with the claims put forward for set theory as a perfectly general theory of higher-order objects as such. (As if the science of colour pigments were to be advanced as an adequate general theory of pictorial art). On the one hand this has had the ill-effect that the higher-order objects themselves, seen by ontologists through the monochrome spectacles of set theory, have tended in consequence to be devalued: it is not only poetry,

we might say, which gets lost in set-theoretic translations. On the other hand there has been the positive effect of the technical developments which have been sparked by these high ontological hopes for set theory: e.g. the development of the theory of equivalence relations motivated by attempts to provide an ontology of abstract objects on the part of Frege and his successors, and the reduction to set theory of mathematics as a whole, of the types which are projected in PM and Quine's 1937, 1951. Some, certainly, of these technical developments will be of service within the generalised set theory (general theory of founding relations) which must form part of the adequate ontology of the future. Many however are of no descriptive mathematical interest at all - they are products whose value is bound up with that of creative mathematics. (For a discussion of the aesthetic and ethical value of creative mathematics see Smith, 1975, Thiel, 1972, § 5.3, and the works of Lorenzen, e.g. his 1968: note that we cannot accept Lorenzen's (interesting) suggestion that the line between what is here called descriptive mathematics and the 'periphery' of purely creative mathematics should be drawn through the point where constructive methods cease to be applicable. The policy of ground-upism (see § 9) is appropriate to ontology only to the limited extent that it allows us to take account of the fact that objects in the universe come into existence or are created at determinate points in time.)

(2) The development of set theory as a creative mathematical discipline has led to a desire to achieve a 'pure' set theory, that is a theory which, like other mathematical disciplines, should not be hampered by the need to consider mundane objects as such (and the sets which they co-constitute) in obtaining its results. Unfortunately this did not lead to the development of a more adequate generalised set theory which would consider founding relations and the objects which instantiate them according to type (where the individual founding relations and objects as such would be dealt with by extra-mathematical disciplines such as theoretical physics). It led, rather to the assumption that it was necessary only to consider sets of sets, that is, in effect, that one considers only the empty set and the sets which can be built up therefrom by finite and infinite cumulation: In this way set theory ceases to have

the status of a descriptive science which would have as its object of study the stratified universe as such: it becomes, rather an aesthetic discipline, which would itself contribute whole new strata (of undeniable beauty) to that universe itself.¹⁶⁸

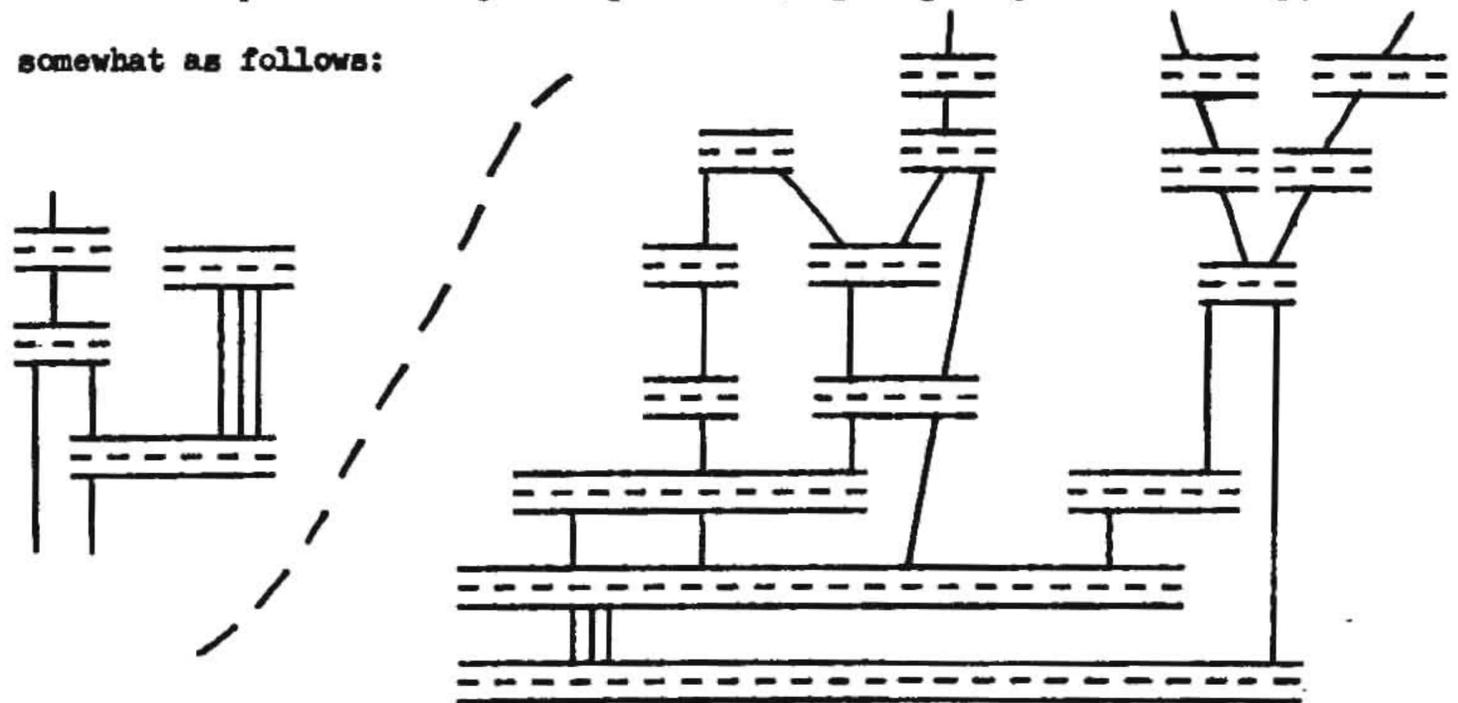
(3) Despite developments such as these in the field of the mathematical theory of sets, the notion of set itself does have a philosophical interest of a type which has made it quite uniquely attractive as a foundation for an extension of the discrete ontology \mathcal{O}^1 which would enable the development within \mathcal{L}^1 of an analogue of a referential theory of attributes. For set theory presents a peculiarly flexible via media between total extensionalism (of e.g. Schröder's Umfangslogik and of whole-part logics such as have been developed by Leśniewski and his school¹⁶⁹), and total intensionalism (e.g. of Inhaltslogik which, with its appeal to conceptual contexts, is mistakenly held to suffer from some form of psychologism). For it is possible to capture an analogue of the distinction, unavailable to the Umfangslogiker, between the Fallen eines Einzelnen unter einem Begriff and the Unterordnung of one Begriff with respect to another (see Angelelli, 1967, Ch.4), in a way which does not require any appeal to intensional entities such as conceptual contents. (This is done simply by distinguishing between the membership of one object (which may be a set) in a set: $\text{Dobbin} \in \{\text{horses}\}$, $\{\text{Dobbin, Neddy, Nijinski}\} \in \{\text{sets of horses}\}$, and the set-theoretic inclusion of one set by another: $\{\text{stallions}\} \subseteq \{\text{horses}\}$, $\{\text{Dobbin, Neddy, Nijinski}\} \subseteq \{\text{horses}\}$). The technical simplicity of the theory of 'attributes' which results is achieved, however, only at the expense of ignoring a complex series of distinctions related to the above, to the extent that any technical gain is many times outweighed by the loss in conceptual refinement. It may be noted that Frege's theory of Wertverläufe is, like his function/object ontology as a whole, in no better shape than set theory to put itself forward as a theory of attributes, and neither is the theory of propositional functions developed by Whitehead and Russell in Principia. Both of these theories are seen, from the point of view of an adequate ontology, to be theories of sets masquerading under other names.

(4) A further quality possessed by sets is that they are what might be

called syntactic entities. (Cf. Husserl's discussion of syntactic categories in *Id, I, § 11*). That is to say, they are entities whose ontological constitution is exhausted by the fact that certain entities are said to belong together in a totality, or, more generally, that particular languages contain the machinery by means of which more or less arbitrary collections of objects may be designated together as forming a common whole.¹⁷⁰ This explains what we referred to above as the 'derivative' nature of the strata of sets, which are associated willy nilly with each and every (linguistically accessible) object stratum, as compared to the non-derivativeness of those strata which contain higher-order objects of other types (e.g. General Motors, the British Milk Marketing Board, the U.S. dollar, the genus equus, and so on). Certainly it is true that many of the latter will consist of objects whose existence (or ontological status as objects) depends more or less mediately upon certain linguistic structures ('Holmes', 'Hamlet', again are the obvious examples, but this is no less true of, say, the River Rhein, which depends upon the establishment of objectively accepted demarcations indicated on various maps and charts.)¹⁷⁰ But in very few cases are higher-order objects other than sets exhausted ontologically in the fact that certain linguistic forms of expression are possible and are engaged in. Thus the complex ontological problems raised by higher-order objects in general and by abstract objects in particular cannot be solved by purely linguistic philosophical means, as is suggested by Dummett (see § 58 below).

Generalised set theory as a general theory of founding relations cannot expect, therefore, to be able to rest content with the relatively simple devices which form the staple of set theory as so far developed. Both intensional and extensional founding-relations will be encountered, and so also will founding-relations which differ from ' \in ' in being double or multiple founding-relations (consider, for example, the founding-relations which will be involved in a higher-order object of the complexity of a performance of a large scale musical or dramatic work). This diversity of founding-relations makes it clear also that the schema considered on p.237, which purports to

represent something of the stratified structure of the universe of objects, incorporates certain important implications which may no longer be overlooked. For the given schema suggests that there is a single rigid hierarchy of strata, which are separated by a series of absolute 'horizontal' divisions, in such a way that it is not excluded that each stratum has a 'width' which is commensurate with the 'width' of the universe, i.e. in such a way that each stratum should include its next lowest stratum and be included by the next highest stratum. The simplified schema thereby ignores the possibility of multiple founding relations, and of 'stratum leaps', by means of which the members of a given stratum k are constituted not only by objects belonging to the next lowest stratum $k-1$, but also by objects belonging to strata below this next lowest stratum. The stratification of the universe is thereby seen to have the character of a diverse series of tree-like structures, which may be relatively disjoint from each other and therefore incomparable as to relative level, implying that there is no unique ordering of the strata by means of an assignment of numerical stratum-'indices'. The diversity involved may be represented, again purely schematically, somewhat as follows:



Here particular strata, or particular groups of closely connected strata, constitute what in more familiar logical parlance are called universes of discourse (domains of specialised scientific investigation). Frege and Russell, who were both conscious, as we shall see, of the requirements of

ontological adequacy for the development of a formal language which can serve the requirements of philosophy, both failed, in different ways, to take adequate account of the stratification in the universe of objects: Frege assumed, in effect, that the universe of objects was a homogeneous totality in its own right, Russell failed to respect the line between ontological stratification which, as will be evident, is far from systematic, with the systematic stratification which is imposed by the linguistic considerations which motivated his type-theoretic hierarchy. What was even more unfortunate was that the lessons which were drawn from the inadequacy of both the Fregean and Russellian approach to ontology were not to the effect that a closer and more respectful inspection of the ontological subject-matter (the universe itself) was required in order to develop an adequate formal language. Rather, the lesson drawn was that the demand of providing a universal formal language was itself something which had to be abandoned. Thus contemporary logic has tended to ignore the cumulative hierarchical arrangement of the various universes of discourse with which logicians may be expected to deal, by considering each domain exclusively in isolation from its neighbours, a tendency which has coincided with the model-theoretic approach to logic as opposed to what was essentially a purely syntactic approach on the part of Frege and Russell. Thus the impression is gained that we are dealing not with hierarchically arranged nodes of a complex set of tree-like structures, but with a number of distinct areas of investigation, each plied by specialists of its own. The stratificational approach suggests a way in which both the universalist approach of Frege and Russell and the model-theoretic approach of Schröder and of mathematical logicians since Gödel, may each provide certain elements of a more complete approach, but these issues can be dealt with only after we have considered some of the consequences of the stratified ontology for the discipline of epistemology.

A Note on Stratified epistemology.

The universe of objects has an 'exposed portion' relative to any given community of subjects, which is just that area of the universe which falls within the scope of what is known by members of the community in question. Each and every epistemic window will possess a stratified structure of its own, which we may designate as an epistemological stratification, related to, though not necessarily perfectly coincident with, the ontological stratification on the side of the entities themselves. Though, again from the realist point of view, it can be assumed that the two sets of stratificational divisions will only peripherally fail to coincide, still such peripheral disparities as do occur will be of the greatest importance from the point of view of epistemology. The most important considerations can be determined by reference to a division between two groups of ontological strata, as follows:

- (I) lower-level strata, which consist of autonomous objects (possessing autonomous properties and relations),
- (II) higher-level strata, whose objects are intentional, depending for their existence upon conscious acts and, e.g., associated frameworks of conventions.¹⁷¹

The task of science can be seen as that of extending the epistemic window in such a way that greater and greater portions of the universe come to be exposed, or in such a way that more and more refined distinctions amongst the entities which are exposed can be recognised and grasped. But in such a process of expansion ontological mistakes can occur. In particular a given domain of group (II) objects may be given as belonging to group (I), something which trivially occurs whenever a work of fiction is read in the belief that it is a work of fact. Conversely a group (I) stratum may (though less frequently) be given as belonging to group (II).

What is important, however, is that ontological mistakes of the given kind can very rarely be analysed simply as a matter of the ontological division between group (I) and group (II) objects, a division which corresponds

to an analogous epistemological division, having been shifted, at various points, in such a way that the group (I)/group (II) opposition which obtains across the epistemic window, fails to coincide with the corresponding ontological opposition. For the domain of epistemically given 'objects' contains not only those 'objects' which are admissible as objects from an external, independent observer's point of view. It contains - or purports to contain - also 'objects' (i.e. in the 'internal' non-ontological sense of positings) which are, from the same external observer's point of view, radically inadmissible. This occurs, e.g. where scientists make the mistake of supposing that there is an intramercorial planet (which may be seen as registering 'itself' on the scientists dials, etc.). Or where explorers lost in the desert make the mistake of supposing that there is an oasis up ahead.

It follows that whilst a two-fold division of strata continues to be sufficient to account for what is known (or believed) concerning the whole domain of epistemically given 'objects' from the 'internal' point of view - the only point of view which can be adopted by the community involved. From the external observer's point of view a three-fold division becomes necessary, epistemic acts (or their noemata), now, being divided into

- (i) acts which involve autonomously existing objects, (objects from a stratum belonging to group (I)) as referents,
- (ii) acts which involve purely intentional objects as their referents, (i.e. which involve objects belonging to a group (II) stratum), and
- (iii) 'purely noematic' acts, which involve no objects at all, although in each and every case the acts will be given as acts of either type (i) or of type (ii). (Compare the act-trichotomy on p.206 above).

This division implies a further task of science, that of seeking out, as far as is possible, those acts which from the external point of view would be seen as belonging to group (iii). And it is a further element of the realist credo here advanced that this task is in large measure capable of being carried out, (though only in a cumulative way: it is a task which will permanently elude completion).¹⁷²

§ 38. Logic as formal ontology.

We have seen that the stratification of the universe of objects implies that the 'discreteness' of the discrete ontology is relativised to particular strata (to particular domains of discourse). Since contemporary logicians, employing logical 'technologies' based almost exclusively on the machinery of standard predicate logic, have adopted the pragmatic approach of restricting each application of formal methods to a single stratum (a single domain of discourse), it has come about that the logical machinery of standard predicate logic, which is associated with the (non-relativised) discrete ontology, has been subjected to little fundamental criticism: Even when this machinery is applied in areas heterogeneous with any domain which could put itself forward as consisting of 'ultimate furniture of the universe', relative discreteness is preserved, and thus the extent to which standard predicate logic is bound up with the discrete ontology is something which is never truly put to the test.

Clearly what is required is an alternative 'non-technological' approach to logic, an approach within which restriction to one domain of discourse at a time, on the grounds that this leads to beneficial results, could no longer be counted as defensible. The crucial insights for such an approach were achieved, we shall suggest, by Frege, in whose work they can be traced, in their turn, to the ambitious projects of Leibniz. (For an account of Leibniz's important influence on Frege's logical thought, see S&R, pp.5-9). Frege shows us that there is an alternative to the pragmatic conception of logic as a special technology, as a 'logic for use', in a conception which has its roots in, (though as we shall see is importantly different from) Leibniz's conception of logic as a universal language or a 'characteristica universalis.' If we spend some time considering how this latter conception - or some equivalent to it - expressed itself in Frege's own work and in the work of his contemporaries, we shall then be able to see the lines along

which a criticism of the foundations of current work in logic could be developed.

To this end it will be useful if we return to consider for a moment Frege's more general position in the history of philosophy. We saw that Frege has to be classified, along with Meinong and Russell, as a member of the realist revolt against German (Hegelian) idealism, (see pp.210ff above, and §§48f). But as Dummett has pointed out, if we leave aside his assault upon psychologism, Frege barely troubled to formulate any explicit attacks on the idealistic position in his writings: he seems rather to have simply 'passed it by' (Dummett, 1967, p.225). What was it that enabled Frege to escape the need to counter idealist positions in the development of his own positive views? Dummett suggests that this was because Frege, almost without noticing the fact, had effected a revolution in philosophy of Cartesian proportions, (loc.cit.). He argues that Frege upset the convention which had been established by Descartes himself and which survived through the idealists to colour even Husserl's work, that the theory of knowledge is the branch of philosophy prior to all others (that is, the branch of philosophy whose problems have to be solved before progress in the other branches can be secured). Frege, looking beyond Descartes to Aristotle and the Scholastics, set logic at the beginning of philosophy arguing, in Dummett's words, that

if we do not get logic right, we shall get nothing else right. Epistemology on the other hand, is not prior to any other branch of philosophy; we can get on with philosophy of mathematics, philosophy of science, metaphysics or whatever interests us without first having undertaken any epistemological inquiry at all. (p.226).

We might express the insight which underlies this rejection of the priority of epistemology as follows: We are not concerned with how we came by the concepts which we employ in each of these areas; our problem is merely to elucidate them. And there is an important lesson in this insight not only for idealists of the more traditional sort, and for Husserl, but also for many empiricist and constructivist philosophers for whom the line between the

pedagogical and the ontological order of things is not always sufficiently decisively drawn. With regard to the second clause of Dummett's argument however, that it is logic which is to replace the theory of knowledge as 'first philosophy', a great deal more precision is required if we are to avoid a fundamental misconception of Frege's (and of Aristotle's) place in philosophy and if we are to make out a case, in particular, for ascribing to Frege a decisive positive contribution to contemporary philosophical thinking.¹⁷³

For it is not at all clear that 'logic' could serve as a philosophia prima: there seems to be a fundamental correctness in the setting aside of the classical triumvirate of logic, ethics and aesthetics from 'substantial' philosophy - not because they are all 'normative' disciplines in any special sense (see § 7 above), but because the principles on which they rest are all principles which are in a certain sense 'already known', they 'go without saying'. An artist can create beauty without the help of the aesthetician, and an individual can be a moral individual without the help of the moralist. So too, we suggest, a philosopher can practice his discipline as well with or without the logician's help.

Certainly there is an admirable 'analytic clarity' which pervades all of Frege's works, but this is not something specific to Frege, nor to those of his successors who were heavily influenced by the Fregean methodology. Indeed the same high standards are met by Husserl's writings of the period of the 1st edition of the Logical Investigations¹⁷⁴, and by all of the mature works of Ingarden, a philosopher who was doubtless conscious of analytic philosophy as it had taken root in Poland between the wars, but who certainly never subscribed to any explicit thesis to the effect that logic could somehow serve as the constant arbiter of his philosophy.¹⁷⁵

Clearly we must conclude that if the notion of a 'Fregean turn' in philosophy is to be rendered defensible, this cannot rest on any conception of logic as a mere 'hygiene of thought'. Frege's logic is more than such a conception would imply, of course, but no precise account of what the dif-

ference consists in is to be found in Dummett's work. Let us therefore consider in some detail three of the more important alternative accounts of this difference which have been put forward in the literature.

(1) The immediate suggestion is that it is linguistic or symbolic innovations which Frege has provided. Indeed Frege's Begriffsschrift does contain important advances over the logical symbolisms which had preceded it, in particular the invention of quantification-theoretic machinery adequate to the presentation of multiple generality, the absence of which had hindered the development of logic for many centuries.(FPL,Ch.2). Frege's other major symbolic innovation however, the analysis of the proposition into function and argument(s) instead of subject and predicate, was far from being undivided in its effect. For whilst such an analysis is appropriate to certain categories of entities (not least within mathematics), to impose the analysis upon all entities and to divide the universe into two exclusive and exhaustive categories of object and function is an indefensible simplification.

Nevertheless Frege's Begriffsschrift, for all its cumbersome to the typographer, does have a pliancy of its own for the practiced user (cf. S&R, p.21,n.22). But the advantages and disadvantages from the user's point of view of one logical symbolism over another have, I think, been overstressed. Even Schröder's logical notation which has received short shrift from Frege-influenced logicians can be shown to be perfectly serviceable for many purposes, and I have found from my own experiments with 'Schröderian' that many of the benefits which are ascribed to it by Löwenheim (1940, p.1) are not without foundation.¹⁷⁶ Only at a later stage (see (3) below) will the various differences between logical symbolisms become crucial, but by then it will no longer be a matter of selecting one symbolic approach in favour of all others.

(2) What is philosophically a more crucial element in Frege's 'logic' derives from the fact that Frege was the first thinker to recognise all of the conditions which are indispensable to the rigorous formulation of a logical

argument, conditions concerning not only the nature of the axioms, and of the rules of inference as turning purely upon the form of the expressions involved, but also concerning the need to distinguish logical and extra-logical primitives and the role of defined terms in the development of a system. One ironic consequence of this quite unique standard of rigour as it was applied by Frege to the development of his 'theory of Wertverhältnisse' in Gg, I, was that it enabled the first fully rigorous derivation of a set-theoretical paradox.¹⁷⁷ Thus Frege's 'set' theory was the first such theory whose inconsistency came to be rigorously demonstrated, despite the fact that, as soon became clear, equivalent paradoxes were derivable also within the naively formulated set theories which had been developed in advance of Frege's work. Only through a certain laxity of formal rigour within the latter theories had it been possible to devise means of shoring up intrinsically insecure foundations - but what must not be assumed is that all such devices were thereby without intrinsic interest.

Frege's achievements under '(2)' can be summarised thus: that he invented the notion of formal system which has become a cardinal notion of mathematical logic since his day. But is this invention something which could support the picture painted by Dummett of a 'Fregean turn' in philosophy? We shall suggest that it is not. For it must be remembered that at least some of the importance of the notion of formal system for modern logic has been that through the investigation of formal systems themselves it has become possible to reveal precisely the limitations of 'formal rigour', limitations which can be identified, for our present purposes, as precisely the limitations of 'mechanical' reasoning. Note that this implies no underestimation of the importance to philosophy of the delimitation of the notion of formal system: even when taken in isolation from his other logical achievements, this delimitation would have been sufficient by itself to ensure Frege a permanent place in the philosophical pantheon. But it is nevertheless nothing more than one contribution, amongst others, to the conceptual resources of the philosopher. It is not a contribution with which we could associate any beneficial

change of direction in the project of philosophy as a whole.

(3) We begin to come closer to determining the quality of Frege's logic which might be held to have set off, or at least given an additional momentum to such a change, if we recognise that Frege's logic was the first mathematical logic. What this involves can be made clear only stage by stage, but we can recognise, first of all, that the demands which Frege set upon his 'logic' were demands which could not be met by the rudimentary symbolisms hitherto developed, which were little more than systems of abbreviations embedded within natural languages. In fact Frege conceived the task of developing a whole new language, 'modelled on the language of arithmetic', which would be an alternative to and not embedded within any natural language. Thus where Frege does use non-formal introductions, explanations and comments in the Begriffsschrift and in Gg, it is only to 'give hints', to help the reader to acquire the new language for himself and thereby to reach the stage, at least in principle, where the 'ladder' of explanations posed in ordinary language can be 'kicked away'. (Cf. van Heijenoort, 1967, p.442 and n.5).

There is no doubt that with the Begriffsschrift Frege succeeded in fashioning an instrument of great power and refinement, by far the closest approximation to a formal language which had at that time been produced. The only thing which is in doubt is the precise purpose which such an instrument may be expected to serve. Leibniz, as is well known, conceived as the purpose of such a language that it would enable scientists of all disciplines (including philosophy) to solve their problems by purely mechanical means. This would be effected by means of some initial decomposition of the concepts used in such sciences to a number of 'simple concepts', represented by means of 'simple signs or characters' in such a way that the individual structure of each complex concept would be perspicuously represented. What is little remarked upon is the extent to which such a project is in fact fundamentally misconceived. And this not least in relation to the aims which Leibniz himself expected it to serve. For science would be hindered, not helped by the restriction to such a language.¹⁷⁸

Even if this were not the case however, and if a universal language thus conceived would somehow be of benefit to science, it is difficult to see why this should be regarded as having any particular philosophical significance, and not, e.g. a purely mnemonic interest. Indeed we shall suggest that Frege was the first thinker to come near to satisfying Leibniz's technical project, precisely because he abandoned, whether consciously or not, Leibniz's background of overambitious philanthropy. This Frege replaced by a less naive assessment of the value of a universal mathematical language, a value which would be restricted to certain branches of philosophy only, in particular to the philosophy of mathematics and to ontology. In order to achieve the requisite technical precision in each of these areas, philosophy finds it necessary to appeal to mathematical methods and to employ mathematically determined concepts in its arguments, something which has been well-expressed by Gödel when he points out that

Mathematical logic, which is nothing else but a precise and complete formulation of formal logic, has two quite different aspects. On the one hand, it is a section of mathematics treating of classes, relations, combinations of symbols, etc., instead of numbers, functions, geometric figures, etc. On the other hand it is a science prior to all others, which contains the ideas and principles underlying all sciences. (1944, p.211).

But in making this appeal to mathematical methods philosophy is no different from e.g. theoretical physics. What distinguishes philosophy from any 'special science',—and we do not claim that the argument which follows is in any sense 'Fregean',—something which entails the indispensability of a mathematical language as an adjunct to philosophy, is that philosophy cannot sanction one likely implication of the introduction of mathematically precise concepts, namely that the objects under investigation should themselves come to be conceived in the image of the formal instruments to which appeal has been made. Philosophy, relative to other sciences, is as it were the custodian of ontological adequacy, which must always seek to counteract the 'reductionist' effects of the introduction of mathematical precision within any given area.

This is especially true, of course, when the area in question is philosophy itself; which implies that here we must take quite peculiar pains to ensure that the formal methods to which appeal is made have no ontologically diluting effect relative to the original content of the concepts which have been made formally precise and of the arguments which have been formalised. But this cannot be ensured by considering, in a piecemeal way, the individual formal elements which are introduced. This is because 'ontological dilution' is something which is effected only through symbolic machinery as a whole. Or rather, to turn the argument around, that the only way to secure against such effects is to employ formal machinery which, taken as a whole, is a language in its own right which preserves the ontological adequacy of the original non-formal language. Only then may the two lend each other mutual support, neither to the detriment of the other.

In order to ensure that adequacy is thus preserved, one must determine the extent to which the formal language satisfies the condition of 'universality' which is imposed upon all natural languages and in particular upon the language of philosophy. This condition may be expressed as follows:

that like all complete natural languages and unlike, say, a technical symbolism developed to represent, e.g. the individual chemical elements and their combinations, a philosophically adequate formal language must be such as to be applicable, or capable of being coherently extended in such a way as to be applicable, to all areas of discourse without restriction.

Formal languages based on standard predicate logic, which have been constructed in such a way as to be applicable to only a single domain of discourse (within a single stratum), fail to satisfy this condition because, as we shall see, their extension to a universal language demands a radical change (a 'hybridisation') of the base logic on which they have been constructed.

Difficulties arise, unfortunately, from the fact that there is more than one conflicting interpretation of the universality condition. Of course, it

can be seen immediately that the universality which is required is not the (unachievable) universality of a language which would contain 'simple signs' for every root concept of all the individual scientific disciplines, as Leibniz supposed. Such a language would be of as little philosophical interest as a language which per impossibile contained proper names for all the objects in the universe. It is a universality, rather, of objects, and of entities belonging to other ontological categories, according to their type. Indeed it is not even demanded that every ontological category should be represented within the ontological language which we are attempting to construct. Such a demand would be tantamount to the assumption of omniscience on the part of the ontologist which he would have no reason to claim: new categories of entities may very well be discovered, either through the extension of that area of the universe which has become accessible to our scientific investigations through increases in refinement in ontology itself (cf. the still rudimentary distinction between universal and particular, concrete and abstract properties discussed on p. 223f above), or because cultural ingenuity has resulted in the creation of abstract entities of a quite unique ontological order:

Phänomenologen sind keine Propheten und wollen es auch nicht sein. (Ingarden, 1975, p.351).

Neither, we suggest, ought we to make prophets of our ontologists. All that is demanded - and hence the requirement of a mathematically ordered syntactic structure of the language under construction - is that that language should show itself to be capable of coherent extension in such a way as to take account of advances in the science of ontology itself. In this way the ontological formal language again establishes itself as a language and not merely as a system of abbreviations which would be tied to one particular stage in the advance of scientific knowledge.

Frege's technical achievements in logic suggest an awareness on his part that the universality of the language of logic was a universality according to type: Frege took the crucial step, for example, of introducing differences in syntactic form to take account of categorial distinctions on the side of

the entities themselves.¹⁷⁹ But his metalogical writings imply, unfortunately, that it is a quite different kind of universality which he has in mind, according to which one and the same base logic could be used in application relative to every domain of discourse; or rather, as Frege would have formulated it: relative to the universe as a whole. This in itself is unobjectionable, until we discover that the base logic involved would be of such simplicity as to imply that formal ontology was itself a relatively simple science. For even Frege was affected by the authority of Aristotle to the extent of supposing that little stood in the way of logic as a completed discipline, and neither he nor Russell succeeded in completely freeing their new mathematical logic from the comparison with (naive or 'epistemological') logic of the kind which had preceded it.¹⁸⁰

Frege and Russell thus failed to grasp the second of the two insights expressed above, namely that formal ontology, like every other science, must partake of piecemeal cumulative advance as a reflection of the temporally determined open-endedness, both of the universe itself and of our knowledge of its individual regions.

Frege's writings nevertheless contain the beginnings of the first clear statement of the relation between the two disciplines of logic and ontology, and it is this, we suggest, which must be seen as the decisive beneficial Fregean contribution to modern philosophy. Thus one might indeed characterise the 'Fregean turn' as something which involved a looking back beyond Descartes to Aristotle, but to Aristotle's unique logico-ontological (logico-metaphysical) methodology rather than to Aristotelian logic, which was itself of little interest to Frege. It will now become clear how important is the conception of Frege as a realist philosopher. For Frege showed us, in effect, the way in which we can conceive logic as a descriptive science, not of proofs, arguments or theories, but of the universe itself; in Russell's words:

Logic is concerned with the real world just as truly as zoology, though with its more abstract and general features. (1919, p.169).

§ 39. The theory of manifolds.

That the notion of 'the universe' could itself have a problematic status was not recognised, either by Frege or by Russell. And they did not see either that it was possible to consider 'other' abstract (e.g. algebraically determined) universes and to consider the validity of the systems which they developed in those universes. Yet such considerations have since become an indispensable tool for mathematical logicians, for whom the notion of validity of the formulae of a system in different 'domains of interpretation' is at least as crucial as the notion of provability of given formulae from specific axioms. Trivially the formula ' $\exists x. x = x$ ' is valid in all but the empty domain (the universe in which there are no objects at all), and the formula

$$' \exists x \exists y (x \neq y \ \& \ \forall z (z = x \vee z = y)) '$$

is valid only in the 2-element domain (i.e. in a universe consisting of just two objects). The crucial question, which could not have been raised by Frege and Russell, is: what are the conditions for a formula to be valid in every domain? This question was raised, and successfully answered for the formulae of the first-order predicate logic, by Löwenheim in his 1915 paper, "Ueber Möglichkeiten im Relativkalkül" which, together with Frege's Begriffsschrift and Ch.v of Herbrand's 1929 thesis (Recherches sur la théorie de la démonstration), has been called by van Heijenoort one of 'the three cornerstones of modern logic' (1967, p.444f). Löwenheim's answer to the question which he had set, was that any formula which is not, like the two formulae considered above, invalidated by some finite domain, is valid in all domains only if it is valid in some denumerable domain (some domain which can be put into one-one correspondence with the natural numbers).

What is of immediate importance for us here however, is not Löwenheim's result, but the fact that he was able to conceive the relevant questions at all, questions which could not arise within the two axiomatic methodologies (of logicism: Frege/Russell, and of proof theory: Hilbert) which were dominant at the time. If we are to understand how Löwenheim came to develop the instru-

ment of 'algebraic' variation of domains as a methodological tool, we must recognise that Löwenheim belonged to neither of these schools, but to the non-axiomatic tradition of Boole, Peirce and Schröder which was itself, we shall suggest, a minor tributary of a much more important tradition to which all too little attention has been paid by modern philosophers of mathematics.

We shall begin with a very brief account of some aspects of Schröder's theory of manifolds which formed the backbone of Schröderian 'logic'. We begin with a universe or manifold, denoted by '1', the nature of which may be left unspecified, for the moment, since we are concerned only with the relations between domains (which Schröder also calls 'classes') which can be picked out within the manifold. Each such domain is denoted by a symbol 'a', 'b', 'c', etc., and between the domains a relation of subsumption is defined; ' $a \in b$ ', that is 'a is subsumed by b' ('b subsumes a'), expresses the fact that a is either equal to or is a proper part of b. Analogues of the processes of addition, subtraction, and multiplication (i.e. the taking of intersections between domains) are then defined, and an empty domain, 0, is introduced, defined as the subtraction of 1 from itself. In general, the following holds for all domains a:

$$0 \in a \quad \text{and} \quad a \in 1. \text{ } ^{181}$$

To fix our ideas we may consider some examples of manifolds which satisfy Schröder's rules. As a first example let 1 = the Euclidean plane, from which it is possible to select, e.g. by circumscription, determinate areas of the plane considered as sets of points. The latter are to constitute the domains of the manifold, and 0 in this case serves as what might be called an empty or null area. The sense in which 0 belongs to (is subsumed in) every non-null area can be seen from the fact that 0 is in each case what is 'left over' when such an area is subtracted from itself. As a second example let us consider the manifold which is the German army. Here the domains which it is possible to select include individual soldiers, classes of soldiers such as regiments and battalions, and even classes of regiments and battalions, and so on. Note that it is a consequence of Schröderian Umfangslogik that the domain

which consists of the 1st and 6th regiments is identical with the domain which consists of the individual soldiers which make up the 1st and 6th regiments: at this level, at least, the two terms 'domain' and 'element' are synonymous.

It is possible to determine domains not only by particular specifications (e.g. by naming the soldiers, battalions or regiments which are involved), but also by means of predicates, that is, in effect, by means of conceptual conditions. Thus one can define the domain of married soldiers, the domain of soldiers more than 2 meters high, the domain of all soldiers (which is identical with the manifold as a whole), and so on. What is at first a puzzling step on Schröder's part is the assumption that a domain which is determined by such a predicate, as assigned to its elements, is itself assigned the predicate in question. Thus if the soldiers in the 8th regiment are all blond-headed, Schröder would assert that the regiment as a whole was itself to be characterised as being blond-headed. (Cf. the puzzlement generated by Lorenzen's argument on p.128 above). A further, even more disconcerting consequence of Schröder's 'theory of predication' is that, since the empty domain is an element of, i.e. is subsumed in every domain, it follows that 0 is the subject of every predicate (Schröder, 1890, p.245). That is that 0 is both blond-headed and red-headed, both taller than 2 meters and smaller than 1 meter, and so on. Such a state of affairs is acceptable for Schröder only because (as we saw on p.137) for him 'is red-headed' can only mean 'is subsumed in the domain of red-heads', and similarly for every other predicate, Schröder being quite prepared for the elimination of conceptual content (Inhalt) which such an interpretation of 'is red-headed' must imply.

We are now in a position to determine, in a more precise way than is usual, the connections between Schröderian logic on the one hand and the 'Fregean' project of logic as formal ontology on the other. Van Heijenoort has proposed the following account:

The universality of logic expresses itself in an important feature of Frege's system. In that system the quantifiers binding individual

variables range over all objects. As is well known, according to Frege, the ontological furniture of the universe divides into objects and functions. Boole has his universe class, and de Morgan his universe of discourse, denoted by '1'. But these have hardly any ontological import. They can be changed at will. The universe of discourse comprehends only what we agree to consider at a certain time, in a certain context. For Frege it cannot be a question of changing universes. One could not even say that he restricts himself to one universe. His universe is the universe. Not necessarily the physical universe, of course, because for Frege some objects are not physical. Frege's universe consists of all that there is, and it is fixed. (1967, p.441).

First of all does the 'universality' of a characteristica universalis (in the acceptable sense of a formal-ontological language) imply the necessity to introduce quantifiers binding individual variables which range over all objects, implying the acceptance of a determinate totality of objects in the universe? And is a system which has this feature even an acceptable system, as van Heijenoort seems to assume? The answer to these questions is provided by van Heijenoort himself when he tells us, without flinching, that Frege's conception of 'universality' yields the consequence

that functions (hence, as a special case, concepts) must be defined for all objects. To take an example, the function '+' is defined not only for the natural numbers, but also for, say, the Moon and I. What the value of the function is in that case is irrelevant here, but this value must exist for every set of arguments chosen from among the objects. (Op.cit., p.441f).

It is perhaps a reflection of the extent to which Frege's conceptual framework underlies current writings on logic that van Heijenoort fails to draw from this consequence any implication to the effect that, as we suggested on p. 76f, there is something very seriously wrong with Frege's quantification theoretic interpretation of the universality condition for formal-ontological languages.

Secondly, is it really the case that Boole's and De Morgan's 'universe classes' and, by implication, Schröder's manifold 1, have 'hardly any ontol-

ogical import'? In coming to this conclusion van Heijenoort seems to be appealing to those elements of Boole's work in which he tries to take account of a feature of thought in general which is all too inadequately represented within either Russell's or Frege's logical machinery, namely that

In every discourse, whether of the mind conversing with its own thoughts, or of the individual in his intercourse with others, there is an assumed or expressed limit within which the subjects of its operation are confined. ... Sometimes, in discoursing of men we imply (without expressing the limitation) that it is of men only under certain circumstances and conditions that we speak, as of civilised men, or of men in the vigour of life, or of men under some other condition or relation. Now whatever may be the extent of the field within which all the objects of our discourse are found, the field may properly be termed the universe of discourse. (1854, p.42).

What we must not overlook are the passages in which Boole expressly admits the possibility of

discourse in which the words we use are understood in the widest possible application, and for them the limits of discourse are co-extensive with those of the universe itself. (Loc.cit.)

Thus although discourse in general is such that the 'universe' which is involved has no ontological import - indeed it may include merely possible 'objects' - still Boole clearly recognises the possibility of a fully ontological style of discourse, the universe class relative to which is determined by Boole in a quite sophisticated realist way as precisely

the actual universe of things, which it always is when our words are taken in their real and literal sense. (Loc.cit.)

In such cases we mean by 'men', 'all men that exist': it is only when

the universe of discourse is limited by any antecedent implied understanding [that it is] of men under the limitation thus introduced that we speak. It is in both cases the business of the word men to direct a certain operation of the mind, by which, from the proper universe of discourse, we select or fix upon the individuals signified. (Loc.cit.)

Thus Boole in effect employs a preferred standard interpretation of the symbol '1' as meaning

"the Universe", since this is the only class in which are found all the individuals that exist in any class. (Op.cit., p.48).

Schröder's use of '1' is more complex: As we shall see, Schröder finds it necessary to reject Boole's universal interpretation, but what must be realised is that this is done precisely in order to uphold the incipient realist approach advanced by Boole - something which is hardly consistent with the suggestion that both Boole's and Schröder's universes are without any ontological import. Of course, in Schröder's works we find various possible prototypes - not only geometrical and military - of manifolds which may satisfy the rules which are laid down for the manifold 1. This is done, first of all, simply to make the more abstract general theory understandable by means of concrete examples. Secondly it is done in various applications of the theory, when the theory itself takes on the guise of a 'calculus ratiocinator'. But even here it is not quite true to say that the various different manifolds 1 which are involved in such applications are without ontological import. Rather we ought to say that they are without any direct ontological import with respect to the category of object-entities. For the given 'manifolds' are, in fact, particular noematic configurations (in the sense established in Part One above). That is to say, they are that through which we organise our experience of the transcendent objects which exist in the various fields of application of the theory.¹⁸²

In his 1877 Schröder accepted, in effect, that the only properly ontological import of the manifold 1 would be to the universe as a whole. But by the time of his Vorlesungen über die Algebra der Logik (Exakte Logik) of 1890 Schröder had discovered that

it is in fact inadmissible to understand by '1' such an all-inclusive ... class as the Boolean "universe of discourse". (1890,p.245).

We might express the core of Schröder's argument somewhat as follows: In the manifold which is the German army it is possible to select, as we have seen, certain domains which are 'equal in width' to the 'universe' itself, e.g. the domains all soldiers, all regiments, and so on.

Now consider the domain determined by the conceptual condition

universe-wide (i.e. relative to the German army as universe). This domain, like all others, has the empty domain 0 subsumed within it. But then it follows that 0 itself is universe-wide! Here we have, be it noted, not merely a peripheral extension of the meaning of the predicate involved, as e.g. we have such a peripheral extension of the meaning of 'dog' if we say that 0 is a dog: The conclusion that 0 is universe-wide would, if we had to accept it, involve us in accepting something which was an outright contradiction. In the given case, however, it might seem that the contradiction can be avoided by arguing that the domain determined by the condition universe-wide, unlike say, the condition soldier of the 8th regiment, does not determine a domain in the manifold which is the German army. It is, so to speak, a non-military condition: it raises issues which are outside the relevant 'universe of discourse'. Such a course is impossible however if we select as our manifold 1 the universe itself, which consists of all objects and of all domains. In this case let us, following Schröder, consider

the class of those classes which are equal to 1, \lceil and this would certainly be permissible if we could include in 1 everything conceivable \rceil , then this class comprises essentially only one object, viz. the symbol 1 itself, respectively: the whole of the manifold which constitutes its reference - but beyond that it contains also "nothing", 0. But now if 1 and 0 constitute the class of those objects which have to be counted as equal to 1, then not only $1 = 1$, but also: $0 = 1$, must be admitted. Since a predicate which belongs to a class \lceil here the predicate being identically equal to 1 \rceil , must also belong to every individual of the class.

In such a manifold, in which $0 = 1$ holds, all possibility of distinguishing two classes or even individuals, is antecedently excluded; everything would here be all the same \llcorner hier wäre dann alles "wurst" \lrcorner . (Schröder, 1890, p.245, the single square brackets are Schröder's).

Thus

It has been shown that it is not permissible to leave the manifold 1 completely undetermined, wholly unlimited or open, since some conceptually possible formations of the predicate-class...prove not to be permissible. How, then, must the manifold 1 be constituted in such a way that the rules of the calculus, when applied to it... could no longer lead to contradiction? (Op.cit.p.246).

This is a most important passage, in which Schröder codifies, perhaps for the first time, the problem which has to be faced by any logic which is to serve as the basis for a universal 'formal ontology' in the sense determined above. What Schröder has recognised, for the specific case of his own manifold-theoretic 'logic', is that that universality can be achieved only at the expense of a complexity of the logic involved of an order hitherto un contemplated. For as will be made clear, only for certain manifolds are the rules of what we might call Schröder's core manifold theory valid: For other manifolds a more complex theory, constructed from the given core, but employing additional logical machinery in such a way that such manifolds become decomposed into manifolds for which various separate expressions of the core theory are valid, has to be brought into play. And in order to determine whether it is the core theory which is applicable or one or other of these more complex hybrid theories an independent 'empirical' investigation of the manifold itself is indispensable. In just the same way, we shall argue, the question whether standard predicate logic (or other systems closely related to it) is applicable to a particular field of investigation, or whether appeal must be made to some more complex, hybrid logic can only be decided by first investigating the structure of the given field of investigation.

For Frege any such suggestion was anathema: he saw Schröder's work as constituting what was in his terms a rejection of the 'universality' of logic, and indeed he describes Schröder's specific proposals as merely an "expedient", which only

belatedly gets the ship off the sandbank; but if she had been properly steered, she could have been kept off it altogether...Whereas elsewhere logic may claim to have laws of unrestricted validity, we are here required to begin by delimiting a manifold with careful tests, and it is only then that we can move around inside it.

(Frege, 1895, p.439f).

Frege, and the logicians who followed him, could preserve their favoured a priori applicability of the 'simple' logics (formal ontologies) which they developed, only at the expense of imposing, in Procrustean fashion, a precisely

equal and opposite 'simplicity' of ontological structure upon the universe itself, such that even today the conceptual complexity of the logical machinery which formal ontology requires is little appreciated.

Schröder's detailed account of the constitution of manifolds for which his 'core' manifold theory would be valid, falls into two parts. The first and more important part of the account may be seen as a partial anticipation of our views on stratification expressed in § 37 above: We are concerned, Schröder writes,

with a manifold of any kind of 'things' - objects of thought in general - as 'elements' or 'individuals'. These may be (in whole or in part) given from the start, or may be (in the other part or in whole) somehow determined only conceptually. But they cannot remain completely undetermined, as we have already shown... the elements of the manifold must be all of them consistent [vereinbar] and "compatible" ["verträglich"] with each other. Only in this case do we symbolise the manifold by 1. (Op.cit., p.246f, trans. in Bochenski, 1961, p.392).

Elements of a manifold are, Schröder tells us, 'compatible' only if they are such that 'we are able to think the manifold as a whole' (op.cit., p.212).

[This may be seen as an anticipation of Cantor's definition of 'set' given on p.238 above; an even more important coincidence of views between Cantor and Schröder will be discussed below.] What this implies, first of all, is that manifolds which are 'too large' will be excluded from the category of manifolds 1; but there is a second implication which may be drawn, namely that elements which would be, in our terms, selected from different strata, do not constitute compatible manifolds. Here the familiar examples will suffice:

<the table on which I write, 10^{27} carbon molecules>
 <Cantor's brain, 1543 gms of nerve cells and nerve fibres>
 <1 army, 8 regiments, 24 battalions, 24,000 soldiers>
 <60,000,000 people, 8 federal states, 1 nation>
 <1 university, 24 colleges,> and so on.

The last manifold, for example, cannot be said to consist of 25 elements, but nor can it be said to consist of only 1, or of only 24 elements. Such examples do not arise only as a result of trivial 'category mistakes' - they can serve

as an indispensable part of the subject-matter, first of all of the individual sciences, and then of the 'generalised (set) theory of founding relations' discussed in § 37. From this point of view the first and second examples are seen as forming part of the field of investigation of, respectively, theoretical physics and theoretical biology, in so far as the scientists in these areas are concerned how the first element is constituted out of the second. And the remaining examples involve particular founding relations which may form an important object of investigation for the 'human sciences' [Geisteswissenschaften: interestingly Schröder tells us that those manifolds which fail to satisfy his compatibility condition arise 'ausschließlich auf geistigem Gebiete' (1890, p.213); the examples which he gives are, however, less sharply determined cases of multi-stratal manifolds than those given here.] We shall return below to consider Schröder's work as a (hesitant) contribution to the generalised theory of founding relations. First however let us consider the second part of Schröder's account of the conditions which must be satisfied for a manifold to be a 'universe'-manifold 1:

Once we have obtained a manifold whose elements are consistent and compatible as determined above, then the elements of such a manifold

can be arbitrarily collected together into systems, 'domains' of its elements which are demarcated within it. In other words, even for the purposes of distributive application, we can bring out [hervorheben] any classes of individuals.

And in particular the individuals themselves belong amongst the classes, which are then designatable as "monadic" or "singular" classes, classes which are crumpled up [zusammenschrumpfen] to just one individual alone.

Through this process of arbitrary selection or bringing out of classes of individuals from the originally established manifold, there arises, [that is to say] is created what is (in general) a new, much more inclusive manifold, namely the manifold of the domains or classes of the first. ...

The new manifolds could be called the 'second power' of the former - or better its 'first derived or derivative manifold' [erste

abgeleitete oder derivierte Mannigfaltigkeit] (Schröder, 1890, p.247f, compare the translation given by Bochenski, op.cit., p.392).

Schröder explicitly countenances the possibility of 'deriving' in this manner a second derived manifold from the first, and so on; and in order that a manifold be such that it satisfies the law $0 \in a$ for all of the domains, a , distinguishable within it Schröder puts forward as a

necessary (and sufficient) condition...that among the elements given as 'individuals' there should be no classes comprising as elements individuals of the same manifold. (Schröder, op.cit., p.248, Bochenski, op.cit., p.393).

The rigid separation of manifolds of successively higher powers from an original compatible manifold is, as has already been recognised in the literature (see Church, 1939, Bochenski, 1961, pp.391-93, Sluga, 1962, 203-5, van Heijenoort, ed., 1967, p.152), an important anticipation of Russell's theory of types. But there are important differences, not all of which accrue to the benefit of Russell:

- (a) Besides the type-theoretic dimension, which Schröder explicitly characterises as 'derived', Schröder distinguishes also a 'horizontal' dimension of compatibility amongst the original zero-type manifolds of objects. This horizontal dimension is, we suggest, an anticipation of the stratificational viewpoint advanced in the present work.
- (b) In recognising that the derived manifold of a given manifold is created [wird geschaffen] by a process of conceptual selection, Schröder introduces an important element of constructivism into his 'type theory' (which finds an echo in certain thought-experiments on the set-theoretic paradoxes made by Husserl, see Haddock, 1973, Ch.VII; cf. also n.180 above).
- (c) Russell developed his ontology, which involved an infinite sequence of higher types, in order to solve specific problems in the foundations of mathematics. In particular he wished to show that it was possible to provide an axiomatisation of 'full' (classical) analysis in a way which was secure against paradoxes, especially of the type which had afflicted Frege's system. For Russell the higher types were an indispensable tool in the 'logicisation'

of classical analysis. But (given the set-theoretic interpretation of Russell's type-hierarchy) such higher types consist exclusively of 'pure' sets, i.e., of sets whose elements are themselves also sets. Thus the Russellian approach to type theory may be seen to have led to an exaggerated importance being given by logicians since Russell (and Zermelo)¹⁸³ to pure sets, despite the fact that such sets are of little significance to descriptive mathematics and are even, it seems, dispensable to the 'set'-theoretic foundation of mathematics. (Cf. Löwenheim, 1940). Schröder's type theory however was formulated in such a way that the principle attention is devoted, correctly, to the lowest manifold, the manifold of individual objects with the uniformly 'impure' sets (domains) definable within it: the hierarchy of derived manifolds can then be conceived merely as an empty possibility which need be of no over-riding ontological significance. This was because Schröderian type theory was not bound up with any axiomatic approach to logic, involving an attempt to reconstruct mathematics from the ground up. Schröder worked, rather, against a background within which mathematics, and especially classical analysis, was already given as a whole (i.e. in the way in which every other cumulative science is given to each generation of scientists who have interiorised the theories involved). This suggests a need to isolate a third non-axiomatic tradition of early foundational research, an alternative both to the logicist/constructivist/intuitionist school on the one hand and to the proof-theoretic/formalist school on the other. The most important members or near-members of this tradition would include Riemann, Bolzano, (who was, however, both geographically and intellectually isolated from the centre of the tradition), Peirce, Schröder, Cantor, Dedekind, Grassmann, Whitehead (of the Universal Algebra), Husserl, (the philosophical apogee of the tradition), and finally Löwenheim. With the successes of the analytic approach this tradition all but died: professional philosophers and mathematicians who were interested in foundational issues devoted their attention almost exclusively to the results obtained from the axiomatic approach, and Schröder's Algebra of Logic became something fit only for schoolteachers, (see Thiel, 1975, for an account of Löwenheim's life).

For the members of this third tradition it was not the provability of

particular formulae from pre-selected or pre-discovered axioms which was important, but rather the validity of formulae and of whole theories relative to particular fields of application; in particular, we might say, the validity of the theory of classical analysis relative to the physical world. Since it is not assumed that the theories involved are given in a specifically axiomatic form, the policy of 'ground-upism' which is dictated by the search for axioms comes to be avoided. We thus obtain a far more acceptable, descriptive account both of the mathematicians and of the physicist's employment of individual theories, from which we may expect to be able to derive the 'direct' theory of applicability which we have argued is lacking from the Fregean approach. (Cf. pp. 93f and n.66). Löwenheim's achievement was that he applied this 'wholistic' (non-axiomatic) approach to logic itself, and thereby extended the notion of 'validity' to logical theories. Löwenheim's 1915 paper is concerned, in effect, with first order predicate logic, but as is pointed out by van Heijenoort (1967, p.443) it does not appeal to axioms; indeed the methodology involved can best be compared to Cantor's 'abstract' or 'naive' approach to the theory of sets, an approach which has maintained its position amongst mathematicians to an extent which is sometimes little realised amongst philosophers of mathematics.

Our own immediate concern is the extent to which this tradition, and in particular the work of Schröder, may provide us with insights toward the development of a generalised theory of founding relations, a theory which, as we have seen, would involve considerations which have largely been ignored by the 'pure' set theories which have been axiomatically developed in the wake of Russell's letter to Frege. What is of great importance is that Schröder not only recognises that there are manifolds for which his compatibility condition is not fulfilled, but he even hints at the possibility of an extended manifold theory which would encompass those

"inconsistent manifolds", whose elements are not all compatible with each other. (Cf. 1890, p.213 and p.247).

Nine years after the appearance of Schroder's work, Cantor was to write, in a

letter to Dedekind that

it is necessary to distinguish... two kinds of multiplicity [Vielheit]...

For a multiplicity can be such that the assumption that all of its elements "are together" leads to a contradiction, so that it is impossible to conceive of the multiplicity as a unity, as "one finished thing". Such multiplicities I call absolutely infinite or inconsistent multiplicities.

As we can readily see, the "totality of everything thinkable", for example, is such a multiplicity; later still other examples will turn up.

If on the other hand the totality of the elements of a multiplicity can be thought of without contradiction as "being together", so that they can be gathered together into "one thing", I call it a consistent multiplicity or a "set". (Cantor, Letter to Dedekind, 1899, as repr. in Cantor, 1932, p. 443; Eng.trans. from van Heijenoort, ed., 1967, p.114).

In the 1920's von Neumann was to develop an axiomatic set theory in which a distinction between 'sets' and 'proper classes', closely related to Schröder's and Cantor's distinction between consistent and inconsistent multiplicities is determined in a formally precise way. (See e.g. Fraenkel, Bar-Hillel, Levy, 1973, II, § 7). Note however that it is not as an anticipation of the 'pure' set theory which followed on from Russell's and Zermelo's work that Schröder's manifold theory ought most fairly to be evaluated.

One incidental consequence of the interpretation of Schröder's manifold theory as an embryo impure set theory, and of his inconsistent manifolds as multi-stratal configurations, is that this provides us with a framework within which we can determine the sense in which Frege's interpretation of the universality condition for a formal-ontological language was inappropriate. Frege assumed, in effect, that the universe of objects was a consistent totality, that all its members were compatible with each other and could be thought together "as one thing", where, as Schröder shows in his argument against Boole's assumption to the same effect, the universe can only be an inconsistent totality, a totality from which stratification is ineradicable. Unfortunately the logical base from which Schröder developed his own onto-

logy was greatly inferior (even) to that of Frege. In particular Schröder failed completely to appreciate the need to recognise different ontological categories e.g. of objects, functions, properties, relations, concepts, and so on, and thus his realism is small meat indeed when set against the much richer ontologies developed by Frege and Russell. [This criticism of Schröder could be applied also to Leśniewski's ontology which is, in this respect, very similar to Schröderian manifold theory.]

In the next section we shall have to determine more precisely why standard predicate logic as normally considered is inadequate to serve as the base logic for formal ontology. As we shall see, the crucial difficulties all turn on the notion of identity¹⁸⁴, and upon the need, in particular, to distinguish the relation of identity (e.g. of Cicero and Tully) from the relation of coincidence (e.g. between this table and the 10^{27} carbon molecules which constitute it). The discrete ontology which underlies standard predicate logic involves the unacceptable assumption that non-identicals are discrete (i.e. that coincidences are identical). The removal of this assumption can be effected only by 'hybridising' predicate logic in a way which allows not only relations within a given stratum but also relations between different strata to be represented in syntactically appropriate ways. Some idea of how this may be done can be gained if we turn our attention to the notion of identity, around which the difficulties can be seen to revolve.

§40. Stratification and identity theory.

Identity, according to the familiar conception, satisfies the following principle, AK, of absoluteness with respect to kinds:

that if a and b are both objects of a given kind K, and if a is the same K as b, then given any other kind K', if a is of kind K', it follows that b is also of kind K' and that a is the same K' as b.

It is a consequence of AK that given any statement of the form 'a is the same K as b' it is possible to ignore the reference to K and to state, simply, that $a = b$; thus the notion of identity to which we appeal is a notion of 'absolute identity', (a term which should not be interpreted as suggesting that any reasonable case can be made out for recognising any weaker notion of identity which would fail to satisfy the given principle)

The first and most obvious examples of identities which one might call to mind suggest that the principle AK could not but be unproblematic. For if Cicero is the same man as Tully, then he and Tully are the same roman orator; and if my dog, Alexius Meinong von Handschuchsheim, is referred to by my enemies as 'Moggy', then since he and Moggy are the same dog, they are thereby also the same animal and the same shoe-eater, and it seems impossible to conceive of a world in which this happy order of things could be disturbed.

Geach, however, finds reason to demand that the absolute notion of identity be rejected in favour of one according to which we

could not object in principle to different A's being one and the same B (1968, p.34),

which is taken as implying that the absolute notion of identity '=' must be given up in favour of a series of relative notions '=' for each kind K, where

'a = b'_K may be read: 'a is the same K as b.'

To understand the basis for such a view - and also to see where it leads into error - we must consider a series of more problematic examples, which we adapt from a critique of Geachian views compiled by Wiggins (see his 1967). The examples will have an additional interest, as we shall see, for the light which they throw on the concept of stratification introduced above.

- (1) A jug is broken into pieces; the fragments could not be the same jug, but they might be the same collection of material bits, or the same collection of molecules.
- (2) The jug-pieces are reassembled to form a coffee pot 'of a quite different shape and order of ugliness' from the original jug; then although the jug may be said to be the same collection of material bits as the pot, it is false that they are the same stencil.
- (3) Whatever is a river is water; the river on which my schooner is now moored is the same river as the river on which I moored it yesterday, but it is not (in spite of the fact that rivers are water) the same water.
- (4) John Doe the boy is the same human being, person (model railway enthusiast) as John Doe the man, but not the same collection of cells.
- (5) A church is, over a period of time, repaired piecemeal until the original fabric is entirely replaced: same church (?), different fabric.
- (6) The stone in Meriden which is inscribed to show that it marks the reputed centre of England is moved to London for exhibition at a festival. We shall certainly have to deal, during the period of exhibition, with one and the same stone, but not with the same landmark.
- (7) At Belgrade Railway Station I point to the Athens Orient Express and say 'That is the same train as the train on which King Alexios of Greece was assassinated in 1899'. Same train, but not the same collection of coaches and locomotives.
- (8) I arrive at the Ministry of Predicate Logic to see the same official as

I saw a year ago, but I will make no complaint if the official I see is not the same man.

(9) The Lord Mayor is not the same public functionary as the Director of the Blagston Street Football Team, but he is one and the same man.

(10) Frege, and many philosophers of mathematics since, have held that it is necessary to find some "objects" for number words to name and with which numbers could be identical, (Benacerraf, 1965, p.64).

A good case can be made out for supposing that the necessary objects would be a system of appropriately constituted sets, for it is known that the whole of the elementary theory of numbers can be recovered on the basis of an identification of numbers with sets. Unfortunately there is more than one (indeed an infinite number) of such systems of identification which can appeal to this 'preservation of results' for its support. For example in NBG set theory (see Fraenkel, Bar-Hillel, Levy, 1973, II, §7) the number 3 is identified with the set $\{\{\emptyset\}\}$, where in ZF set theory (op.cit., II, §§ 1-6) this number is identified as the set $\{\emptyset, \{\emptyset, \{\emptyset\}\}\}$. Thus whilst $\{\{\emptyset\}\}$ and $\{\emptyset, \{\emptyset, \{\emptyset\}\}\}$ are certainly not the same set, it may well be said that they are the same number.

If such examples could be taken at their face value, as counterexamples to the principle AK, they would each lead to a contradiction in conjunction with an appeal to the absolute notion of identity determined above. But do we really have to deal here with what are genuine counterexamples to this principle? Certainly the given list contains more than its fair share of ambiguity and logical sleight of hand, but still certain of its items, and here we would single out items (1) to (5) and the rather special case of (10), do succeed in capturing what is a substantial difficulty. What we wish to suggest, however, is that it is not, as Geach would claim, a difficulty which lies with the absolute notion of identity. The latter is a notion which has been, and which deserves to remain uncontroversial. In fact it is possible to hold fast to the principle AK, and to resolve the contradictions which threaten to arise by suitable choice of background logic for each of the

problematic cases isolated.

We have already suggested that the latter may be seen to rest on a running-together of the relation of identity - which holds only between members of one and the same stratum - with one or other relation of 'coincidence' between objects of different strata. [Wiggins himself draws attention to a distinction between the 'is' of identity and the 'is' of constitution (1967, pp.10-13) and much of his (Aristotelian) approach would be consistent with the stratification theory which is here advanced.] What we now wish to argue is that this running-together of identity and coincidence is something which is encouraged by the employment of standard predicate logic. This is not, indeed, encouragement in any positive sense, as would be implied, e.g. by a logic which rested on some 'axiom of discreteness', say:

$$x \neq y \rightarrow x \# y \quad (\text{with its contraposition : } \neg x \# y \rightarrow x = y),$$

to the effect that non-identicals are discrete from each other. The encouragement merely consists in the fact that relations of coincidence, backed up by appropriate relations of constitution, all of which would run in tandem with the intra-stratal relation of identity, have not themselves been distinguished in the logical systems currently employed. Their absence has not made itself felt (i) because of the relation between standard predicate logic and the reductionist approach of the discrete ontology, and

(ii) because of the restriction by practical logicians to one domain of discourse (one stratum) at a time, implying the abandonment of the 'universality' of logic which had characterised Frege's and Russell's work.

We can now recognise that what leads to contradictions in the examples above is not something which is at fault in the notion of absolute identity, but rather in the application of standard predicate logic (or one of its derivatives) simultaneously to more than one interrelated stratum or domain of discourse, but in such a way that the identity-relation is

extended to do duty for the different types of constitution-relations which hold between those domains. Thus it is asserted that the jug is the collection of fragments or molecules, where what ought to be asserted is that the jug is made up of (is constituted by) those fragments or molecules.¹⁸⁵ The immense variety of possible constitution relations (discussed on p.237f) is reflected in what seems at first to be a lack of any uniform distinguishing feature amongst the examples isolated as admissible. Only within the stratification-theoretic framework does it seem possible to provide a uniform account of all such examples. But the stratificational approach is useful also because it allows us to explain why the remaining examples ((6) to (9)), and the large number of variant cases which could easily be constructed, are not problematic at all. The latter are distinguished by the use of forms of speech which seem to suggest a stratificational difference which, on closer inspection, is seen not to be correlatable with any stratum difference on the side of the objects themselves. Landmarks, for example, given the present arrangement of our affairs, do not seem to be justifiably characterisable as objects which lie on a stratum above the stratum of mere stones, steeples, towers and so on. Such a characterisation can be justified however for the closely related case of works of art: one and the same lump of soldered metal may well constitute (found) two different works of art, e.g. if it should be discarded by its original sculptor-creator, and then resubmitted for exhibition at a later date by a practitioner in the art of objets trouvées who is ignorant of its origins.

What must not be ignored however is that there may be difficulties in deciding which is correct of the two alternative accounts which may be advanced for any given case, i.e.

(i) that we have to deal with one stratum, some of whose members are distinguished as possessing attributes which are signified by means of noun-like predicates ('is a landmark', 'is a roadsign', 'is an official'), or

(ii) that more than one distinct object-stratum can indeed be distinguished. But that we should have this difficulty is not to be surprised at: It is merely a reflection of the fact that objects belonging to

different higher-level strata are dependent for their status as objects (that is in general, for their 'demarcation') e.g. upon frameworks of conventions, which may take time to become fully established, or which may stabilise in a position of partial establishment. Thus we can easily imagine circumstances in which it would be necessary to develop a fully operative framework of conventions which would yield a new stratum consisting of self-subsistent objects called landmarks, distinguished from the stones or steeples which would continue to form their ontological support in much the same way that works of art may be distinguished from canvas and pigment. The establishment of the given stratum would then be signalled by the use of stable forms of expression referring to landmarks qua objects, and in particular by the use of predicates applying to landmarks but not to the underlying objects which support them, (for example: that landmark A 'marks land more efficiently than', 'in more varied weather conditions than', 'from a greater distance than', landmark B, or that landmark A 'is unfamiliar to the Germans', and so on). Some such forms of expression have indeed entered the language ('same landmark', and so on), but they do not constitute what might be called a 'complete vocabulary of predicates' (see §§ 58-9 below); they exist in the language side by side with linguistic conventions which resist the acceptance of other noun-like constructions relating to the same subject-matter, something which would not be possible if a corresponding stratum of objects had truly been established...

Example (10) is distinguished from the other examples acknowledged as resting upon true stratificational differences by the fact that in this example there are no autonomous constitution relations between either ZF sets, or NBG sets and the numbers which they are represented as supporting. Thus the two domains involved e.g. in the ZF reduction: the domain of ZF sets and the domain of numbers (qua abstract objects), are not related ontologically, i.e. in virtue of their own individual make-up; they are related only epistemologically, in the sense that the constitution relation involved is a purely intentional one, imposed by the foundation-theorist upon the given subject-matter for his own particular purposes. ¹⁸⁶

In order to distinguish between the numbers themselves, 0, 1, 2, etc., and their Doppelgänger within the ZF set-universe we shall refer to the latter as 0_{ZF} , 1_{ZF} , 2_{ZF} , etc. Now there are functions and relations, signified, by $+_{ZF}$, $<_{ZF}$, \leq_{ZF} , etc. which are definable within the theory of ZF in such a way as to reflect precisely, when applied to set-theoretic Doppelgänger, the effects of +, <, \leq , etc. as applied to numbers themselves. This is the sense in which it is possible to "do arithmetic" within ZF, ZF numbers being treated relative to these defined operations in just the same way that numbers themselves are treated relative to the operations of arithmetic. What this means in ontological (here in stratum theoretic) terms is that, by establishing the analogue of an arithmetical language within ZF we have demonstrated the possibility of conceiving the numbers themselves as belonging to a stratum supported by the stratum of ZF sets and as dependent upon the latter for their ontological status. It is as if the region of mathematical objects which was brought into being by the laying down of the ZF axioms has been shown to have the "path" of natural numbers running through it. But we can experience the objects which form this "path" only as possessing "unnatural" aspects, imposed by these principles of ZF which are not analogous to any corresponding arithmetical principles. For example 3 is given in the unnatural aspect that it is a member of 4, in virtue of the fact that

$$3_{ZF} \in 4_{ZF}, \text{ i.e. } \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} \in \{\{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}, \{\emptyset, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}\}$$

is a theorem of ZF. Thus it becomes clear that within ZF the natural numbers themselves have been removed, as it were, from direct vision, we receive only sufficient information concerning them as will enable us to obtain certain theorems which they satisfy. (Cf. Smith, 1975, §§4,5).

There is another field in which objects belonging to one stratum are represented, in an intentional way, as objects which are supported ontologically by a second stratum. This occurs in works of fiction, e.g. when Julius Caesar or Henry IV is represented as 'supported by' the actors of a play by Shakespeare, or when Stephen Dedalus (or rather, Joyce himself) is represented within a novel. Here again it is possible that there are conflicting

representation-systems operating in different works of fiction. Indeed if, by analogy with the notation introduced above, we refer to Stephen in Portrait of the Artist as 'Joyce_{PA}', and to Stephen in Ulysses as 'Joyce_U', then we can see very clear parallels between the relation between Joyce himself and the intentional objects Joyce_{PA} and Joyce_U on the one hand, and the relation between \mathfrak{J} 'itself' and the intentional objects \mathfrak{J}_{ZF} and \mathfrak{J}_{NBG} on the other.

Differences arise, most importantly, in virtue of the fact that \mathfrak{J} , unlike Joyce, is primarily a meaning-entity. This implies that the identification of numbers with particular sets has to serve a double purpose (i) as a reduction of ontological commitment on the part of the foundation-theorist, and (ii) as one amongst many other possible referentialisations of number, which is carried out in order to make arithmetic as a positive science (a science of objects) possible at all. But (i) and (ii) are not necessarily compatible, since what we are seeking is, so to speak, a 'minimal' referentialisation of numbers, one which introduces as few 'unnatural aspects' as possible: The doubts which have been expressed in the literature concerning set-theoretic reductions turn on the fact that the latter do not present themselves as promising candidates in this regard. Paradoxically, we can move towards the necessary minimal referentialisation only by appealing to objects of a much higher degree of ontological complexity than is characteristic of sets.

§41. The confusion of predicate and attribute.

Analytic philosophy favours a purely linguistic approach to the investigation of properties (and of attributes in general) via an investigation of the logico-grammatical behaviour of corresponding predicate-expressions. One highly unacceptable consequence of this linguistic approach is that credence has been lent to the assumption that wherever we have a grammatical predicate (or some expression of a logically equivalent form) there we can legitimately speak of a corresponding property or relation. The only problems which, it is supposed, might be raised by such an assumption concern merely certain 'singularities', (e.g., the 'heterologicality' predicate , which is assigned to all predicates which are not assigned to themselves). Certainly there are many instances of coincidence between grammatical predicates and ontological properties. We can safely assume, for example, that there is an autonomously existing property of being red, which applies to certain perceptible objects. Similarly we can assume that there is an autonomous property of being a horse, which applies to certain animals. Such areas of coincidence arise in virtue of the fact that the features of our language rest, in the end, upon the manner in which our experience of objects and of their properties, relations, and so on, has shaped the kind of statements which we are called upon to make. (Simons, 1975, esp.Ch.3). But such coincidence is never more than an imperfect approximation, in virtue of the fact that the complexity of the universe must outrun our knowledge of it: language, whether natural or artificial, is therefore never more than an inadequate reflection of ontology. To put the matter in a perhaps oversimple way: we are behoven always to attempt direct ontological investigations of the entities involved in any given area of experience, in order to ensure that the linguistic formations adequate to one area continue to be adequate when extrapolated beyond this area to new categories of entities which may be less "well-behaved".

This demand is especially pressing when we are concerned with the relations between predicate expressions and any corresponding entities or moments on the side of the objects themselves. For consider, as a preliminary example, the predicate expression: 'is a horse and $2 + 2 = 5$ '. This expression fails to determine or correspond to an autonomous property, first of all for the reason that it is true of no object, indeed the example has been selected such that there is no object of which it could be true. But it fails also for the reason that it suffers from a 'verbal' artificiality, the like of which we could have no reason to impute to any autonomously existing entity. The same kind of capriciousness excludes also what is, perhaps, a more familiar example, namely the predicate 'is at present green and it is not yet 2000 AD or is at present blue and 2000 AD has past', and it excludes the predicate 'is a horse and $2 + 2 = 4$ ' even though both of these predicates are clearly true of a large number of autonomous objects.

The suggestion that failure of a given predicate expression to determine any corresponding property may arise in virtue of an arbitrary complexity of structure of the predicate is a view which receives support from certain arguments put forward by Grossmann, a philosopher who also denies that there is any automatic relationship between predicates and properties. (See his 1972, p.161, text to note 12). Grossmann's arguments turn, initially, on a recognition of certain paradoxical consequences which ensue from the supposition that every predicate determines a property, i.e. consequences of the 'singularity' type already mentioned. The most important of these, due to Russell (see his 1902) has already been discussed in its set-theoretic form. In its predicational form it may be expressed as follows: Suppose the predicate expression 'does not apply to itself' determines a property, say R (for Russell). Now suppose that $\neg R(R)$, i.e. that R does not apply to itself. Then, by the definition of R, it follows that R does apply to itself, but then R does not apply to itself, which is a contradiction.

Russell's own preferred method of resolving this (and similar) paradoxes he expressed in the form of the so-called vicious-circle principle, which is such as to exclude from the totality of those predicates which determine properties only those expressions which possess a certain kind of circularity in their definition. (Cf. Gödel, 1944, p.219). Grossmann however draws the much more sweeping conclusion that it is the complexity rather than the circularity in the definition of 'R' which implies that that predicate must be excluded from the category of predicates for which there are corresponding properties. Hence Grossmann excludes not only 'R' and our 'is a horse and $2 + 2 = 5$ ' but also, say, 'is green and is round'.

In defence of this approach he argues as follows:

Assume that the individual thing A is both green and round; that it is a fact that A is green and round. Then A most certainly exists; and so do the two properties of being green and of being round. But there is no third property, no other entity belonging to the category of properties, that could be called the property of being green and round. (Op.cit.p.161).

Grossmann even excludes, for the case of any such arbitrarily pre-selected individual object A, predicates such as 'is identical with A' and 'is situated to the left of A', whose logical complexity derives from their individual reference. For:

If it is a fact that A is identical with A, then there exists the entity A and also - at least in my view - the relation of identity. But there is no such thing as the property of being identical with A. If it is a fact that A is to the left of B, then the relation of being to the left of as well as A and B exist, but there is no such thing as the property of being to the left of B. To believe that there exist such entities as the property of being both green and round, the property of being identical with A, and the property of being to the left of B is sheer ontological superstition... (Loc.cit.)

Grossmann does not fail to recognise that one can of course

abbreviate the expression 'A is green and A is round' by, say, 'A is ground'. But one must not conclude that 'ground' is here the name of a complex property. If it were the name of such a property, then 'A is ground' could not possibly be an abbreviation for 'A is green

and A is round'; for then, as we just saw, 'A is ground' would represent a quite different fact [state of affairs, Sachverhalt, in our Ingardenian terminology] from 'A is green and A is round', since the former fact contains the property ground, while the latter does not. (Op.cit., p. 161f).

Thus it is not suggested that logical simplicity relative to any currently existing language could ever serve as a sufficient criterion for a predicate's being associated with an (autonomous) property. In order to move closer toward achieving such a criterion we have to move out of the purely linguistic or logico-linguistic sphere to the sphere of the objects themselves. There we find not only a stratification amongst objects but also an even more complex stratification amongst properties, amongst relations, and indeed amongst all of the other categories of object-entities which may be distinguished. This implies that the task of providing sufficient criteria becomes both easier - since we no longer have to search for any absolutely simple, lowest-level properties from which all higher-level properties would be constructible without remainder. And it becomes more difficult, since quite different problems occur, relative to those purported properties which are admissible, in each of the different kinds of strata which we have to consider. The nature of some of these difficulties will be sketched in the section which follows.

§ 42 Attribute-stratification and the law of excluded middle.

We made clear in our schemata on pp. 237 and 242 above, that the two dimensions of 'types' and of 'strata' are interrelated in such a way that stratification is exhibited not only amongst objects but also amongst attributes (properties and relations) which they instantiate.¹⁸⁷ Just as we find different 'levels of stability' amongst objects, so we find such levels of stability amongst attributes, stability which is reflected in the success achieved in the operation of various higher-level sciences, each of which has the function of determining the properties which are exhibited, in effect, on one particular stratum. This success reveals, as Wang points out, 'an orderliness of the way the world is which we had no good reasons to expect before we acquired this knowledge.' (1974, p.2).

This stratified ordering of attributes has been recognised by some analytic philosophers, but this has usually been against the background of the linguistic approach to the investigation of properties discussed above. Thus ontological stratification has been recognised only indirectly, as something which concerns ranges of significance of given predicates. An apple, it is claimed, can be significantly said to be red, to be coloured, to be darker in colour when ripe, and so on. But such predicates cannot be significantly applied to an electron, a number, or a violin concerto. Any validity which may be attributed to claims of this sort, resting on the consideration of various significance-ranges, can only be granted in cases where such significance-ranges depend upon prior ontological facts, namely upon the stratification both of objects and of the properties which they may possess. Thus the validity of the apple/electron opposition just mentioned depends upon the fact that colour attributes are instantiated only in those strata which contain perceptible objects or such perceptible near-objects as rainbows and aurora.

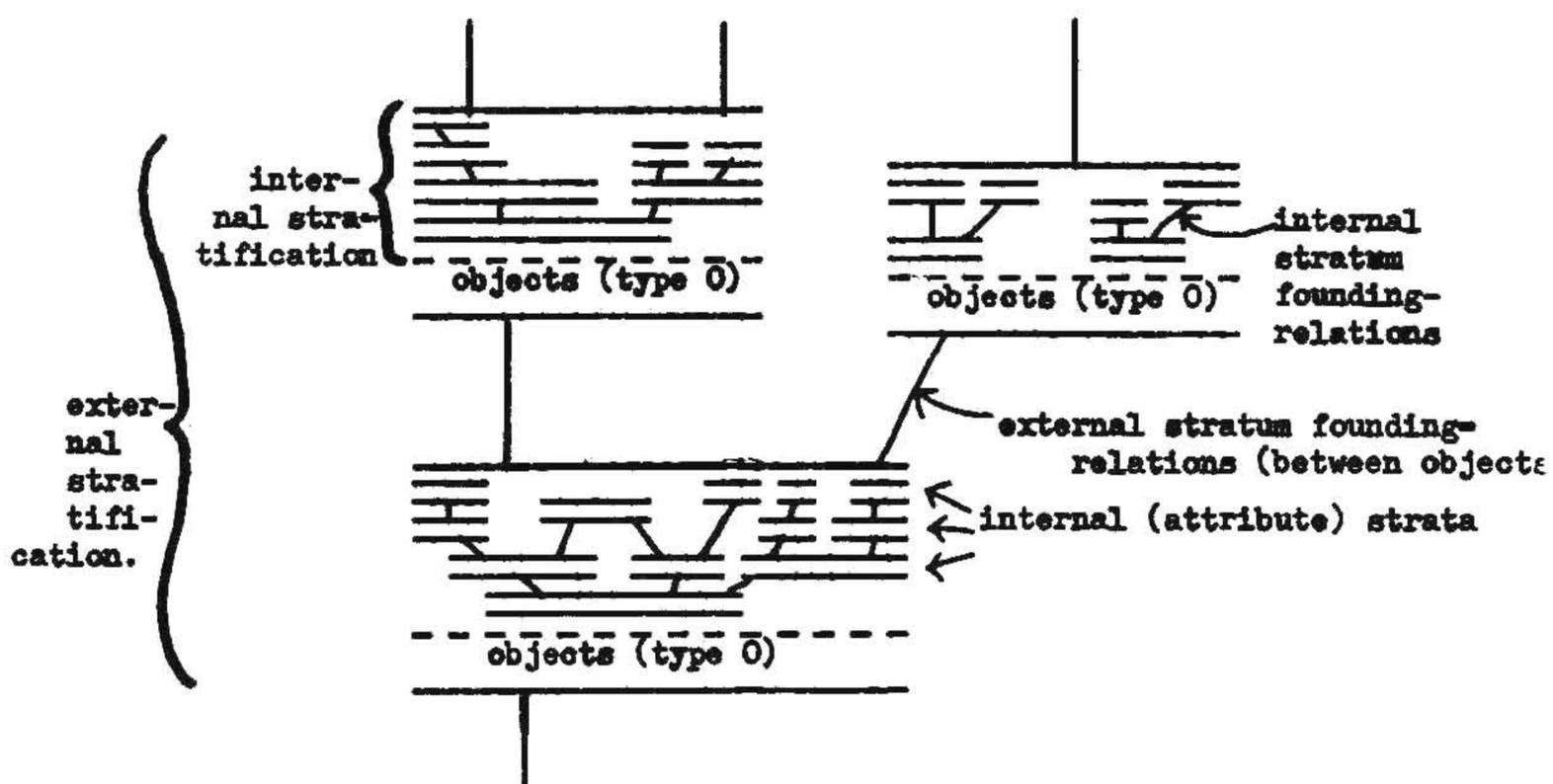
Hence if an opposition between 'simple properties' and 'complex (purported) properties' is to be made defensible, it must be recognised as an oppos-

ition which is relative to any particular object-stratum with which we have to deal, in virtue of the fact that properties which are simple relative to one stratum may well be complex relative to another, lower stratum. A similar point is made by Cocchiarella, who also exploits an opposition between simple and complex properties in the development of a formal-ontological philosophical language. Cocchiarella argues that whilst perhaps

every level of ontological analysis must presuppose simples amongst its individuals as well as among its properties and relations [,still] relative to a deeper level of analysis these same simples may turn out to be complex (cp. an analysis of macro-physical objects and their basic properties and relations in terms of micro-physical objects and their basic micro-states). We need not assume that this descent into ever deeper levels of ontological analysis must reach a final deepest level with absolutely simple objects and properties and relations. (1974, p.44, n.1).

Here we can do no more than sketch in a provisional way an account of the nature of properties (and of attributes generally) which is implied by the recognition of such stratification. The account is provisional only, as a reflection of the immense complexity of the subject-matter: consider, for example, the variety of physical, chemical, biological, even aesthetical properties which may be possessed by a simple real material object such as a carrot or a cart. This variety of properties and of interrelationships which can be discerned between them, suggests that even within one and the same object-stratum there is, besides the formal or structural division of attributes into types (attributes of objects having type 1, attributes of type k attributes having type k+1, etc.), also a fuller and more complex ontological division of attributes, into what might be called 'internal' strata. Such internal strata fall "between" the 'external' stratificational division dictated by the stratification of objects. And whilst these internal, attributional strata will be conceived by analogy with the broader stratum-divisions amongst attributes in general which are dictated by the stratification amongst objects, they will differ from the latter in that objects belonging to a single stratum - or even single individual objects - are seen as themselves possessing a

whole range of attributes at various different levels.¹⁸⁸ Thus a given apple may be appetising, it may be coloured, it may be red, it may reflect light rays with a wavelength in the region of 700μ , and it may attract ravenous predators, and each of the given attributes would then lie, we claim, upon different attributional sub-strata. (Cf. Ingarden's theory of primary and derivative properties, discussed by Swiderski, 1975, p.85). Diagrammatically, therefore, we must refine the structure of our schema of p.242 in such a way that each particular node exhibits its own internal stratification, somewhat as follows: (Note that now we ignore attributes of type > 1 within each external stratum, and note also that the schema represents only one small region of the schema of p.242).



Whether a given property P is possessed by an object a is something which is either true or false, independently of whether it is recognised which of these two alternatives is true, and even independently of whether this could be recognised given e.g. the linguistic materials which we have at our disposal. This realist principle of the law of excluded middle is, as a logical principle, unaffected by any delineation of stratificational inter-relationships amongst properties which has an ontological significance only. In contrast the given law is threatened by the 'stratification' theory of 'ranges of significance' discussed above. For the given theory seems to

imply that for a given predicate 'A' and object a, there is, besides the two alternatives; Aa and $\neg Aa$, also a third alternative, namely that A is not significantly applicable to a, i.e. that 'Aa' is meaningless (in the sense of the 'Widersinn' of 'colourless green sleeps furiously' or 'his discussion paper could not drive a car' (Cf. LU, Investigation IV, § 14)). If 'Aa' is meaningless then so, of course, is the compound ' $Aa \vee \neg Aa$ ', something which would seem to threaten the universal validity of the logical law of excluded middle.

Can such an argument really be accepted however? First of all it will be useful to consider the general considerations which have led many logicians to accept the possibility of 'truth value gaps', of 'three-valued logics', even of 'n-valued logics' for arbitrary (finite or infinite) n. Such considerations derive much of their force, I believe, from an inappropriate paradigm of what truth values are, which had been bequeathed to philosophy by Frege. Frege held, of course, that truth values were objects named by sentences. Yet such a conception is seen to be absurd on even the most trivial reflection upon the role which truth and falsehood play in our actual thought and argument. Such reflection shows that it is not spurious logical objects which serve as the referents of our sentences, but, to the extent that the notion of reference is applicable to sentences at all, appropriately constituted states of affairs. Once we recognise the role which is played by states of affairs, however, it becomes clear why there are and can only be two 'truth values', as the objectivisations, respectively, of the presence and of the absence of a referent for any given sentence. Thus if 'A' and 'a' have incompatible 'ranges of significance' it follows that 'Aa' lacks a referent, i.e. that ' $\neg Aa$ ' (which is just another way of saying 'Aa' lacks a referent) is true. These remarks suggest a close connection, developed in §§ 61f below, between truth for sentences containing singular terms of particular types, and existence of states of affairs corresponding to those sentences and of the individual objects denoted by their constituent singular terms. Thus, just as there are empty singular terms such as 'Vulcan'

and 'the present king of France', so there are 'empty sentences' (' $2+2=5$ ', 'Denmark is the capital of Copenhagen', 'the round square is less salubrious than the golden mountain'), which is just to say that there are sentences which are false.¹⁸⁹

We have reached the point where we can deal with the specific argument in favour of 'truth value gaps' which is put forward on the basis of considerations of 'ranges of significance' of various predicates. Clearly, by the argument above, 'insignificant' sentences such as 'the square root of 2 is learning how to swim', must be counted as false for the same reason that 'this page is red in colour' must be counted as false, namely that they both lack any corresponding state of affairs to which they would 'refer'. But we can begin to see also what might be the basis for the doubts expressed as to the validity of the law of excluded middle in the given area, namely that the two sets of examples seem to 'lack states of affairs' for importantly different reasons. This can be accounted for by defining a non-logical (but ontological) predicate negation, say ' \bar{P} ', by:

$\bar{P}x$ iff $\neg Px$ & x belongs to the stratum of application of P .

Then clearly the law

$$\forall x \forall X^1 (X^1 x \vee \bar{X}^1 x),$$

which is no longer a logical but rather an ontological law of excluded middle, fails to be universally valid, simply in virtue of the fact that not all properties are applicable to all strata.¹⁹⁰