Mereotopology: A Theory of Parts and Boundaries

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1. Introduction

The term ‘ontology’ has recently acquired a certain currency within the knowledge engineering community, especially in relation to the ARPA knowledge-sharing initiative (see Gruber (to appear), Mars (ed.) 1994, Guarino 1994, Guarino, Carrara and Giaretta 1994, 1994a). The term is used in a number of different senses, however, not all of them clear or mutually compatible. Here I follow philosophical tradition in conceiving ontology as the science which deals with the nature and the organisation of reality. Ontology thus conceived may be formal, in the sense that it is directed towards formal structures and relations in reality. This formal ontology is contrasted with the various material ontologies (of physics, chemistry, medicine, and so on) which study the nature and organisation of specific sub-regions of reality. Formal structures, for example the structures governing the relation of part to whole, are shared in common by all material domains. Both formal and material ontologies may be pursued with the aid of the machinery of axiomatic theories, and it is axiomatic formal ontology that has proved to be of most interest for the ontology-building purposes of the knowledge engineer.

The term ‘formal ontology’ was introduced by Edmund Husserl in his Logical Investigations (1970, 1st German edition 1900/01), and the mereology or formal theory of part and whole there developed by Husserl is still, of all the component disciplines of formal ontology, that which has received the most developed axiomatic treatment. Mereotopology, the subject of the present essay, is built up out of mereology together with a topological component, thereby allowing the formulation of ontological laws pertaining to the boundaries and interiors of wholes, to relations of contact and connectedness, to the concepts of surface, point, neighbourhood, and so on.

Our understanding of mereotopological principles rests on philosophical and logical studies both classical – here Aristotle, Brentano and Whitehead deserve special mention (see also Menger

1940 and Tarski 1956) – and modern (Adams 1973ff, Smith 1982ff., Chisholm 1984, Simons 1987, Stroll 1988, Bochman 1990, Eschenbach 1994, Varzi 1994, Fine 1995, Eschenbach and Heydrich (to appear)). Too much of the artificial intelligence literature in the areas of formal ontology and naive physics has however not drawn from these sources, but has rather been dominated by the use of set-theoretical instruments not conducive to the direct representation of mereological and topological structures. The work of Cohn and his associates (Randell, Cui and Cohn 1992, Gotts 1994f, Cohn and Gotts 1994; see also Aurnague and Vieu 1993) is an exception to this rule, and has done much to demonstrate the fruitfulness of the mereotopological alternative for knowledge-engineering purposes. This work is however based on the Whitehead-inspired system of Clarke (1981, 1985) which is problematic for at least the following reasons:

1. The system has a single primitive, that of connection, in terms of which the notion of part is defined by means of what, intuitively, appears to be a logical trick. This means that the mereological and topological components of the resultant theories are difficult or impossible to separate formally. The power of the approach is thus reduced, since experiments in axiom-adjustment at different points in the theory cannot be carried out in controlled fashion. Moreover, there are associated formal difficulties with the system (discussed by Varzi in his paper in this volume) which contradict the goal of formal ontology as a realistic, descriptive enterprise.

2. The system rests on a no longer fashionable conception of formal ontology (embraced by Lesniewski and his followers, by Carnap, Goodman, and others) according to which the goal of minimizing the number of non-logical primitives (ideally to the point where a system would have precisely one such primitive) is taken to override other goals, such as intuitive plausibility of definitions, easy testability of axioms, and so on. More recent experience in the construction of formal-ontological systems, for example for the purposes of naive physics (Hayes 1985), has suggested that systems capable of describing real-world phenomena will require large numbers of non-logical primitives, no group of which will be capable of being eliminated formally in favour of any other group.

The axiomatic version of mereotopology here presented is designed to serve as the starting point of a formal-ontological system which will be free of these defects. It rests on the two non-logical primitives of *part* (P) and *interior part* (IP) respectively. Connection (C) is then defined in terms of P and IP. Other versions of this basic theory have been proposed (Smith (forthcoming), Varzi 1993f., Casati and Varzi 1994 (Appendix)), and a survey of the whole field is presented in Eschenbach et al. (eds.) 1994, and in Zelaniec (ed.) 1995.

2. Constituency

Classical first order logic with identity and descriptions will be assumed without ceremony. In a complete account we should have to employ the resources of a free logic, perhaps along the lines of Simons 1991, to take account of the fact that the term-forming operator ‘σ‘ introduced below is not defined for every predicate. Variables x, y, z, etc. will range over entities (particulars, individuals) in general. Here the term ‘entity’ is to be understood as ranging over *realia* of all sorts. Our quantifiers are otherwise unrestricted, embracing *inter alia* my left foot and the interstellar vacuum, my present headache and the three-dimensionally extended colour of this green glass cube. They embrace what is continuous or discontinuous, bounded or unbounded, connected or non-connected; and they embrace also volumes of space and intervals of time, as well as three-dimensional material things and their parts and moments.

We adopt as mereological primitive the relation of parthood or constituency. We say x is a **part** of y, and write 'x P y', when x is any sort of part of y, including an improper part (so x P y will be consistent with x's being identical to y). Three further purely mereological notions can be defined immediately:

DP1 x **overlaps** y: $xOy := \exists z(zPx \wedge zPy)$

DP2 x is **discrete** from y: $xDy := \neg xOy$

DP3 x is a **point**: $Pt(x) := \forall y(yPx \rightarrow y = x)$

As axioms governing P we shall assume the universal closures of:

AP1 $xPy \equiv \forall z(zOx \rightarrow zOy)$

AP2 $xPy \wedge yPx. \rightarrow x = y$

(Generally speaking we suppress all initial universal quantifiers in our statements of axioms and theorems.) From **AP1** and **AP2** and the usual axioms of identity it follows that our system of mereology is extensional (Simons 1987, ch. 1), and in particular that $x{=}y{\equiv}\forall z(zPx{\equiv}zPy)$. From **AP1** it follows also that:

TP1 xPx P is reflexive

TP2 $xPy{\wedge}yPz. \rightarrow xPz$ P is transitive

We say that a condition 'φ' in a single free variable 'x' is **satisfied** iff the sentence 'φx' is true for at least one value of 'x'. Intuitively we are to suppose that each satisfied condition 'φ' picks out a certain unique entity, the **sum** (fusion or join) of all those entities in the world which φ, an entity which we shall represent by '$\sigma x(\varphi x)$'. Note that the sum of φers is to be distinguished from the extension of the concept φ: not everything that is in the sum of φers need itself be such as to φ(thus my leg is in the sum of Britons, but it is not itself a Briton).

The sum of φers can be defined as that entity y which is such that, given any entity w, w overlaps with y if and only if w overlaps with something that φs. That is:

DP4 $\sigma x(\varphi x) := \iota y(\forall w(wOy \equiv \exists v(\varphi v \wedge wOv)))$

We can then prove

TP3 $y = \sigma x(\varphi x) \rightarrow \forall x(\varphi x \rightarrow xPy)$

Empty sums do not exist (they are not a part of reality). Thus if φ is a non-satisfied condition, then '$\sigma x(\varphi x)$' is undefined. The uniqueness of sums, where they are defined, is guaranteed by **AP1**. We stipulate further that:

AP3 $\exists x\varphi x \rightarrow \exists y\forall w(w\text{O}y \equiv \exists v(\varphi v \wedge w\text{O}v))$

which guarantees the existence of sums for satisfied conditions.

From the usual axioms for identity we have $\exists x(x = x)$, from which we can prove a theorem to the effect that the universe exists:

TP4 $\exists x\forall y(y\text{P}x)$

Further:

TP5 $y\text{P}\sigma x(\varphi x) \equiv \forall w(w\text{P}y \rightarrow \exists v(\varphi v \wedge w\text{O}v)$

y is a part of the sum of φers if and only if all parts of y overlap with some φer.

We have already noted that not all parts of the whole $\sigma x(\varphi x)$ need be such as to φ. When $y\text{P}\sigma x(\varphi x)$ iff φy, then we say that φ is a **distributive** condition, and we can prove that $\varphi(\sigma x(\varphi x))$. Examples of distributive conditions are (for some fixed entity t): *is a part of* t, *is a boundary of* t, and *is an interior part of* t.

We can prove further a theorem to the effect that we can form arbitrary finite unions in the following sense:

TP6 $\exists z\forall w(z\text{O}w \equiv z\text{O}x \vee z\text{O}y)$

We define:

$\mathbf{1}\text{:} = \sigma x(x = x)$	universe
$x \cup y\text{:} = \sigma z(z\text{P}x \vee z\text{P}y)$	union
$x \cap y\text{:} = \sigma z(z\text{P}x \wedge z\text{P}y)$	intersection
$x'\text{:} = \sigma z(z\text{D}x)$	complement
$x\text{-}y\text{:} = \sigma z(z\text{P}x \wedge x\text{D}y)$	difference

Note that all set-theoretical associations of these terms are to be resolutely suppressed. Note, also, that intersections, complements and differences are not always defined. We can however prove the following remainder principle:

TP7 $x\text{P}y \wedge x \neq y \rightarrow \exists z(z = y\text{-}x)$

3. Interior Parts

As topological primitive we select the relation *is an interior part of,* which can be elucidated, roughly, as follows. Some entities are what we might call tangential to, i.e. such as to touch or cross the exterior boundaries of, other entities. Some entities are themselves boundaries of other entities, though we note that the boundary of an entity may be outside the entity it bounds (as for example in the case of an open interval in the real line). When x is a part of y that is *off* – which is to say: shares no parts in common with – the boundary of y, i.e. is neither tangential nor itself a boundary, we say that x is an **interior part** of y and write 'x IP y'. We then stipulate:

AIP1	$x\mathrm{IP}y \rightarrow x\mathrm{P}y$	IP is a special sort of P
AIP2a	$x\mathrm{IP}y \wedge y\mathrm{P}z \rightarrow x\mathrm{IP}z$	left monotonicity
AIP2b	$x\mathrm{P}y \wedge y\mathrm{IP}z \rightarrow x\mathrm{IP}z$	right monotonicity
AIP3	$x\mathrm{IP}y \wedge x\mathrm{IP}z \rightarrow x\mathrm{IP}(y \cap z)$	condition on finite intersections
AIP4	$\exists x(\varphi x) \wedge \forall x(\varphi x \rightarrow x\mathrm{IP}y) \rightarrow \sigma x(\varphi x)\mathrm{IP}y$	condition on arbitrary unions
AIP5	$\exists y(x\mathrm{IP}y)$	
AIP6	$x\mathrm{IP}y \rightarrow x\mathrm{IP}\sigma\mathrm{t}(\mathrm{tIP}y)$	

All of which follow from the usual axioms for a topological space. **AIP5** is very strong, and allows us to infer an initially counterintuitive-seeming theorem to the effect that the universe is an interior part of itself:

TIP1 $\mathbf{1}\mathrm{IP}\mathbf{1}$

The universe is, as we might also say, 'unbounded'. Indeed we can prove that:

TIP2 $\forall x(x\mathrm{IP}\mathbf{1})$

Every entity is an interior part of the universe. From **AIP4** it follows that IP determines a distributive condition, i.e. that:

TIP3 $t\mathrm{P}\sigma x(x\mathrm{IP}y) \equiv t\mathrm{IP}y$

Hence also we have:

$$t\mathrm{P}\sigma x(x\mathrm{IP}y) \equiv t\mathrm{IP}\sigma x(x\mathrm{IP}y)$$

and:

TIP4 $\sigma x(x\mathrm{IP}y)\mathrm{IP}y$

4. Boundaries

As a first step towards defining what it is for x to be a boundary of y, we define 'x X y' (x **crosses** y) by:

DP5 $x\mathrm{X}y := \neg x\mathrm{P}y \wedge \neg x\mathrm{D}y$

or, equivalently, for y $\neq$ **1**,

$$xXy: = xOy \wedge xO(\mathbf{1}\text{-}y)$$

i.e., *x* overlaps both *y* and its complement. From this it follows trivially that no entity crosses itself and that the universe crosses every entity not identical with the universe itself. We now define '*x* St *y*' (*x* **straddles** y) by:

DIP1 $xSty: = \forall z(xIPz \rightarrow zXy)$

An entity *x* straddles an entity *y* whenever *x* is such that everything of which it is an interior part crosses *y*. The definitions then yield immediately that $xSty \rightarrow \neg\, xIPy$, from which we can prove:

TIP5 $xPy \rightarrow xIPy \vee xSty$

Every part of *y* is either an interior part of *y* or such as to straddle *y*. This follows from **AIP1**, **AIP2a** and definitions. As a theorem we also have: $\neg xIPx \rightarrow xStx$.

The entities straddling a given entity can be divided, intuitively, into two classes. On the one hand are those which include among their parts a boundary of the straddled entity. On the other hand there are those – characteristically non-connected – which include no such boundary. We shall refer to the first group as **tangents**. As an example of a non-tangential entity straddling *y*, consider the sum of two points, both off the boundary of some three-dimensional solid *y*, one of which is interior to *y*, the other exterior. If we examine case V, where *x* is not merely such as to straddle *y* but is in fact a boundary of *y*, then we see that what is characteristic of this case is that here *x* is such that not merely it but also all its parts are such as to straddle the bounded entity. Accordingly we can define **boundary** as follows:

DIP2 $xBy: = \forall z(zPx \rightarrow zSty)$

We can now define what it is for *x* to be a **tangent** of *y*:

DIP3 $xTy: = \exists z(zPx \wedge zBy)$

i.e a tangent of *y* is any entity which has as part a boundary of *y*. From this definition we can prove that tangents are straddlers, and also that every boundary of *y* is a tangent of *y* and is thereby also not an interior part of *y*. We can prove further, by inspection of the definitions, that:

TB1 $xBy \equiv \forall z(zPx \rightarrow zTy)$

so that, as required, all parts of boundaries of *y* are not merely straddlers but in fact tangents of *y*.

Closure

We can prove further:

TB2 $xBy \wedge yBz \rightarrow xBz$ transitivity

We also have:

TB3 $xPy \wedge yBz \rightarrow xBz$

TB4 $xT(y \cup z) \rightarrow xTy \vee xTz$ splitting

We can prove also the following collection principle for boundaries:

TB5 $\forall x(\varphi x \rightarrow xBy) \rightarrow \sigma x(\varphi x)By$

5. Topology

These theorems enable us to show that the system so far established defines a topological space in the standard sense, by defining the closure cl(x) of $x \neq \mathbf{1}$ as the union of x with all its boundaries:

DIP4 $\mathrm{cl}(x) := x \cup \sigma y(yBx)$

Closure thus defined satisfies the usual Kuratowski axioms:

I. $xP\mathrm{cl}(x)$
II. $\mathrm{cl}(\mathrm{cl}(x)) = \mathrm{cl}(x)$
III. $\mathrm{cl}(x \cup y) = \mathrm{cl}(x) \cup \mathrm{cl}(y)$

An entity is called **closed** iff it is identical with its closure. cl(x) as defined above can be shown to be identical to the standard topological closure defined equivalently as the union of x with its accumulation points (see below) or as the intersection of all closed entities containing x. A **dense** entity, standardly, is an entity x for which $\mathrm{cl}(x) = \mathbf{1}$.

The **maximal boundary** of x, defined by:

DIP5 $\mathrm{bdy}(x) := \sigma y(yBx)$

now corresponds to the standard topological boundary, defined as the intersection of the closure of an entity with the closure of its complement. Further our **interior**, defined by:

DIP6 $\mathrm{int}(x) := \sigma y(yIPx)$

corresponds to the standard topological interior, defined as the difference between an entity and its boundary.

An entity is called **open** iff it is identical with its interior. From this we can prove that an entity is open if and only if its complement is closed.

6. Dependent Existence and Brentano's Thesis

The remarks above are non-controversial reformulations of standard topological ideas on a mereological basis. Now, however, we wish to go further and capture mathematically certain ontological intuitions pertaining to ordinary material objects extended in three-dimensional space and enjoying qualities of for example shape and colour. We wish to capture, if one will, the mathematical structures characteristic of the mesoscopic world of everyday perception and action. Three layers of such intuitions can be distinguished:

1. the layer corresponding to general topological notions of boundary, interior, etc., which has been treated above;
2. the layer corresponding to the general properties of three-dimensional space as we conceive it; this space is 'real' in the sense that it is not an abstract construction; thus it allows no space-filling curves, no objects of fractional dimension, etc.
3. the layer corresponding to the special topological properties of material objects and their associated qualities.

What follows is a provisional attempt to formulate some of the principles underlying 3. It is provisional not least because the definitive statement of such principles must await a more adequate understanding of the general properties of space.

Intuitively, we wish it to be the case that every entity smaller than the universe has a boundary:

AB1 $y \neq \mathbf{1} \rightarrow \exists x(x\mathrm{B}y)$

This does not imply that the only *open* entity is **1**. Rather, it tells us that every open entity smaller than the universe is bounded, as it were, on at least one side or in at least one place (consider the case of the Western hemisphere of the universe or of the interstellar vacuum). The boundary itself need then not necessarily be a part of the entity bounded, and indeed that this should be the case in general is ruled out by:

TIP6 $x\mathrm{B}y \wedge y\mathrm{IP}z \rightarrow x\mathrm{B}(z\text{-}y)$

From this and **TIP2** it follows in particular that every boundary of y is also a boundary of the complement of y. From **TIP6** it follows trivially that:

$$x\mathrm{B}x \wedge x\mathrm{IP}y \rightarrow x\mathrm{B}(y\text{-}x)$$

Imagine x is a point in the interior of a three-dimensional solid y. Then y-x is here the result of deleting this point in such a way as to produce an entity which has a non-constituent boundary *within its interior*. The opposition between exterior and interior boundaries will receive more detailed attention in what follows.

From **TIP6** and **TIP1** it follows no less trivially that

TIP7 $x\mathrm{B}x \rightarrow x\mathrm{B}(\mathbf{1}\text{–}x)$

whence also we can infer that, for any x, $\sigma y(x\mathrm{B}y) = \mathbf{1}$, whence also we can infer that B does not define a distributive condition in the first argument.

We can prove further that an entity x is self-bounding (i.e. that x B x) if and only if it has no interior parts:

TIP8 $\mathrm{B}x \equiv \neg\exists t(t\mathrm{IP}x)$

We can now prove that every boundary which is a part of that which it bounds is also self-bounding:

TIP9 $\mathrm{B}y \wedge x\mathrm{P}y \rightarrow x\mathrm{B}x$

This does not imply, however, that we are defending a position which stands in conflict with the commonsensical intuition to the effect that that which bounds e.g. a surface is the *outer form* or *edge* of the surface. That the surface is self-bounding is consistent with its having as boundary in addition some proper part of itself, including its outer form.

From **AB1** and **TIP8** we could then prove that boundaries have no interior parts. From **TB5** we can prove:

TIP10 $x\mathrm{B}z \wedge y\mathrm{B}z \rightarrow (x \cup y)\mathrm{B}z$

And we have also:

TIP11 $x\mathrm{P}y \rightarrow x\mathrm{B}y \vee x\mathrm{IP}y \vee \exists uv(u\mathrm{B}y \wedge u\mathrm{IP}y \wedge u \cup v = x)$

Every part is either a boundary or an interior part or the union of a boundary and an interior part (where the disjunctions are of course exclusive).

7. Variants of Brentano's Thesis

We wish now to capture the commonsensical intuition to the effect that boundaries exist only *as* boundaries, i.e. that boundaries are dependent particulars: entities which are such that, as a matter of necessity, they do not exist independently of the entities they bound (Brentano 1988, Part One; Chisholm 1984; Smith 1992). This thesis – which stands opposed to the set-theoretic conception of boundaries as, effectively, sets of points, each one of which can exist though all around it be annihilated – has a number of possible interpretations. One general statement of the thesis would assert that the existence of any boundary is such as to imply the existence of some entity of higher dimension which it bounds. Here, though, we may content ourselves with a simpler thesis, one whose formulation does not rest on the tricky notion of dimension, to the effect that every boundary is such that we can find an entity which it bounds of which it is a part and which is such as to have interior parts. (Note that analogous theses could be formulated with respect to other neglected or metaphysically contested categories of entities: for example holes, shadows, colours, universal forms, thoughts, minds, etc.) Define first of all the predicate *is a boundary* by means of:

DIP7 $\mathrm{Bd}(x) := \exists y(x\mathrm{B}y)$

We can then write:

AB2 $\mathrm{Bd}(x) \to \exists z\mathrm{t}(x\mathrm{B}z \wedge x\mathrm{P}z \wedge \mathrm{tIP}z)$ First Brentanian Thesis

From this the theorem to the effect that all boundaries are self-bounding can be inferred immediately via **TIP9**. **AB2** is not very strong, however. For it seems that we have:

$$x\mathrm{B}y \to x\mathrm{B}(y \cup \mathrm{t})$$

for any arbitrary t that is separate from the closure of y. Thus **AB2** is satisfied by choosing t such that t IP t and setting z equal to the scattered object $x \cup \mathrm{t}$.

A Brentanian thesis of the required strength must impose on z in **AB2** at least the additional requirement of connectedness. To this end we define, for $x \neq \mathbf{1}$ and $y \neq \mathbf{1}$:

DCn1 $x\mathrm{S}y := \mathrm{cl}(x)\mathrm{D}y \wedge x\mathrm{D}\mathrm{cl}(y)$

We then say that $\mathbf{1}- (x \cup y)$ separates x from y. Thus $\mathrm{bdy}(x)$ separates $\mathrm{int}(x)$ from $\mathrm{int}(\mathbf{1}-x)$ in the given sense. We can then prove:

TS1 $x\mathrm{S}y \wedge w\mathrm{P}x \wedge u\mathrm{P}y \to w\mathrm{S}u$

Further we know that disjoint entities are separate if either both are open or both are closed.

Define **connected**:

DCn2 $\mathrm{Cn}(x) := x \neq \mathbf{1} \wedge \neg\exists yz(y\mathrm{S}z \wedge x = y \cup z)$

We then have a new Brentanian thesis affirming, for connected boundaries, the existence of connected wholes which they are the boundaries of:

AB3 $\mathrm{Bd}(x) \wedge \mathrm{Cn}(x) \to \exists z\mathrm{t}(x\mathrm{P}z \wedge x\mathrm{B}z \wedge \mathrm{Cn}(z) \wedge \mathrm{tIP}z)$ Second Brentanian Thesis.

Note that **DIP2** yields no guarantee that boundaries are connected in the sense here defined.

8. Exterior and Interior Boundaries

Intuitively, boundaries can be divided into exterior and interior (See Brentano 1988, Part One, I; Smith 1992). The exterior boundaries of x are, as it were, boundaries which separate x from the remainder of the universe. Exterior boundaries in this sense may or may not be parts of the things (or other entities) they bound, and they may or may not be on the exterior of the relevant entity in the normal understanding of this phrase. Thus they may be the boundaries of holes, including internal cavities; see, on the wealth of possibilities in this respect, Casati and Varzi 1994. We can distinguish also however **interior boundaries** – the boundaries which would result, intuitively, if interior parts of *x* were exposed to the light of day by annihilation of what stands between them and *x*'s exterior. Interior boundaries are in this sense potential exterior boundaries; they are those parts of *x* which are boundaries of interior parts of *x* but not themselves, or not yet, boundaries of *x*. We define:

DIB1 $x\mathrm{IB}y := x\mathrm{IP}y \wedge x\mathrm{B}x$

We might consider also in this connection the idea of a slicing principle to the effect that, in those cases where x B y results from the fact that x is a deleted region inside some $z = y\text{-}x$, we can slice z along x to produce one or more entities of which x is both exterior boundary and part.

Points

We can prove:

TPt1 $\forall y(y\mathrm{B}x \equiv x = y) \rightarrow \mathrm{Pt}(x)$

A point is that which has no parts other than itself (**DP3**). We can now stipulate that a point has no boundaries other than itself (a condition which might also have been used as a definition of 'point'):

APt1 $\mathrm{Pt}(x) \rightarrow \forall y(y\mathrm{B}x \equiv x = y)$

This is equivalent to the proposition:

$$\mathrm{Pt}(x) \rightarrow x = \mathrm{cl}(x)$$

which is one (mereological) formulation of the usual condition on a T1 topological space. A more standard formulation would be:

$$\neg\forall x\forall y(x \neq y \wedge \mathrm{Pt}(x) \wedge \mathrm{Pt}(y) \rightarrow \exists z((x\mathrm{IP}z \wedge \neg y\mathrm{IP}z) \vee (y\mathrm{IP}z \wedge \neg x\mathrm{IP}z)))$$

From **APt1** it follows further that:

TPt2 $\mathrm{Pt}(x) \wedge x\mathrm{B}y \wedge x \neq y \rightarrow \neg\mathrm{Pt}(y)$

and, by setting $y = \mathbf{1}\text{-}x$:

TPt3 $\mathrm{Pt}(x) \rightarrow \exists y(x \neq y \wedge x\mathrm{B}y)$

This goes some way towards capturing the anti-set-theoretical intuition to the effect that there are, in reality, no isolated points.

A **neighbourhood** of a point *x* is any entity *y* of which *x* is an interior part. A punctured neighbourhood of *x* is a neighbourhood with *x* deleted. An **accumulation point** may now be defined as follows:

DA1 $x\mathrm{A}y := \mathrm{Pt}(x) \wedge \forall z(x\mathrm{IP}z \wedge x \neq z \rightarrow (z\text{-}x)\mathrm{O}y)$

i.e., an accumulation point of *y* is any point *x* which is such that any punctured neighbourhood of *x* overlaps *y*. We now prove:

y is closed $\rightarrow \sigma x(x\mathrm{A}y)\mathrm{P}y$

From the definitions we can prove

TPt4 $x\mathrm{A}y \rightarrow x\mathrm{B}y \vee x\mathrm{IP}y$

By **TP11** we can prove generally that: $\mathrm{Pt}(x) \wedge x\mathrm{P}y \rightarrow x\mathrm{B}y \vee x\mathrm{IP}y$

TPt5 $x\mathrm{B}y \wedge x\mathrm{D}y \wedge \mathrm{Pt}(x). \rightarrow x\mathrm{A}y$

We may now go on to define **interior points** and **boundary points** as follows:

DPt1 $x\mathrm{IPt}y := \mathrm{Pt}(x) \wedge x\mathrm{IP}y$

DPt2 $x\mathrm{BPt}y := \mathrm{Pt}(x) \wedge x\mathrm{B}y$

Using axiom **AIP3** we can prove further that interior points are accumulation points.

TPt6 $x\mathrm{IPt}y \rightarrow x\mathrm{A}y$

Exploiting an analogy with Brentano's notion of the 'full plerosis of an internal boundary' (Brentano 1988, Part One, I) we may define further:

DA2 $x\mathrm{FA}y := \mathrm{Pt}(x) \wedge \exists z(x\mathrm{B}z \wedge x \neq z \rightarrow \exists \mathrm{t}(\mathrm{tIP}y \wedge \mathrm{tP}z \wedge x\mathrm{At}))$

x is a **full accumulation point** for *y* iff it is an accumulation point to *y* in all the directions in which *x* can serve as boundary (*x* is, as it were, the centre of a spherical ball within *y*).

TPt7 $x\mathrm{FA}y \rightarrow x\mathrm{A}y$

9. Things

Return, once again, to the Second Brentanian Thesis:

AB3 $\mathrm{Bd}(x) \wedge \mathrm{Cn}(x) \rightarrow \exists z\mathrm{t}(x\mathrm{P}z \wedge x\mathrm{B}z \wedge \mathrm{Cn}(z) \wedge \mathrm{tIP}z)$

This is still too weak if we wish to capture the intuition to the effect that exterior boundaries in the real material world are boundaries of *things*. For we require at least a further requirement to the effect that the entity z in question is the object bounded and not its complement. By **TIP6** each boundary behaves symmetrically in relation to the object and its complement. From the perspective of common sense, however, the boundary (of, say, this stone) is much more intrinsically connected to the stone than it is to the rest of the universe. To capture this notion formally would require (what we do not yet have) an adequate formal account of *things*, which we can characterize briefly as three-dimensional material entities which are at the same time maximally connected. Thus my arm is three-dimensional and material but it is not a thing, and similarly the scattered whole consisting of my arm and this pen is three-dimensional and material but it, too, is not a thing (Smith 1992). To this end we shall define the notion of a ‘component’ or *maximally connected entity*. For values of x such that $\mathrm{Cn}(x)$ we set:

DCn3 $\mathrm{cm}(x) := \sigma y(x\mathrm{P}y \wedge \mathrm{Cn}(y))$

The component of x is the maximal connected entity containing x. We can then prove:

TCn1 $z = \mathrm{cm}(x) \rightarrow \forall y(\mathrm{Cn}(y) \wedge z\mathrm{P}y \rightarrow y = z)$

Components are, if one will, those natural units from out of which the world is built. Such natural units can be found not only in the realm of three-dimensional material things, but also e.g. in the temporal dimension (salutes, weddings, lives, are natural units in the realm of events and processes). To deal with these matters, here, however, as also with the concepts of dimension (edge, surface) and with the relations between units and their underlying stuffs, all of this would lead us too far afield.

10. From Cognitive Linguistics to Ontology

Concepts and theories derived from mereology and topology have been utilized already in a variety of ways not only in the field of ontological engineering but also in a range of other cognitive science disciplines. Examples of such work include:

– analyses of natural language, especially of object-categorization, verb-aspect and the mass-count distinction: Mourelatos 1981, Habel 1990, Descläs 1989, Ojeda 1993, Habel, Pribbenow and Simmons 1993, Aurnague and Vieu 1993; Pianesi and Varzi 1994 and (to appear);

– work in the theory of geographic information systems: Mark and Frank 1991, Herring 1991, Frank and Campari (eds.), 1993, Frank, Campari and Formentini (eds.), 1992;

– investigations of spatial perception, and of the spatial properties of mental representations; studies in image-processing, for example in automatic analysis of X-ray images for medical and other purposes: Randell, Cui and Cohn 1992, Freksa 1992, Varzi 1993, Smith 1993, Casati and Varzi 1994;

– contributions to qualitative or 'naive' physics and to the field of common-sense reasoning: Forbus 1984, Hayes 1985, Hager 1985, Thom 1990, Petitot and Smith 1990, Randell, Cui and Cohn 1992, 1992a, Smith 1992, Smith and Casati 1994;

It is, however, in work in the area of cognitive linguistics on the part of Lakoff, Talmy, Langacker, Jackendoff and others that topological notions have been most explicitly and systematically applied. (See especially Talmy 1977ff., Brugman and Lakoff 1988, Lakoff 1989, Jackendoff 1991, and the related work of Petitot 1982ff., and Wildgen 1982f.) The importance of topology to the conceptual structuring effected by language is illustrated most easily in the case of prepositions. As Talmy notes, a preposition such as 'in' is magnitude neutral (in a thimble, in a volcano), shape neutral (in a well, in a trench), closure-neutral (in a bowl, in a ball); it is not however discontinuity neutral (in a bell-jar, in a bird cage). The task of formally defining the precise nature of the transformation which maps one *in*-structure onto another under these conditions and in such a way as to do justice to the features illuminated by Talmys many examples, is a difficult one; in the absence of appropriate formal theories of the mereotopological sort, however, the work of the cognitive linguists will remain subject to the charge that it has not gone beyond the stage of narrative evidence-gathering. As Wildgen points out in criticizing Talmy: The quasi-formal symbols in Talmys descriptions come from algebra, geometry, topology and vector-calculus, but the mathematical properties of these concepts are neither exploited nor respected. (1994, p. 32) Similar criticisms could be marshaled also against Lakoff and his associates, and it is perhaps above all in helping to meet such criticisms that the mereotopological framework can be of most immediate benefit to progress in cognitive science.

The formal resources of mereotopology can be used, however, to generalize the project of cognitive linguistics beyond the linguistic sphere. We can conceive this project as one of providing an account of the ways in which we impose conceptualizations or categorizations or structurings on reality through our uses of natural language. The more general project, which bears comparison with recent work by Talmy, would consist in providing an account of conceptualizations and categorizations in general, i.e. not merely those imposed via uses of language but also those generated by other cognitive modes of access to reality, including perception, scientific theories, the map-making activities of the geographer, knowledge-sharing systems, and so on – a move, in other words, from cognitive linguistics to general ontology. Such partition-systems may be applied on different levels (thus atoms, molecules, cells, organisms, populations can all be conceived as products of partition in the highly general sense here at issue). In addition, partition-systems may involve boundaries of different sorts: for example, boundaries of different dimensions, boundaries which are more or less determinate, boundaries which are more or less enduring, connected and non-connected boundaries, interior and exterior boundaries, and so on. Perhaps the most important typological division amongst boundaries from our present point of view, however, is that between 1. natural or autonomous boundaries – those which reflect genuine divisions or heterogeneities in reality, and which thus exist independently of all conceptualizing or categorizing activity on our part: for example the boundaries around my body, heart, lungs, cells, and so on; and 2. boundaries which correspond to no genuine local heterogeneity or natural divisions on the side of the bounded entities themselves: for example the boundaries between the Northern and Southern hemispheres or between one calendar month and the next. Let us call boundaries of the first sort genuine or *bona fide* boundaries, boundaries of the second sort *fiat* boundaries, a terminology that is designed to draw attention to the sense in which the latter owe their existence to acts of human decision or fiat or to cognitive processes of similar sorts. (Smith 1994) Complete boundaries within a partition yield *objects*: countries on the globe, months on the calendar. Such objects, too, may be either *bona fide* or fiat objects in our suggested terminology. Examples of genuine objects are: you and me, the planet earth. Examples of fiat objects are: all geographical entities demarcated in ways which do not track qualitative differentiations or physical discontinuities in the underlying territory. Clearly geographical fiat objects will in general have boundaries which involve a combination of *bona fide* and fiat elements, for such objects will in most cases owe their existence not merely to human fiat but also to associated real properties of the relevant factual material. Some ontologists, and most cognitive linguists, have embraced a thesis to the effect that everything is a fiat object, that there are no partition-independent realities to which our humanly constructed partitions or conceptualizations would correspond. According to this view – a linguistic version of standard Kantian metaphysics which we might call linguistic idealism – the reality which exists independently of our experiences is unknowable as such. That to which we are related when we use language correctly is a humanly or culturally *shaped and contoured reality*, never the amorphous and unknowable matter of reality (in) itself. Two versions of linguistic idealism can be distinguished: a global version to the effect that, in the terminology introduced above, all objects (or all cognitively relevant objects) are fiat objects; and a local version to the effect that fiat-object-status is to be assigned only to objects in specific and limited areas – for example in the area of conventional units of measure. The local thesis has been employed especially in relation to social and conventional terms such as democracy, liberalism, obligation, claim, right, etc. Consider a term like just, as in John is just, John's decision was just and so on. For the linguistic idealist the given term does not refer to some pre-existent *species* or *quality* of justice, but is rather a linguistic device by which we *articulate the impressions* which, say, a certain man's decision makes on us – or the way in which it affects us – in judging it to be just ... [Justice] does not inhere in John *simpliciter* ("on its own grounds") but only in relation to a person judging it, and only *qua* mediated by this relation. (Delius 1980, 109f) But does not a view of this sort come too close to the quite unacceptable thesis that there is no justice except where people find there to be justice or feel that there is justice? Do we not much rather have to ask in virtue of what it is that we can properly or correctly conceptualize our experience of Johns decision *as* an experience of a just decision? If there is nothing on the side of John and the decision which justifies this conceptualization, then the employment of the given predicate is surely completely arbitrary. If, on the other hand, there *is* something on the side of John which makes our statement true, then it is surely this *fundamentum in re* which we *mean* by justice: it is this which provides the necessary exterior friction which enables language of the given sort to gain a purchase on reality in the first place. There are, moreover, a number of further reasons why the general thesis of linguistic idealism must be rejected: how would the thesis that all objects are fiat objects be applied to the conceptualizers themselves, the demarcating subjects who construct the relevant systems of fiat boundaries? And how would fiat demarcation be possible if there were no genuine landmarks which we (or the first fiat demarcators) were able to discover, and in relation to which fiat demarcation becomes possible and objectively communicable? How would cumulative fiat demarcations be possible unless prior fiat demarcations had become allied, to some degree, with real perceptible differences, including differences in phonemes and graphemes, in relation to which later fiat demarcations could be drawn? In fact, as in geography so in relation to the demarcations we impose upon reality through language and in other ways, we have typically to deal with partitions which involve a mixture of fiat and *bona fide* demarcations. If I say blood flowed from his nose or he was bleeding from the nose then I am capturing a genuine event, genuinely distinct from other events (of the postman ringing the doorbell, the cat barking), but I am capturing this event via two distinct articulations, one or both of which must involve some fiat component. As Husserl already stressed in his *Logical Investigations*, only certain determinate parts of our expressions can have something corresponding to them in sensibly perceivable reality. When we consider the various simple judgment forms: *A is P*, *An S is P*, *The S is P*, *All S are P*, and so on, it is easy to see that only at the places indicated by letter-symbols ... can meanings stand that are fulfilled in perception itself. (1970, 779) As Husserl also points out, and his remarks here can be applied to our general topic of the nature of conceptualizations and categorizations of reality, each given object can be grasped in explicating fashion: in acts of articulation we put its parts `into relief', in relational acts we bring the relieved parts into relation, whether to one another or to the whole. And only through these new modes of conception do the connected and related members gain the character of parts or of wholes. (Husserl 1970, 792) The world, from the realist, Husserlian perspective we are defending, has a certain sensible, material stuff. Within this stuff we can pick out via our use of language and by other means many different sorts of objects by drawing fiat boundaries within the realm of matter. By means of suitable acts of relating or of setting into relief we can make for ourselves a range of different sorts of structures and we can carve out for ourselves new objects in cumulation by cleaving the relevant matters along different sorts of contour lines. This drawing of fiat boundaries is a purely intellectual affair. But the objects it picks out are not denizens of any separate, purely intellectual realm. It is, rather, as if the corresponding object-boundaries arise in the same fashion as political boundaries come to be delineated within areas of virgin territory, i.e. in such a way as to leave all sensible, material structures and all real unities at least initially unaffected. Ontological conceptualizations or categorizations can now be defined, quite generally, as *systems of complete boundaries which partition a given domain into objects or regions or elements of different sorts*. More generally, they are systems of boundaries which generate, from a given whole as starting point, another whole with more or fewer or different parts. The framework of mereotopology is surely not sufficient to provide a coherent formal expression for all distinctions which are of importance for the general theory of conceptualizations and categorizations. For the purposes of the cognitive scientist and of the ontological engineer, however, it can provide a uniquely fertile starting-point.

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