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SOME FORMAL MOMENTS OF TRUTH*

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Correspondence theories of truth have, since the time of the *Tractatus* and of *The Philosophy of Logical Atomism*, fallen out of favour amongst logically minded philosophers. Theorists of truth have indeed normally avoided the issue of correspondence by averting their attentions from the basic truth-relation between individual sentence and world, and concentrating instead upon logically structured *sets* of sentences and artificially constructed set-theoretical models. In the present note I wish to give a brief account of some more or less obvious formal characteristics of this almost forgotten basic truth-relation. I shall then attempt to show how this account may be extended to provide elements of a theory of truth which is in keeping with the spirit of the *Tractatus.*

One principal reason why the correspondence theory has lost so much of its former attraction rests on the fact that philosophers have, in the last few decades, extended their interests beyond the narrow corpus of sentences for which the question of correspondence is most obviously appropriate, i.e. simple declarative sentences about the spatio-temporal world. They have sought instead to construct theories of truth which take account of the characteristic properties of sentences of other types, for example of mathematical or fictional sentences, of counterfactuals, or of sentences involving intentional or modal operators. Such experiments are of considerable interest, and they demonstrate the power and scope of set-theoretical semantics. Here however, in keeping with our more modest task, we shall restrict ourselves entirely to descriptive sentences like 'John has a headache' or 'atom *a* [at some specific instant] strikes atom *b*', which are used to make assertions about objects or object-configurations in the real material world.

Simons has argued that most, if not all such sentences are made true by entities which, following Husserl, he calls 'moments'.[1] Wittgenstein and Russell on the other hand saw the role of truth maker as being filled by specific kinds of complexes—states of affairs, or facts—complexes which may involve moments, but which manifest an essentially different ontological structure. The ideas sketched in the present note are in fact consistent with either approach and with a range of possible variants. We shall need to assume only that the relations which hold amongst truth-makers (be they moments, states of affairs, or objects of other kinds) are *ontological* rather than *logical*; that, in other words, such relations are radically distinct from the kinds of relations which hold amongst sentences, propositions, or other candidate bearers of truth. This assumption reflects a principle of the heterogeneity of logic and ontology, which has been formulated in order to forestall any too ready imputation of logical structure to the objects of the material world.[2]

The ontological relation which is most important for our present purposes is the relation of (proper or improper) part to whole. Thus it seems clear that if *a* makes *p* true, then every *b* which includes *a* as a part will also make *p* true. In symbols:

(1) $a \models p \rightarrow \forall b\, (a \leqslant b \rightarrow b \models p)$.[3]

The relation $\leqslant$ obtains only between objects (names for which occur to the left of the $\models$-connective). It has no analogue amongst sentences or propositions—in complete accordance with the principle of the heterogeneity of logic and ontology.

We can now go on to affirm that

(2) $a \models p. \rightarrow p.$

But is the converse,

$$(3)\quad p \rightarrow \exists a \,.\, a \models p,$$

also true? Consider, say, the sentence: 'Jack and Jill went up the hill [i.e. not necessarily together]', or, alternatively: 'There were three eclipses of the moon in Erna's lifetime'. It would surely be wrong to assume that there are any single composite objects, events, or states of affairs—a *Jack's running up the hill* fused, mereologically, with a *Jill's running up the hill*, or a three-fold eclipse-fusion—which would make these sentences true. Rather we should accept, quite naively, that the given sentences are made true by a relevant manifold or plurality of truth-makers. Such a manifold is not a new, conjunctive entity. (It is not, for example, a set.) There are no conjunctive entities, any more than there are disjunctive, negative or implicative entities. A manifold is, rather, nothing more than the objects it comprehends; a manifold comprehending a single object is therefore simply that object itself.[4]

Manifold truth-makers may be represented by means of non-empty lists, '$a, b, c, \ldots, k$', of names of individual truth-makers. 'Γ', 'Δ', etc., will be used to stand in for lists of this kind. '$a \in \Gamma$' will signify that Γ comprehends or includes a or, in other words, that a name for a appears as an item in the list 'Γ'. We define the relation $\leqslant$ of mereological inclusion between manifolds as follows:

$$D1.\quad \Gamma \subseteq \Delta := \forall a \in \Gamma \,.\, \exists b \in \Delta \,.\, a \leqslant b.$$

As possible axioms for the $\models$ -relation we might now propose:

$$A1.\quad \Gamma \models p. \rightarrow p.$$

$$A2.\quad p \rightarrow \exists \Gamma \,.\, \Gamma \models p.$$

$$A3.\quad \Gamma \models p. \rightarrow \forall \Delta\, (\Gamma \subseteq \Delta \rightarrow \Delta \models p),$$

(a generalisation of (1) above). *A3.* implies in particular a rule of thinning:

$$(4)\quad \Gamma \models p. \rightarrow \forall \Delta \,.\, \Gamma, \Delta \models p.$$

$$A4.\quad (\Gamma \models p. \wedge \Delta \models q) \rightarrow \Gamma, \Delta \models p \wedge q.$$

$$A5.\quad (\Gamma \models p. \wedge p \rightarrow q) \rightarrow \exists \Delta\, (\Delta \subseteq \Gamma \,.\wedge \Delta \models q).$$

By *A3.* and *A5.* we have:

$$(5)\quad (\Gamma \models p. \wedge p \rightarrow q) \rightarrow \Gamma \models q,$$

whence, in particular:

$$(6)\quad \Gamma \models p. \rightarrow \Gamma \models p \vee q,$$

from which we can infer:

$$(7)\quad \Gamma \models p. \vee \Gamma \models q. \rightarrow \Gamma \models p \vee q,$$

the converse of which we affirm as an axiom:

$$A6.\quad \Gamma \models p \vee q. \rightarrow \Gamma \models p. \vee \Gamma \models q.$$

By (5) we have:

$$(8)\quad \Gamma \models p \wedge q. \rightarrow \Gamma \models p. \wedge \Gamma \models q,$$

and by *A4.* we have also:

$$(9)\quad \Gamma \models p. \wedge \Gamma \models q. \rightarrow \Gamma \models p \wedge q.$$

We can affirm as axioms:

$$A7.\quad \Gamma \models \exists a\,.\,p : \leftrightarrow \exists a\,.\,\Gamma \models p;$$

$$A8.\quad \Gamma \models \forall a\,.\,p : \leftrightarrow \forall a\,.\,\Gamma \models p.$$

Logically compound sentences involving negation raise more serious problems (as may be expected within the framework of a theory constructed on the belief that there are no negative entities to which negated sentences or sentence-parts may correspond).[5] Wittgenstein solves this most fundamental of all problems facing a correspondence theory of truth by embracing two distinct types of correspondence. We have first of all the relation of *direct depiction* between *Elementarsätze* (all of which are positive) and states of affairs; and secondly a higher-level truth relation between sentences and facts. The classes of sentences and *Elementarsätze*, like the classes of facts and states of affairs, are mutually exclusive. Sentences are obtained from *Elementarsätze* by successive applications of the logical functions *it is true that* and *it is false that*. Facts are obtained from states of affairs by successive applications of the ontological functions *the existence of* and *the non-existence of*. A fact is, that is to say, the existence or non-existence of states of affairs (cf. *TLP* 2, 2.06, 2.062, 2.11).[6]

It would take us too far afield to provide a detailed account of Wittgenstein's theory here. We shall content ourselves instead with an approximation to the theory which can be constructed within the framework developed above. We define, first of all, a counterpart of Wittgenstein's relation of direct depiction, which we shall signify by $\models\!\!\equiv$.

$$D2.\quad a \models\!\!\equiv p := \text{(i) } a \models p. \wedge$$
$$\text{(ii) } \forall q\,(a \models q. \rightarrow .p \rightarrow q)\,. \wedge$$
$$\text{(iii) } \forall \Gamma\,(\Gamma \models p. \rightarrow a \subseteq \Gamma).$$

Clause (i) asserts that direct depiction (being made true *elementarily*) is merely one special case of being made true. Clause (ii) is intended to capture the immediacy of the relation $\models\!\!\equiv$ between the sentence p and object-configuration a. It tells us that a proposition made true elementarily involves no logical redundancy or roundaboutness (thus no disjunctive sentence, for example, is elementarily made true). Clause (iii) signifies that the atomic object or state of affairs a which elementarily makes p true is a unique common factor of all truth-makers for p.[7] Note that $\models\!\!\equiv$ is defined only in relation to atomic or individual truth-makers: no sentence is elementarily made true by a manifold.

From *D2.* and the axioms above we may infer:

$$(10)\quad a \models\!\!\equiv p. \wedge a \models\!\!\equiv q. \rightarrow p \leftrightarrow q.$$

(We cannot however affirm

$$(11) \quad a \mathrel{|\!\!\equiv} p \wedge q. \rightarrow p \leftrightarrow q.$$

Consider the case where a is the mereological sum of two discrete individuals b and c, such that $b \mathrel{|\!\!\equiv} p$ and $c \mathrel{|\!\!\equiv} q$.)

$$(12) \quad a \mathrel{|\!\!\equiv} p. \wedge b \mathrel{|\!\!\equiv} p. \rightarrow a = b.$$

$$(13) \quad a \mathrel{|\!\!\equiv} p. \wedge b \models p. \rightarrow a \leqslant b.$$

$$(14) \quad a \mathrel{|\!\!\equiv} p. \wedge \Gamma \mathrel{|\!\!\equiv} p. \rightarrow a \subseteq \Gamma.$$

The following now suggests itself as a possible definition of the concept *elementary proposition*:

$$D3. \quad \text{elem}\,(p) := p \rightarrow \exists a\,.\, a \mathrel{|\!\!\equiv} p,$$

whence by clause (iii) in *D2.*,

$$(15) \quad p \wedge \text{elem}\,(p)\,. \rightarrow \exists ! a\,.\, a \mathrel{|\!\!\equiv} p,$$

—every true elementary proposition has a unique (minimal) truth-maker.

In a fully developed system we should wish to recognise also theorems like:

$$(16) \quad a \mathrel{|\!\!\equiv} p. \rightarrow \Diamond \sim \mathrm{E}\,(a)$$

—elementary truth-makers exist only contingently.

$$(17) \quad a \mathrel{|\!\!\equiv} p. \rightarrow \Diamond \sim p,$$

a theorem which has no analogue for $\models$. (Consider, for example, the sentence 'It is raining or it is not raining'. It is not possible that this sentence be false, yet it is made true—though not, of course, elementarily—by the relevant contingently existing condition of the weather.) And:

$$(18) \quad p \wedge \Diamond \sim p. \rightarrow \exists a\,.\, \exists q\,(a \mathrel{|\!\!\equiv} q. \wedge p \rightarrow q).$$

But there are more radical extensions of the basic theory which suggest themselves. We might, for example, wish to consider the formal moments of the relations: *is possibly true in virtue of*, *is necessarily true in virtue of*, *is made empirically probable by*, and so on. And we might wish to consider the interconnections between formal correspondence theory and the characteristic issues of epistemology. These questions must however be postponed until another place.

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NOTES

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1 See P. Simons, "Moments as Truth Makers" (in this volume).

2 I have defended this principle elsewhere: see B. Smith, "An Essay in Formal Ontology", *Grazer Philosophische Studien*, Vol. 6 (1978), pp. 39—62 and B. Smith, "Logic, Form and Matter", *Proceedings of the Aristotelian Society*, Supplementary Vol. 55 (1981), pp. 47—62.

3 $\rightarrow$, here, is at least as strong as strict implication. Our theory of part-whole relations is a generalisation of the mereology of Leśniewski; see Smith and Mulligan, "Pieces of a Theory", in: B. Smith (ed.), *Parts and Moments: Studies in Logic and Formal Ontology* (Munich 1982) and Smith and Mulligan, "Framework for Formal Ontology", *Topoi* (Special Issue on Leśniewski, forthcoming.)

4 See P. M. Simons, "Number and Manifolds", in: B. Smith (1982).

5 A sentence like 'There is no God' (if it is true) is perhaps made true by everything.

6 This interpretation of the *Tractatus*, according to which no fact is a state of affairs, is defended most rigorously by R.-A. Dietrich, *Sprache und Wirklichkeit in Wittgensteins Tractatus* (Tübingen 1974).

7 Clause (iii) is perhaps too strong, since for certain kinds of sentences we may wish to allow for a plurality of partially coincident elementary truth makers. Consider, for example, the sentence 'This tone is middle C', which may be seen as being made true elementarily not only by the tone as a whole, but also by the specific pitch from out of which (along with a specific individual timbre and loudness) the tone is constituted: See Smith and Mulligan, "Pieces of a Theory", and also Smith, Mulligan and Simons, "Truth-Makers", *Philosophy and Phenomenological Research* (forthcoming).

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