SNAP and SPAN: Towards Dynamic Spatial Ontology

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We propose a modular ontology of the dynamic features of reality. This amounts, on the one hand, to a purely spatial ontology supporting snapshot views of the world at successive instants of time and, on the other hand, to a purely spatiotemporal ontology of change and process. We argue that dynamic spatial ontology must combine these two distinct types of inventory of the entities and relationships in reality, and we provide characterizations of spatiotemporal reasoning in the light of the interconnections between them.

Keywords: Ontology, Processes, Objects vs. Fields, Spatial Reasoning, Dynamic GIS.

Reality is essentially dynamic. Faced with this fact, two alternative views may be distinguished, which have traditionally been perceived as incompatible. On the one hand are views which take the dichotomy between space and time ontologically seriously. On the other hand are views which see reality in terms of a single unified ontological theory of the spatiotemporal. These two views correspond to two different sorts of spatiotemporal reasoning. The former deals with successions of instantaneous snapshots of the world, the latter with changes and processes as such.

We here outline a theory that is designed to do justice to what is of value in both of these approaches. Our position is that a good ontology must be capable
of accounting for spatial reality both synchronically (as it exists at a time) and diachronically (as it unfolds through time), but that these are two different tasks. Our approach thus rests on what might be described as a joint venture between the so-called three-dimensionalist and four-dimensionalist perspectives current in contemporary philosophical ontology.

The paper is structured as follow: We lay out the methodological and philosophical foundations of our approach and we present the main elements of its formalization. Our two main ontological theories, namely SNAP and SPAN, are presented in sections 3 and 4, and section 5 discusses the structures which link them together. Finally we apply our framework to the domain of geography, culminating in the outline of an ontologically well-founded and comprehensive dynamic geospatial ontology.

1 Philosophical Background

Ontology concerns itself with the question of what there is. Briefly, it purports to produce an account of the token entities existing in the world, of the types or categories under which these token entities fall, and of the different sorts of relations which hold between them.

This philosophical task of working out the types and relationships among entities must of course at some point join up with the work of scientists. The full task of ontology is then a matter of going back and forth between the formulation of philosophical theories and the testing of such theories against what we know about reality from the work of scientists.

Ontology produces theories about the world formalized in some logical language. The virtue of formalization is first of all that of enforcing a certain degree of clarity. Another virtue is that it makes theories readily accessible, evaluable, and re-usable by other communities of researchers. Additionally, formalization makes it possible for us to exploit some of the power of logic when using ontologies in reasoning systems. In this paper, we provide the basic framework for a complete axiomatized theory of our modular ontology.

Our methodology, defended in (Smith, 2003) and (Grenon, 2003a), is realist, perspectivalist, fallibilist, and adequatist. Realism asserts that reality and its constituents exist independently of our (linguistic, conceptual, theoretical, cultural) representations thereof. Perspectivalism maintains that there may be alternative, equally legitimate perspectives on reality. But perspectivalism is constrained by realism: thus it does not amount to the thesis that just any view of reality is legitimate. To establish which views are legitimate we must weigh them against their ability to survive critical tests when confronted with reality, for example via scientific experiments. Those perspectives which survive are deemed to be transparent to reality. This is however in a way that is always subject to further correction. It is a fact that sciences change with time, and that thus all assertions must be understood against the background of fallibilism, which accepts that both theories and classifications can be subject to revision.
Adequatism, finally, is opposed to reductionism in philosophy. The reductionist affirms that, among the plurality of alternative views of reality there is some one, basic view to which all the others ought to be reduced. We, in contrast, affirm that there are many views of reality which are equally veridical. These are views of entities in different domains or of entities as seen from different perspectives, or they are views of what exists on different levels of granularity (microscopic, mesoscopic, geographic). Adequatism is the doctrine that a plurality of such views is needed if we are to do justice to reality as a whole.

Basic Formal Ontology
It was Husserl in his *Logical Investigations* (1913/21) who first drew a clear distinction between two kinds of ontological inquiry. On the one hand is what he called formal ontology, which is a theory at the highest and most domain-neutral level. Formal ontology deals with the categories and relations which appear in all domains and which are in principle applicable to reality under any perspective (with some possible additions or subtractions in specific domains or levels). Examples of such categories and relations include: object, relation, group, number, part-of, identical-to. On the other hand are what Husserl called material or regional ontologies, which are the ontologies of specific domains. There are as many ontologies in this sense as there are subject matters or domains of inquiry. Examples of such domains were for Husserl the domain of space, time and physical things; the domain of organisms; the domain of mind; and the domain of societies.

Basic Formal Ontology (BFO) is a theory of the basic structures of reality currently being developed at the Institute for Formal Ontology and Medical Information Science (IFOMIS) in the University of Leipzig. BFO is a formal ontology in the sense of Husserl and its construction follows the methodological maxims presented above. The enterprise of building BFO is thus motivated on the one hand by the desire to be faithful to reality, and on the other hand by the need to accept a multiplicity of perspectives upon reality which may be skew to each other. IFOMIS and its associates are developing a series of material or regional ontologies, including: MedO (for Medical Ontology), GeO (for Geographical Ontology), and DisReO (for Disaster Relief Ontology). BFO is designed to serve as a reusable template, which can (with some modifications) be used in constructing material ontologies for any and all domains of entities.

Temporal Modes of Being
The central dichotomy among the perspectives represented in BFO concerns the modes of existence in time of the entities populating the world. BFO endorses first of all a view according to which there are entities that have continuous existence and a capacity to endure (persist self-identically) through time even while undergoing different sorts of changes. We will henceforth use the terms ‘continuant’ and ‘endurant’ interchangeably for such entities. These entities come in several kinds. Examples are: you, the planet Earth, a piece of rock; but
also: your suntan, a rabbit-hole, Leeds. All of these entities exist in full in any instant of time at which they exist at all and they preserve their identity over time through a variety of different sorts of changes. You are the same person today as you were yesterday.

In addition, however, BFO endorses a view according to which the world contains *occurrences*, more familiarly referred to as processes, events, activities, changes. Occurrences include: your smiling, her walking, the landing of an aircraft, the passage of a rainstorm over a forest, the rotting of fallen leaves. These entities are four-dimensional. They occur in time and they unfold themselves through a period of time.

Occurrences are all *bound in time* in the way described by Zemach (1970). This means that each portion of the time during which an occurrence occurs can be associated with a corresponding temporal portion of the occurrence. This is because occurrences exist only in their successive temporal parts or phases. Some occurrences – for example beginnings and endings (the initial and terminal boundaries of processes) – are instantaneous.

The term *perdurant* is more precisely used for those occurrences which persist (perdur) in time, in other words for those which are extended and not instantaneous. For more concerning these notions see (Lowe, 1998; Sider, 2001).

Following (Ingarden, 1964), we shall reserve the term ‘process’ for temporally extended occurrences. The beginnings and endings of processes and the crossings of transition thresholds within processes – all entities which exhaust themselves in single instants of time – we shall call ‘events’.

Spatiotemporal Ontologies in BFO

Continuants and occurrences exist in time in different ways. The challenge is to build a unified framework within which we can do justice to both of these modes of being equally. This framework needs to keep the two corresponding groups of entities clearly separate, since no single inventory can embrace them both. At the same time however we have to find a way of bringing them together: continuants are themselves subject to constant change; occurrences depend on continuant objects as their bearers. In particular, there is an important correspondence between continuants and those special types of occurrences which are their *lives*.

Here again we draw on an intuition of Zemach (1970) to the effect that distinct modes of being generate distinct ontologies. The difference in mode of being of continuants and occurrences corresponds to an opposition between two different ways of existing in time. Accordingly, we distinguish two main kinds of ontologies, called SNAP and SPAN, one for continuants, the other for occurrences. Relations between continuants and occurrences are thus trans-ontological – they transcend the SNAP-SPAN divide. The resulting framework is a combination, in the spirit of adequatism, of a three-dimensionalist and a four-dimensionalist perspective, positions which are normally seen as representing mutually incompatible views of reality.
A reductionist four-dimensionalist asserts that it is possible to translate all talk about three-dimensional entities into talk which refers exclusively to processes. She holds that we can eliminate continuants in favour of four-dimensional spatiotemporal worms. Thus she will accept just the SPAN part of our present framework. A reductionist three-dimensionalist, in contrast, asserts that it is possible to describe the whole of reality by referring exclusively to endurant entities. She can take advantage, accordingly, only of the SNAP part of our framework. Here, however, we shall insist that, if we want to do justice to the whole of reality in non-reductionistic fashion, then we need both types of component. We need SNAP to do justice to the world of three-dimensional bodies, as also to the spatial regions at which they are located as well as to the qualities, powers, functions, roles and other entities which exist self-identically from one moment to the next. We need SPAN to do justice to the processes in which such enduring entities are involved and to the spatiotemporal volumes within which such processes occur.

Granularity and Ontological Zooming

Reality can be carved up ontologically in many different ways. A rock can be apprehended as an object in its own right, or as a structured group of molecules. Each material application of BFO is restricted to some given level of granularity, and each such granular ontology will respect BFO’s two-component SNAP-SPAN structure. Here we understand levels of granularity as roughly equivalent to levels of details (Hornsby, 2001; Hornsby and Egenhofer, 2002). In practice, granularity often reflects the specific ways of carving up domains of reality we associate with different scientific theories. Where issues of granularity arise in what follows we shall assume as underlying framework the theory of granular partitions presented in (Bittner and Smith, 2002).

Taxonomies of SNAP and SPAN

It might be useful to give a schematic representation of the taxonomies of SNAP. These will serve here the purpose of representing the subsumption relations among top-level categories (for lack of place, we will not give axioms).

Figure 1. The main formal categories of SNAP entities.
In figures 1 and 2, the italicized items stand for categories which are both
dissective and cumulative. Items picked out in bold are categories of basic
entities. These notions will be defined in 2.2. Meanwhile, figure 3 provides a list
of primitive relations used in the sequel and indicates their signatures.

<table>
<thead>
<tr>
<th>Constituent: Entity x Ontology</th>
<th>TemporalIndex: Ontology x TimeRegion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part: Entity x Entity</td>
<td>SpatialLocation: SnapEntity x SpaceRegion</td>
</tr>
<tr>
<td>InheresIn: SnapDependent x Substantial</td>
<td>TemporalLocation: SpanEntity x SpacetimeRegion</td>
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<tr>
<td>TemporalLocation: SpanEntity x TimeRegion</td>
<td>TemporalLocation: SpanEntity x SpacetimeRegion</td>
</tr>
<tr>
<td>Exists-At: SnapEntity x TimeInstant</td>
<td>ParticipatesIn: Substantial x Processual</td>
</tr>
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2  Formal Framework for BFO

Given the two-ontology structure of BFO, we need some framework within
which to express the SNAP and SPAN perspectives and to represent their
interconnections. Our framework draws on ideas introduced in (Grenon, 2003b;
2003c). The basic idea is that the variables of the theory range over both entities
and the ontologies they recognize. Entities and ontologies are not however on a
par. Entities are in the world of what happens and is the case. Ontologies are
analogous to mathematical structures such as groups and vector spaces. They
are understood as ways in which reality and the entities in reality are organ-
ized or structured.

We use a first-order language for our formalization, employing the following
symbols: ¬ for negation, ∧ for conjunction, ∨ for disjunction, → for material
implication, ↔ for material equivalence, ∃ and ∀ for the existential and universal
quantifiers. x, y, z, … are variables ranging over spatio-temporally existing
entities (continuants, occurrents, spatial, temporal and spatiotemporal regions of
various kinds, and material universals), and a, b, c are constants denoting such
entities. ω, ω′, ω″, … are variables ranging over ontologies, and α, β, γ, … are
constants denoting ontologies. Despite appearances, we are not using a sorted logic; entities and ontologies form one single domain, and it is only for the purposes of making the formulae more readable that we use different kinds of variables. We introduce special constants denoting certain designated entities, including, *space*, *time*, *earth*, *mountain*, *lake*. Functor symbols include: *spatial-location*, *surface*, *life*, etc.

All symbols for predicates in what follows have initial capital letters, and all predicates correspond to formal categories and relations. We use monadic predicates (Substance, SpatialRegion, Entity) to express membership in formal categories. We use polyadic predicates such as Identity, Part and Instantiation exclusively in order to capture formal relations.

We assume all the tautologies and the standard rules of inference of classical first-order logic. ‘A’ next to a number indicates that the formula is an axiom, ‘D’ that it is a definition, ‘T’ that it is a theorem. Throughout this paper unbound variables are assumed to be within the scope of universal quantifiers.

2.1 Entities and Ontologies

Everything which exists in the spatio-temporal world is an *entity*. ‘Entity(x)’ stands for: ‘x exists in the spatio-temporal world’.

An *ontology* is captured by depiction of the entities which exist within a given portion of the world at a given level of generality. It includes a taxonomy of the types of entities and relations which exist in the world under a given perspective. ‘Ω(α)’ stands for: ‘α is an ontology’.

A *constituent* of an ontology α is an entity whose existence is recognized by α. ‘Constituent(x, α)’ stands for: ‘x is a constituent of α’.

Here are a few of the basic axioms constraining these notions:

(A1)  ¬ (Ω(ω) ∧ Entity(x) ∧ ω = x)

(A2)  Ω(ω) → ∃x (Entity(x) ∧ Constituent(x, ω))

(A3)  Entity(x) → ∃ω (Ω(ω) ∧ Constituent(x, ω))

A1 enforces the separation between entities and ontologies. By A2, any ontology has at least one constituent. By A3, any entity is a constituent of some ontology.

Ontologies and Ontology Forms

Each depiction of the world is such that the entities depicted fall under certain formal categories such as Quality or SpatialRegion. The latter are related together in a specific way, which we call an *ontology form*. Each ontology instantiates a certain ontology form. So far we have distinguished two ontology forms, of SNAP and SPAN, respectively. Different SNAP and SPAN ontologies can be indexed in various ways, reflecting the different sorts of perspectives to which they correspond. Each SNAP ontology is indexed by a single time instant; each SPAN ontology by a single time interval. The default time-index for SPAN
ontologies is the whole of time. We write ‘TemporalIndex(\(\alpha, t\))’ for: ‘\(t\) is the temporal index of the ontology \(\alpha\)’. TimeRegion is the predicate applying to regions of time of arbitrary length, including temporal instants. We then have:

\[(A4) \quad \text{TemporalIndex}(\alpha, t) \land \text{TemporalIndex}(\alpha, t') \rightarrow t = t'\]

Both SNAP and SPAN ontologies are indexed also by domain and by level of granularity. The default SNAP domain is: everything, the universe in its entirety and its parts, i.e., all continuants existing at some given instant of time. The default SPAN domain is the history of the universe in its entirety. All SNAP ontologies instantiate the same ontology form, though they differ in the material they recognize and in the perspectives they are associated with. A SNAP ontology of the zoological domain with index now would contain no entity instantiating the material universal dinosaur.

Material Universals

Material universals correspond to the real invariants or patterns in the world apprehended by the material sciences. They are the ontological foundation for typing, classifying or sorting entities. They are not concepts but rather entities in the world to which our material concepts (when they are transparent) correspond. Material universals are multiply instantiated: they exist in toto at different places and different times in the different particulars which instantiate them (Smith, 1997; Grenon and Johansson, 2003). For instance, the material universal mountain is instantiated by Mont Blanc in France and by Grossglockner in Austria. The two mountains are numerically distinct entities, but it is the very same universal which exists in these two different places. The relation between universals and particulars is one of instantiation. ‘Instantiation(\(a, u\))’ stands for: ‘particular \(a\) is an instance of the material universal \(u\)’.

Given any material universal \(u\) we can form a constructed pseudo-predicate \(U\) such that an entity falls under \(U\) if and only if it is an instance of \(u\). Formally, we write:

\[U(x) =_{\text{def}} \text{Instantiation}(x, u)\]

We refer to material universals by means of terms such as ‘mountain’, ‘lake’, and so on. While it is common in formalized theories to allow the unrestricted definition of ever more complex predicates via Boolean composition, we claim that the realm of universals is not structured in this Boolean fashion. There may be mountains and there may be oceans, but there is no universal mountain-or-ocean. We thus accept a sparse theory of universals, a theory of the type embraced by realist ontologists from Aristotle to Armstrong (1978) and Lewis (1983), who hold that the question as to which universals exist in reality is a matter for scientists – not for ontologists, logicians or linguists – to determine.

Taxonomies of Universals in BFO

Each SNAP or SPAN ontology may be understood as opening a window on a certain portion of the world. We can look through this window in a way which
selects particulars or in a way which selects universals. Thus each SNAP and SPAN ontology is an ontology of particulars, but it is also, when we change our direction of focus, an ontology of the universals which such particulars instantiate. The Foundational Model of Anatomy talks in this connection of an opposition between canonical and instantiated anatomy (Rosse and Mejino, 2003).

A taxonomy is a tree, organized into nodes of greater and lesser generality, these nodes correspond to material universals. We use the terms 'genus' and 'species' to refer informally to universals at higher and lower positions within such trees. A geological taxonomy recognizes the genus rock and the species igneous rock, metamorphic rock, sedimentary rock. Each of the latter is an immediate sub-species of the single genus rock. The species on any given level are pairwise disjoint (no rock falls under more than one of these species). In addition, species jointly exhaust their genus (each rock falls under one of these species). When all species meet these requirements the taxonomy is a partitioning tree. To see how the phenomenon of vagueness can be dealt with in this context see (Bittner and Smith, 2002).

2.2 Mereology

Mereology, the theory of the part-whole relationship, will be our basic tool for understanding the domains of SNAP and SPAN ontologies (Simons, 1987). 'Part(a, b)' stands for 'a is a (proper or improper) part of b'. The parthood relation is reflexive (A5), transitive (A6), and antisymmetric (A7). ProperPart is the irreflexive form of parthood (D8). Overlap obtains when two entities have a part in common (D9). The ternary relation Sum defined in (D10) obtains when the first relata is the sum of the two other relata. The sum of two entities, when it exists, is the smallest entity which overlaps anything which overlaps either entity.

(A5) \( \text{Part}(x, x) \)

(A6) \( (\text{Part}(x, y) \land \text{Part}(y, z)) \rightarrow \text{Part}(x, z) \)

(A7) \( (\text{Part}(x, y) \land \text{Part}(y, x)) \rightarrow x = y \)

(D8) \( \text{ProperPart}(x, y) \equiv \text{def} \text{Part}(x, y) \land x \neq y \)

(D9) \( \text{Overlap}(x, y) \equiv \text{def} \exists z (\text{Part}(z, x) \land \text{Part}(z, y)) \)

(D10) \( \text{Sum}(z, x, y) \equiv \text{def} \forall w ((\text{Overlap}(w, x) \lor \text{Overlap}(w, y)) \leftrightarrow \text{Overlap}(w, z)) \land \forall z' \forall w ((\text{Overlap}(w, x) \lor \text{Overlap}(w, y)) \leftrightarrow \text{Overlap}(w, z')) \rightarrow z = z' \)

We do not assert a general existential axiom for sums because we want to postulate sums of entities within particular categories only, namely, within those of substances, SNAP dependent entities, processes, and spatial, temporal, and spatiotemporal regions. Our full formal theory will contain axioms for each of these categories similar to this one for substances:

(A11) \( (\text{Substance}(x) \land \text{Substance}(y) \land \text{Constituent}(x, o)) \land \text{Constituent}(y, o)) \rightarrow \exists z (\text{Constituent}(z, o) \land \text{Sum}(z, x, y)) \)
Thus for any two substances in an ontology, there is a constituent of this ontology which is their sum.

Using the schematic letter ‘P’ as a stand-in for a given (possibly constructed) predicate, we give the following definitions (adapted from Simons, 1987):

\[ (D12) \quad \text{P is dissective when all parts of any instances of P also fall under P:} \]
\[ (P(x) \land \text{Part}(y, x)) \rightarrow P(y). \]

\[ (D13) \quad \text{P is cumulative when the sum of any two instances of P also falls under P:} \]
\[ (\text{Sum}(z, x, y) \land P(x) \land P(y)) \rightarrow P(z). \]

The predicate corresponding to the universal cup is neither dissective (a part of a cup is generally not a cup) nor cumulative (the sum of two cups is not a cup). However, the predicate corresponding to water is both, though its dissectivity holds only down to parts of a certain level of granularity. Entity is dissective.

(For lack of place, we will not write these claims formally, but they ought to be seen as axioms instantiating the axiom schemas D12-13.)

Mereology and Universals

For each material domain, each level of granularity, and each perspective, we can distinguish a number of basic universals whose instances are the basic building blocks of reality as captured by the corresponding SNAP and SPAN ontologies. Generally, these universals – for example molecule, cell, organism, city, nation – will be neither cumulative nor dissective. Instances of basic universals we call basic entities. In addition to the basic entities, there are also aggregates, fiat parts (Smith, 2001b), and boundaries of basic entities. You are a basic entity in the SNAP ontology of common sense; your digestive system is, within this ontology, a fiat part (of you). You and your hat are basic entities, the sum of you and your hat is an aggregate. The categories of aggregate, fiat part, and boundary are formal categories which appear in an ontology as soon as the basic entities themselves are recognized. However, aggregates, fiat parts, and boundaries are not themselves extra entities; rather they reflect merely different possibilities of slicing up a given domain on a given level of granularity.

2.3 Formal Relations

Our framework comprehends a number of different types of formal relations. Here we shall focus on binary relations, but the treatment can be extended to relations of higher arity. The most important types of relations are:

(i) Intra-ontological. A relation between constituents of a single ontology is called intra-ontological. An example is the relation of part to whole between entities of a single granularity. An object is never part of a process, a process is never part of an object. Since in BFO there are two main types of ontology, SNAP and SPAN, there are also two main types of intra-ontological relations.

(ii) Trans-ontological. A relation between entities that are constituents of distinct ontologies is called trans-ontological. Consider the relation of participa-
tion between an object and a process (as when an army participates in manoeuvres). We distinguish four main types of binary trans-ontological relations, having signatures: (SNAP, SPAN), (SNAP, SNAP), (SPAN, SNAP), and (SPAN, SPAN), respectively.

(iii) Meta-ontological. A relation that obtains between ontologies or between an ontology and an entity is called meta-ontological. Examples are the relation of constituency introduced above or the relations of temporal order between ontologies of different temporal indices.

Ontological Indices of Relations.
Entities recognized by distinct ontologies may be differently related in each. In one ontology, for example, the relation of being on between the cup and the desk is recognized, in another it is not.

When a relation $R$ obtains between entities $a$ and $b$ in a single ontology $\alpha$, we write '$R(a, b, \alpha)$'. In the case of trans-ontological relations, there might be as many ontologies mentioned as there are relata. We then write for example '$R(a, \alpha, b, \beta)$' for: '$R$ obtains between $a$ in $\alpha$ and $b$ in $\beta$'. We then assume the following constraints:

\[
R(x, y, \omega) \rightarrow (\text{Constituent}(x, \omega) \land \text{Constituent}(y, \omega))
\]

\[
R(x, \omega, y, \omega') \rightarrow (\text{Constituent}(x, \omega) \land \text{Constituent}(y, \omega'))
\]

In practice, when the context is clear we shall omit ontological indices.

3 SNAP

A SNAP ontology is formed through the depiction of enduring entities existing at a given time (in a given domain at a given level of granularity). Such an ontology is analogous to a snapshot, or to the results of a process of sampling. Constituents of SNAP ontologies (Snap$\Omega$) are SNAP entities. The predicate SnapEntity is both dissective and cumulative (parts and sums of SNAP entities are SNAP entities themselves). Each SNAP ontology is indexed to some specific instant of time (for which we shall use the predicate TimeInstant):

\[
(A14) \quad (\text{TemporalIndex}(\alpha, x) \land \text{Snap}(\omega)) \rightarrow \text{TimeInstant}(x)
\]

A SNAP ontology recognizes only enduring entities existing at the time of its index, entities which typically have existed already for some time in the past and will go on existing in the future. SNAP entities are in other words not instantaneous entities, and nor are they sums of instantaneous slices. Rather, we assume a variant of the philosophical doctrine of presentism propounded by (Brentano, 1976) and (Chisholm, 1989), which holds that all entities which exist exist at the present time. Each SNAP ontology is a presentist ontology relative to some specific instant. BFO’s ability to handle a multiplicity of SNAP ontologies with different indices means however that it is not presentist in the narrow sense.
Being a constituent of a SNAP ontology amounts to existing (as a continuant) at the time of the ontology’s index. ‘ExistsAt(a, t)’ stands for: ‘the SNAP entity a exists at instant t’.

(D15) \[ \text{ExistsAt(a, t)} \equiv_{df} \exists \omega (\text{Snap}\Omega(\omega) \land \text{TemporalIndex}(\omega, t) \land \text{Constituent}(a, \omega)) \]

Enduring entities exist continuously during extended periods of time even while undergoing certain sorts of changes (for instance, changes of temperature or mass or height). We may thus speak about existence during a period of time by integrating across the instants of time in which endurants are captured by corresponding SNAP ontologies. We define ExistsDuring as follows:

(D16) \[ \text{ExistsDuring(x, y)} \equiv_{df} \text{TimeRegion(y)} \land \forall z ((\text{TimeInstant}(z) \land \text{ProperPart}(z, y)) \rightarrow \text{ExistsAt}(x, z)) \]

3.1 Spatial Regions

Reasoning within a single SNAP ontology is prototypically (but not exclusively) a form of spatial reasoning. We embrace a so-called absolutist view of space, sometimes called a container view, which holds that spatial regions are entities in their own rights. Moreover, they are endurants of such a sort that other SNAP entities can be located at or in them. (The phenomenon of vacuum shows that some regions need not have any entity located at or in them.) There is one distinguished spatial region, namely space or the entire spatial universe (the maximal spatial region). All spatial regions are parts of space. Thus we may define the predicate SpatialRegion as follows:

(D17) \[ \text{SpatialRegion}(x) \equiv_{df} \text{Part}(x, \text{space}) \]

SpatialRegion is trivially dissective and cumulative.

We may study space both topologically and geometrically. In addition, space supports a rich variety of qualitative reasoning. Thus, we may extend the taxonomy of spatial regions in a variety of ways, for example by reference to qualitative topological categories such as self-connected, convex, holed, and so on. The mereotopology of spatial regions also gives us the resources to speak about the relations of connectedness and separation between spatial regions, flat boundaries, dimensionality, and other related notions (Cohn, Bennett, Gooday, and Gotts, 1997; Smith, 1997; Smith and Varzi, 2000; Cohn and Hazarika, 2001a; Galton, 2001; Cohn and Varzi, 2003).

Spatial regions can serve as locations for SNAP entities. To this end, we introduce a new primitive relation of spatial location, symbolized: ‘SpatialLocation’ (it corresponds to the notion of exact location in (Casati and Varzi, 1996)).

(A18) \[ (\text{SnapEntity}(x) \land \text{Constituent}(x, \omega)) \rightarrow \exists y \text{SpatialLocation}(x, y, \omega) \]

(A19) \[ (\text{SpatialLocation}(x, y, \omega) \land \text{SpatialLocation}(x, y', \omega)) \rightarrow y = y' \]

SpatialLocation is a functional predicate. We can denote the spatial location of an entity a in an ontology \( \omega \) via the expression ‘\( \text{spatial-location}(a, \omega) \)’.
3.2 Substantial Entities

The category of *substantial entities* is a non-basic formal category comprehending the categories of: substances, their fiat parts, boundaries, aggregates, and sites. It is the generalization of the category of substances which results from the relaxation of the mereotopological constraints of boundedness and connectedness. The predicate Substantial is both dissective and cumulative.

*Substances* are maximal connected substantial entities. They are those substantial entities which enjoy a certain rounded-offness or natural completeness. In addition to material objects such as cups, rocks, and planets, organisms are the prototypical examples of substances. We treat substances in BFO along the lines presented in (Smith, 1997). Substances have the following principal features (these are all necessary conditions for being a substance; their conjunction comes close to forming a sufficient condition):

(i) Substances do not depend for their continued existence upon other entities.

(ii) Substances are the bearers of qualities and are subject to qualitative change.

(iii) Substances are enduring entities: they preserve their identity over time and through changes of various sorts.

(iv) Substances have a location in space.

(v) Substances are self-connected wholes with *bona fide* boundaries (Smith and Varzi, 2000; Smith and Brogaard, 2003).

The predicate Substance is neither dissective nor cumulative. On any given level of granularity, the parts and aggregates of substances are not themselves substances. Note well that the term ‘substance’ is thus used here as a count noun, in keeping with the philosophical tradition stretching back to Aristotle. In ordinary English the term ‘substance’ has of course a mass sense, the full treatment of which is here however left to one side as it involves issues of granularity.

*Fiat parts* are discussed in (Smith 1995; Smith and Varzi, 2000; Smith 2001b). It is possible to carve out zero-, one-, two- and three-dimensional fiat parts of the Earth, as when the North Pole or the Equator are demarcated, when a parcel of land is demarcated on a cadastral map, or when a certain volume beneath the Earth’s surface is demarcated for purposes of mining exploration.

*Boundaries* are lower-dimensional parts of spatial entities (Smith, 1997; Smith and Varzi, 2000). The boundary of the Earth is a closed two-dimensional surface.

*Aggregates* of substances are mereological sums comprehending separate substances as parts. In contradistinction to substances, aggregates may be scattered and thus have non-connected boundaries. Aggregates on one level of granularity (for example a collection of soldiers) may correspond to substances (for example an infantry battalion) on another level of granularity.
Sites are for example a room in a house, or a landing strip, or the interior of an airline cockpit. The corner of a room is a site, as also is your alimentary tract. Many sites are holes, cavities or similar entities dependent on physical hosts (Casati and Varzi, 1994). Some sites, for example the Reichstag in Berlin, are formed by sums of holes and cavities together with the substantial entities which surround or bound them. The addresses where people live and work are labels not of spatial regions but rather of the sites which they characteristically occupy. Sites sometimes act as surrounding spaces for other entities (Smith and Varzi, 2002). They are generally filled by a medium such as air or water. The medium is then itself one of the parts of the site.

As substances are located in space (at spatial regions), so also are the sites associated with them. But substantial entities also occupy sites or parts of sites, so that we have a three-term relation between substantial entities, sites and spatial regions. Occupies itself is a two-term relation between a substantial entity and a site. It is definable in terms of the notions of location and internal part (Smith and Varzi, 1999). For present purposes it is sufficient to point that in any ontology in which a case of the occupies relation obtains, i) the substantial entity and site which are joined by this relation do not overlap and neither do their respective locations; ii) the substantial entity’s location is an internal part of the location of the sum of this entity with the site which it occupies.

SNAP can thus do justice to a certain feature of the relational approach to space – in which spatiality is essentially conceived in term of system of relations between objects (Galton, 2001). Sites will call for a mereotopology in which both fiat and bona fide boundaries play an important role, and also the ability to deal with the surfaces of substances (Stroll, 1988; Casati and Varzi, 1994).

We can consider more specific kinds of sites:

(a) Boundary-Free and Physically Bounded Sites: Sites are typically compounds of rigid surroundings and fluid medium. Some sites, for instance air traffic corridors, are defined exclusively in terms of fiat boundaries. Sites whose boundaries are partly or wholly solid we call ‘physically bounded’; others we call ‘boundary-free’. Your office, when the door and windows are closed, is a wholly physically bounded site. Your nostril is a partly physically bounded site. The determination of boundaries of sites often involves a factor of vagueness (Smith and Varzi, 2002), which is however here left out of account.

(b) Niches or Environments: Niches (Smith and Varzi, 1999) are those special sorts of sites, marked out by their capacity to be occupied by substances of special sorts, especially organisms.

3.3 SNAP Dependent Entities

Examples of SNAP dependent entities include: particularized qualities (the colour of this tomato, the ambient temperature in this room), functions (the function of the canal to enable transportation), roles (as student, as captain of a soccer team), dispositions (of vegetables, that they are prone to rotting), powers (of a refrigerator, to slow the decay of vegetables), liabilities (of the vegetables,
to be affected by the functioning of the refrigerator). Other types of SNAP dependent entity include: states or conditions, shapes, plans, norms, tasks, laws.

All of these entities have in common the feature of enduring through time and of requiring a basis in SNAP independent entities in order to exist. (The particular colour of this tomato would not exist without the tomato it is the colour of.) However, if endurance and dependence are necessary conditions for SNAP dependent entities, they are not sufficient conditions. The distinguishing feature of these entities is that they inhere in substances. InheresIn is an intra-ontological relation between a SNAP dependent entity and its substantial bearer.

The redness of the ball inhere in the ball; the elevation of a summit inhere in the summit; the shape of a landform inhere in the landform. Inherence is a form of existential dependence (Husserl, 1913/21; Simons, 1987; Smith, 1997). The latter is such that the first of its relata (in the case of inherence, the SNAP dependent entity) exists only in virtue of the existence of the second (the bearer). There are other forms of dependence relations (e.g., between processes and their participants). Thus dependence alone does not suffice for the relation of inherence to obtain, though we will not pursue this matter here. Inherence is also a form of spatial subsumption, i.e., the spatial location of the inhering entity is a part of its bearer’s:

\[(A21) \quad \text{InheresIn}(x, y, \omega) \land \text{SpatialLocation}(x, x', \omega) \land \text{SpatialLocation}(y, y', \omega) \rightarrow \text{Part}(x', y', \omega)\]

There are both monadic and polyadic or relational SNAP dependent entities. Examples of monadic SNAP dependent entities include qualities dependent on one substance only, such as the mass of this apple. Examples of relational SNAP dependent entities include entities which depend upon a multiplicity of substances, which they serve to relate together, such as a treaty between two countries. The treaty itself depends for its existence upon the countries involved. The treaty document itself is a SNAP independent entity.

Given the multiplicity of kinds of dependence relations, the terminology is here very hard to fix and the types we have distinguished by way of example are not themselves clearly separated from each other. We avoid the terms ‘property’ and ‘attribute’ and cognate terms, because the latter are too closely associated with a traditional reading of the term ‘predicate’ in predicate logic – a reading which rides roughshod over the distinctions of importance to us here. We reject also Aristotle’s ‘accident’ and Husserl’s ‘moment’ and the term ‘trope’ as used for instance in (Campbell, 1990; Simons, 1994; Lowe, 2002). This is because each of these terms refers to a mixed group of entities which traverse what is for us the crucially important divide between SNAP and SPAN. In what follows we shall sometimes use ‘quality’ as a convenient shorthand term for ‘SNAP dependent entity’.
3.4 Universals

Going downward in the hierarchy of kinds of being, we can introduce further predicates which are specializations of those surveyed here. Eventually, we move from the domain of formal categories to the domains of material universals belonging to different material regions apprehended at different levels of granularity. Finding out which are the genuine formal categories is the job of ontologists. Finding out which are the genuine material universals is the job of specialist scientists.

SNAP material universals fall into two main families:

(i) **Substantial universals.** These are the universals instantiated by substantial entities. For instance, substance universals such as ball, body, planet; site universals such as valley, bay, cave; universals under which fiat parts of substances fall such as leg, hilltop, mound; universals instantiated by aggregates of substantials such as: family, mountain-range, fleet.

(ii) **Quality universals.** These are the universals instantiated by SNAP dependent entities. Recall that on our view SNAP dependent entities — qualities, conditions, functions, etc. — are particulars, which instantiate corresponding kinds of universals. The universals colour or shape are instantiated by the colour and the shape of this ball. The universal red is a determinate of the determinable universal colour (Johansson, 2000).

By combining substantial and quality universals (or their associated defined predicates), we may build constructed predicates a volo (Grenon and Johansson, 2003). For instance RedBall, which corresponds to no material universal, applies by definition to all those instances of the substance universal ball in whose surface there inheres an instance of the quality universal red.

4 SPAN

SPAN is the ontological theory of those entities which unfold themselves through time in their successive temporal parts. (SNAP entities do not have temporal parts: your youth is not a part of you, but rather a part of your life.) A SPAN ontology (SpanΩ) is obtained by depicting the reality constituted by the entities that unfold themselves within some determinate interval of time, in some given domain of reality, and at some level of granularity. As ontologies of endurants are analogous to snapshots of reality, so ontologies of perdurants are analogous to videos spanning time (though the latter analogy has to be taken with a large grain of salt).

In regard to SPAN, a form of methodological eternalism is appropriate, i.e., of the philosophical doctrine according to which all times (past, present, and future) exist on a par. An all-encompassing SPAN ontology has time itself as constituent. Any SPAN ontology has some temporal regions as constituents.

\[ \text{SpanΩ}(ω) \rightarrow \exists x (\text{TimeRegion}(x) \land \text{Constituent}(x, ω)) \]
Occurrents are structured also along the spatial dimension; however, the real substrate for location here is no longer space but rather spacetime, which is itself a SPAN entity. SPAN entities are not located in space – the assumption that they are so located derived from the fact that each region of space may be put in correspondence with a particular portion of an instantaneous slice of spacetime. SPAN ontologies comprehend spatiotemporally extended regions and the occurrences (including changes, as entities in their own right) located at such regions.

4.1 Processual Entities

These are entities which stand to processes as substantial entities stand to substances. Processuals are occurrences or happenings; they involve participants of a SNAP kind, and they are dependent on their participants. They are located at spatiotemporal regions and temporal regions. (We here leave open the question whether this location relation is determinate, or whether there are also cases of vague location of processual entities in time.) Processual entities include: processes, fiat parts, aggregates and boundaries thereof. The predicate Processual is both dissective and cumulative. Processual entities may have more or less complex structures. However, in contradistinction to what obtains in the realm of substances, there are few clean joints (bona fide boundaries) in the realm of processual entities. This is because processes merge with each other in a variety of ways to make larger process-wholes, to the degree that the SPAN realm is in large part a realm of flux. Thus when you breathe and your friend talks these two processes become merged together in a way that has no counterpart in the SNAP realm of substances. When talking about processes as unitary entities we are often in fact engaging in the sort of gerrymandering alluded to in (Quine, 1960; §36, p. 171). Notice, however, that we here depart from Quine, who propounded an exclusively four-dimensionalistic ontology. Processes are precisely occurrent entities in which substances and other SNAP entities participate. Processes are the entities revealed when we adopt a certain special (SPAN) perspective on reality. But the orthogonal (SNAP) perspective is no less faithful to what exists.

Processues are those extended processual entities which are self-connected wholes. On the one hand, they have beginnings and endings corresponding to real discontinuities, which are their bona fide boundaries. On the other hand, they involve no temporal or spatiotemporal gaps in their interiors. In particular, a given process may not be occurring at two distinct times without occurring also at every time in the interval between them. Similarly a process may not occupy two spatiotemporal regions separated topologically along the spatial dimension without occupying also a continuous spatiotemporal bridging region which lies between them (Cohn and Varzi, 2003).

If a given processual entity is affected by gaps of either of these sorts, then we are dealing not with one single process but rather with an aggregate of processes. As we see when examining for example movements of air fronts,
tornadoes, combustion processes within a forest fire, troop movements, epidemic transmissions of diseases, what is a process and what is an aggregate of processes (thus what is connected and what is non-connected, what is a fiat and what is a bona fide boundary) may depend on the level of granularity chosen.

All the proper parts of a process on the same level of granularity as the process itself are by definition fiat parts; for if processes are connected entities, then any division into parts must of necessity be a fiat division. Examples are: the first day of the Six Day war, the displacement of an anticyclone during some given 20 minute period. However, when we move to a lower level of granularity, then we often find it possible to recognize bona fide process parts (for example, the movements of individual droplets of water in a rainstorm).

Events are instantaneous boundaries of processes and instantaneous transitions within processes. They are more generally bona fide and fiat instantaneous temporal parts of processes. Examples are: the beginning of a conflict; the ceasing to exist of a country as a result of annexation; the detaching of a portion of rock as a result of erosion. Bona fide events are the temporal analogues in SPAN of the spatial boundaries of substances in SNAP. Instantaneous transitions within the interiors of processes – for example the transition in your life which marked your becoming 4 years old – are fiat temporal boundaries.

Aggregates of processes. Examples of aggregates of processes are: the aggregate of all wars and ethnic conflicts in Central Africa in a given year; the aggregate of all movements of troops in the jungle in a given night. Again: aggregates of processes at one level of granularity may appear as processes in their own right at coarser grains.

Settings (spatiotemporal environments). Just as we distinguished sites within SNAP, so we can distinguish settings (Barker, 1978) and their four-dimensional counterparts within SPAN. The Hundred Years War forms a setting for the burning of Joan of Arc.

4.3 Temporal Regions

Time, the maximal temporal region, is a perdurant, and thus a SPAN entity. A temporal region is a part of time. TimeRegion may now be defined as follows:

\[(D23) \text{TimeRegion}(x) \equiv \underset{=} {\text{Part}}(x, \text{time})\]

Instants of time are zero-dimensional boundaries of extended temporal regions. TimeInstant is a specialization of the predicate TimeRegion.

Our apparatus will require a theory of temporal order reflecting the structure of time as a linear continuum. We follow traditional usage and take as our primitive the relation Before, a strict total order which holds between two time instants when the first is earlier than the second.

Every SPAN entity can be assigned a temporal location. TemporalLocation is a primitive relation between an entity and an (instantaneous or extended, connected or disconnected) region of time (‘TemporalLocation(a, t)’ stands for: ‘a is located at region of time t’).
(A24) \( \text{SpanEntity}(x) \rightarrow \exists y \ (\text{TimeRegion}(y) \land \text{TemporalLocation}(x, y)) \)

The temporal location of each SPAN entity is unique and TemporalLocation is therefore functional:

(A25) \( (\text{TemporalLocation}(x, y) \land \text{TemporalLocation}(x, z)) \rightarrow z = y \)

We may thus introduce a \textit{temporal-location} function analogous to \textit{spatial-location} introduced earlier, though without an ontology as an argument. Processual entities do not change their locations in time in the way in which substantial entities may change their locations in space.

Instantaneous SPAN entities are located at instants of time. We define the intra-ontological relation \textit{AtTime}:

(D26) \( \text{AtTime}(x, y) \equiv (\text{TemporalLocation}(x, y) \land \text{TimeInstant}(y)) \)

We can define a number of temporal relations holding between processuals in virtue of the ordering and mereotopological relations which exist between their respective temporal locations. In particular, we could define the relation Cotemporal holding between SPAN entities which have as temporal locations exactly the same region of time.

(D27) \( \text{Cotemporal}(x, y) \equiv (\text{temporal-location}(x) = \text{temporal-location}(z)) \)

The relation TemporalPart (between SPAN entities) can then be defined as follows:

(D28) \( \text{TemporalPart}(x, y) \equiv (\text{Part}(x, y) \land \forall z ((\text{Part}(z, y) \land \text{Cotemporal}(x, z)) \rightarrow \text{Part}(z, x))) \)

Each temporal part is thus the sum of all cotemporal parts of a SPAN entity located within a given region of time. Instantaneous temporal parts we call ‘temporal slices’:

(D29) \( \text{TemporalSlice}(x, y) \equiv (\text{TemporalPart}(x, y) \land \exists t \ \text{AtTime}(x, t)) \)

We may now define the predicate \textit{Event} holding of instantaneous processuals:

(D30) \( \text{Event}(x) \equiv \exists y \ (\text{Processual}(y) \land \text{TemporalSlice}(x, y)) \)

By definition of \textnormal{TemporalSlice}, events are located at an instant of time. Notice that this definition allows us to capture events of both the fiat and bona fide sorts. We may now define the relation \textit{OccursAt} between a processual and a time. It is the relation which holds between a processual entity and an instant of time at which a temporal slice of this processual is located:

(D31) \( \text{OccursAt}(x, t) \equiv (\exists y \ (\text{TemporalSlice}(y, x) \land \text{AtTime}(y, t))) \)

Obviously, in the case of an event \( e \), \( \text{OccursAt}(e, t) \) is equivalent to \( \text{AtTime}(e, t) \). A proper mereotopological apparatus will allow us to define self-connected and scattered SPAN entities, and maximally connected parts of SPAN entities.
4.4 Spatiotemporal Regions

The totality of spatiotemporal regions reflects the totality of possible fiat demarcations of that maximal region we call *spacetime*. In accordance with the eternalism of SPAN, all spatiotemporal regions exist on a par with each other within the spatiotemporal universe. Our framework calls for a fiat mereotopology for spacetime regions, with:

\[(D32) \text{SpacetimeRegion}(x) \equiv_{\text{def}} \text{Part}(x, \text{spacetime})\]

Some of the instances of SpacetimeRegion are the spatiotemporal extensions of processual entities. Processuals then stand to spatiotemporal regions in SPAN ontologies in a way analogous to the way in which substantials stand to spatial regions in SNAP ontologies. In other words, *spacetime* constitutes the universal substratum for all SPAN entities, and location in SPAN has to be understood in relation to this spatiotemporal framework. SpatiotemporalLocation, defined by analogy to TemporalLocation, is the relation between a SPAN entity and the region of *spacetime* at which it is located. SpatiotemporalLocation, too, is functional and so allows the definition of the functor *spatiotemporal-location*.

Spatiotemporal regions are entities which exist in their own right, independently of any processuals which may be located at or within them. Moreover, partitions of the spatiotemporal regions at which processuals are located generate corresponding delineations of the processuals themselves. Spatiotemporal regions have specific sorts of four-dimensional shapes – for example the characteristic shapes of army manoeuvres or of hurricane movements – and these may be used in the formulation of taxonomies of their processual occupants.

As in the spatial and temporal cases, a number of spatiotemporal relations hold between processuals, including those which hold in virtue of the relations between the regions of *spacetime* at which they are located. Thus there are mereological and mereotopological relations as well as relations of co-incidence or locational overlap which obtain when two entities occupy the same spatiotemporal region or when their respective spatiotemporal regions share a part.

5 Trans-Ontology in BFO

Trans-ontology is the element in our framework which serves to integrate distinct ontologies. We have distinguished different kinds of trans-ontological relations and we can now characterize different varieties of trans-ontological reasoning of the forms: SNAP-SNAP, SPAN-SPAN, and SNAP-SPAN (or SPAN-SNAP). Trans-ontology is involved also in reasoning with multiple levels of granularity, but we shall not focus on this issue here.

**SNAP-SNAP Trans-Ontology**

This is reasoning paradigmatically involving discrepancies between temporally separated snapshots. A depiction of the world over time from the SNAP perspective involves taking a succession of temporally indexed SNAP ontologies
into account. As single SNAP ontologies are akin to snapshots of reality, so a succession of such ontologies is akin to a slide show.

It is important to emphasize that changes themselves are not entities according to the SNAP point of view. Rather, they appear, here, as trans-ontological structures or patterns. (Only in the SPAN ontology are changes entities in their own right.) Here we review from the SNAP perspective three main types of change: qualitative, locational, and substantial. We shall focus on changes of the qualitative sort, as these exhibit some of the peculiar features of SNAP-SNAP trans-ontological reasoning.

Qualitative Change
At any given time certain qualities inhere in each given substance. At any subsequent time a number of different qualities may inhere in the same substance. Pace (Loux, 1979) there need be no inconsistency in ascribing contradictory properties to the same object providing such ascriptions pertain to distinct ontologies and such a way that there is no single perspective within which they conjunctively obtain. There are various modes of qualitative change:

(i) Change in determinables: In many cases a SNAP dependent entity will instantiate different determinates of the same determinable quality universal at different times. The colour of this table becomes tarnished over time. The colour changes; yet there is still something which remains the same and which is the subject of such change: the token determinable remains the same while transitioning through successive token determinates or values. Some quantitative changes, for example changes in temperature, belong under this heading also.

(ii) Qualitative creation: A SNAP dependent entity that is not present in one ontology appears in later ontologies. This is what happens when Jacques Chirac takes on the role of President of France, or when a desert area is irrigated and thereby acquires a new dispositional quality of supporting vegetation coverage.

(iii) Qualitative destruction: A substance has a certain SNAP dependent entity for a certain interval of time but not at later times. This is what happens when a substance loses powers or functions, as when Jacques Chirac ceases to be President or when a ground area is classified as not suitable for development.

Spatial and Locational Change
In one SNAP ontology the cup is on the table, in a later SNAP ontology it is on the floor. The cup underwent locational change. In one SNAP ontology Danzig is a part of Germany; in a later SNAP ontology it is part of Poland. Here, Germany and Poland undergo spatial change.

Substantial Change
Substantial change occurs when substances are created or destroyed, as when a substance is divided up so as to produce a plurality of substances or when a plurality of substances is fused or merged. The Kingdom of Saxony was once one nation; currently its territory is divided among several administrative re-
regions of a new and more comprehensive state. We can distinguish further simple
types of substance formation following the list provided by (Smith and Bro-
gaard, 2003), including budding, absorption, separation, unification, and so on.

Genidentity
As these kinds of change illustrate, there are trans-ontological filiations and
 correspondences between entities recognized in successive SNAP ontologies. In
 order to account for such kinds of changes it is useful to introduce the trans-
ontological relation of genidentity, which is a relation in which one entity stands
to another when the latter is such-as-to-have-come-forth-from the former (cut a
chunk of rubber into two pieces and the sum of the separated pieces is exactly
physically genidentical to the chunk as it existed before the cutting). On the
varieties of genidentity see (Lewin, 1922) and (Smith and Mulligan, 1982).

Reasoning with SPAN Ontologies
Reasoning about change can be conducted within a single SPAN ontology.
However, it is useful to introduce the notion of a temporal restriction of a SPAN
ontology motivated by temporal partitioning of the time line. This amounts to
forming partial SPAN ontologies by restriction to specific intervals of time. If a
SPAN ontology \( \beta \) is such that its constituents are the constituents of a SPAN
ontology \( \alpha \), where the maximal temporal region of \( \alpha \) is a part of that of \( \beta \), then
we say that \( \alpha \) is a temporal restriction of \( \beta \), and write: TemporalRestriction(\( \alpha \),
\( \beta \)). When the index of the restriction is an instant of time, we call \( \beta \): ‘the cross-
section of \( \alpha \)’ and write: Section(\( \alpha \), \( \beta \)).

\begin{align*}
(D33) \quad \text{TemporalRestriction}(\alpha, \alpha') & \equiv \exists \exists t \exists t'(\text{TemporalIndex}(\alpha, t) \\
& \land \text{TemporalIndex}(\alpha', t') \land \text{Part}(t, t') \land (\forall x (\text{Constituent}(x, \alpha) \\
& \leftrightarrow (\text{Constituent}(x, \alpha') \land \text{Part}(\text{temporal-location}(x), t))))
\end{align*}

\begin{align*}
(D34) \quad \text{Section}(\alpha, \alpha') & \equiv \exists t (\text{TemporalRestriction}(\alpha, \alpha') \\
& \land \text{TemporalIndex}(\alpha, t) \land \text{TimeInstant}(t))
\end{align*}

Notice that the uniqueness of temporal location and the putative granularity
of ontologies, requires that any ontology which has a given SPAN entity as a
constituent has its corresponding temporal location as a constituent (hence, the
granularity of processuals is transmitted to the granularity of temporal regions).

\begin{align*}
(A35) \quad (\text{Constituent}(x, \alpha) \land \text{TemporalLocation}(x, y)) & \rightarrow \text{Constituent}(y, \alpha)
\end{align*}

SNAP-SPAN Trans-ontology
Each SNAP entity is related to the unique SPAN entity which is its history or
life. This is true of both substantial and dependent SNAP entities: each has a life,
with a beginning and an end. The relation between you and your life is of course
difficult to define. We shall assume that it is a functional relation (since your life
is unique), and use a functional notation, life, in order to denote the life of a
SNAP entity. The term “history” is also used in the context of SNAP trans-
ontology (e.g., in (Bennett, 2001; 2002)) to denote a mapping from time points to states of the physical world. Our approach seems closer to that of (Cohn and Hazarika, 2001b), with the qualification that they focus on regions and intend to work exclusively within a four-dimensional framework for handling material objects as occurrents. Histories might best be understood as four-dimensional settings along the line introduced already above.

There is a family of other trans-ontological relations between SNAP and SPAN entities, of which not only participation but also initiation, termination, creation, destruction, sustenance, deterioration, facilitation, hindrance are examples (for details, see Grenon, 2003; Smith and Grenon, 2003). Here, we describe the basic intuitions of the framework, taking as prototypical the example of participation.

A participant in a process exists during a time which overlaps the temporal location of the process. We then have:

\[(A36)\] \text{ParticipatesIn}(a, \omega, b, \omega') \rightarrow (\text{Substantial}(x) \land \text{Processual}(y))

\[(A37)\] \text{ParticipatesIn}(a, \omega, b, \omega') \land \text{ExistsDuring}(x, t) \land \text{TemporalLocation}(y, s, \omega') \rightarrow \text{Overlap}(t, s, \omega')

Notice, first, that both the instant at which \(a\) exists and the processual entity in which \(a\) participates are on the side of SPAN. Those SNAP entities which exist at an instant may be related to non-instantaneous SPAN entities, e.g., to processes or time intervals, in virtue of the fact that, at the instant at which they exist, there is located a temporal slice of the corresponding extended SPAN entity. Thus, we see that SNAP-SPAN trans-ontological relations are rooted in a form of cotemporality. This cotemporality between entities is reflected by yet another kind of cotemporality, namely between a SNAP ontology and a section of that SPAN ontology which recognizes the processual entity involved in the pertinent relation of participation. The temporal slice of the process acts as a mediator, while instants of time play a cardinal role as medium of connection between SNAP and SPAN entities and ontologies.

If we know that \(a\) participates in \(b\) at two separate time instants \(t_1\) and \(t_2\), then we are entitled to infer that \(a\) exists during \([t_1, t_2]\). (Recall that SNAP entities have a continuous existence in time.) As concerns \(b\), strictly speaking our framework guarantees only that it has two temporal slices \(t_1\) and \(t_2\) (by definition of OccursAt). For instance, \(b\) might be a scattered processual entity in which \(a\) participates only in each of the beginning slices of its successive phases.

That \(a\) participates in \(b\) over a region of time \(t\) means that there is a series of SNAP ontologies \(\alpha_i\ (1 \leq i)\) covering \(t\), and a SPAN ontology \(\beta\) whose temporal index contains \(t\), such that \(\text{ParticipatesIn}(a, \alpha_i, b, \beta)\) is true for each \(i\). However, the temporal location of \(b\) and the lifespan of \(a\) may enter in a variety of relations, corresponding to the range of possible situations (e.g., initiation of \(b\) by \(a\) process or destruction of \(a\) through \(b\)). In the case of the relation between a SNAP entity and its life, we will always have exact temporal coincidence. In
other words, SNAP entities exist for the entirety of the period of time at which their life is located, and conversely.

There is finally a crucial form of trans-ontological relation between spatial and spatiotemporal regions. Instants of time delineate a cross-section of *spacetime* which is super-imposable on *space* as apprehended by the corresponding SNAP ontology. The relation between the two is then not one of identity – no continuant is identical with any occurrent – but rather a *sui generis* relation of superposition. *Space* in other words is not a mere temporal slice of *spacetime*. We will come back to this in the following section.

### 6 Case Study: The Ontology of Geodynamic

We have laid down a comprehensive framework for spatial ontology, discussing the many kinds of changes which take place in this domain. We also considered changes as entity in their own right, and argued that they require an ontological treatment in their own right as processual entities. We propose now to apply this general approach to the domain of geography. In order to avoid redundancy, we will take for granted all that has been said above and discuss in detail here only some of the specific features of geographical ontology.

**Geographical Ontology**

A geographical ontology is a theory of those coarse-grained entities and kinds of entities and relations we find in the geographical realm. Geographical entities are those entities appearing at a certain level of granularity and which have a certain relation to the Earth. These entities – both continuants and occurrents – are as numerous as the objects of the different branches of geography, including not only physical and human and political geography, but also geology, geomorphology, climatology, meteorology and so forth.

A complete geographical ontology should include in addition not only those entities which fall under the purview of such geographical sciences but also those entities at geographical scales which are encountered by human beings in the everyday business of life (Egenhofer and Mark, 1995). Geographical scale is here understood as encompassing Frank’s cityscapes as well as geographic landscapes (2003). In geographical reality we find first of all continuants such as mountains and lakes, valleys and deltas, land parcels and cities; armies and their areas of operation; commuters and their patterns of commuting; hospital systems and their patterns of provision of medical services. These are all SNAP geographical entities.

Mountains, cities and armies are entities with which human beings are involved in their everyday interactions with the world, which are themselves SPAN geographical entities. A platoon climbs a hill and fights to defend its summit. Winds blow along the coast creating dunes. Ambulances fan out to deal with the victims of disasters. Our basic assumption in light of these examples is
that reality is made up of objects: platoons, hills, dunes, ambulances, which are involved in a continuous series of processes: climbing, blowing, fighting.

At the levels of granularity with which geographical ontologies are concerned, reality is essentially dynamic. It involves a continuous succession of processes; but these are in every case processes in or involving objects. A geographical ontology can thus be limited neither to objects nor to processes, but must make room for both. Indeed, geographical sciences are interested precisely in those SPAN entities which are patterns of development of geographical objects over time.

It is however a limitation of most mainstream approaches to geographical ontology, in particular (as noted in Worboys, 2001) in the GIS context, that they rest on an essentially static view of the geographical realm. They focus essentially on snapshots of the world and concentrate on handling discrepancies between different states of the world at different times, reflecting the fact that geographic information systems have their roots in printed maps. There is certainly a growing body of literature working towards developing the tools for handling dynamic geographic information (Hornsby and Egenhofer, 2000; Worboys, 2001; Frank, 2003; Galton, 2003). But still, the main paradigm remains what we earlier characterized as SNAP trans-ontological reasoning coupled with what we might also call the ‘spatialization’ of occurrences.

The notion of lifeline (Hornsby and Egenhofer, 2002), for instance, is a way of tracking objects through a series of changes conceived in terms of transitions of discrepancies between successive states; the notion of lifestyle (Medak, 2001) is a way of building types of objects according to the common features of their lifelines. Lifelines, however, are abstract constructs. Geographical changes are entities in their own right. The approach to change and non-change in terms of lifelines can thus do some justice to the logical properties of reasoning about such changes, but it cannot do justice to the ontology of changes themselves. Treatments of change still predominate which centre on the category of objects and construct changes as abstract entities out of these, e.g., (Frank, 2003).

The assumption that it is possible to construct spatiotemporal entities, or abstract counterparts thereof, on the basis of enduring entities arises from the fact that it is possible to use the same coordinate system in a space and time approach as in a spacetime approach. This artificial indistinguishability has its bright side as it provides the foundation for managing the trans-ontological co-incidence of space at a time with a corresponding temporal slice of spacetime.

GeO, our ontology of the geographical realm, reproduces in its structure the formal-ontological divide between SNAP and SPAN. Geographical regions are at the heart of our theory of geographical entities. We will start by characterizing the former before turning toward other SNAP and SPAN geographical entities.

6.1 Georegions and Geo-Ontologies

We have used ‘region’ as a generic term embracing regions of space, of time, and of spacetime. The term ‘georegion’ will have the same generic character
here. Common to all georegions is the fact that they are of geographical scale and are locations of entities which are or occur at or near the surface of the Earth. A theory of granularity is thus needed for a fully fledged ontology of geographic entities, since the latter will provide analyses of the two concepts of existing or occurring in proximity to the surface of the Earth and of being of geographical scale. Here, however, we shall take these notions as primitive. Note that since we will here be dealing with ontologies focused on coarse-grained entities at or near the surface of the Earth, we can here set aside issues of the relativistic frame-dependence of simultaneity.

Geospatial Regions.
The Earth itself is not a geospatial region and neither is its surface. Rather, it is a substance, and its parts (mountains, cliffs, plateaux) are substantial entities (they are flat parts of the substance Earth). We use henceforth ‘earth’ in our formulas to designate this substance. The Earth is located at a certain geospatial region at any time at which it exists. Geospatial regions are regions of space at or in which are located parts of the Earth or of the Earth’s atmosphere at or near its surface.

The framework for three-dimensional spatial reasoning presented above will need to be extended in the geographical context in such a way as to include the means to define for any substance, its surface, interior, and exterior, together also with the concepts of near-surface-interior and near-surface-exterior.

Since the relation between a substance and its (maximal, and therefore unique) surface is functional, we use the functional expression surface in order to denote a substance’s surface. The entity surface(earth) is a SNAP substantial entity existentially dependent upon earth, and it has a specific spatial location at any given time. We may now define surface geospatial regions as spatial locations of parts of the surface of the Earth:

\[
(D38) \text{SurfaceGeoSpatialRegion}(x, \omega) \equiv \text{def} \ \text{SpatialRegion}(x) \\
\wedge \exists y (\text{Part}(y, \text{surface(earth, } \omega)) \wedge \text{SpatialLocation}(y, x, \omega))
\]

Geospatial regions are transient in nature; in other words, the regions of space which are geospatial are such in a given SNAP ontology only and their role is carried on from one time index to the next by numerically different regions. (This is the reason behind the addition of an ontological index in the definiendum of D41.) More generally, geospatial and surface geospatial regions are, within any given SNAP ontology, those regions of space which have a geographical significance.

This implies that there is a further sort of spatial reasoning in the geographical context, relating to the way geographical entities are located with respect to surface(earth). This is the type of reasoning we employ when working with two-dimensional maps. The functional relation SurfaceLocation relates a spatial entity to its corresponding surface location. (This is in fact a ternary relation between a SNAP entity, a spatial region and a substance acting as a reference
body. Here, however, we can leave the third term out of account, since we
assume in all geographical contexts that the relevant reference body is the
Earth.) The surface location of a geographical entity is that portion of the loca-
tion of the Earth’s surface onto which the spatial location of the entity projects
along the vertical axis at a given time.

Geographical entities are in every case spatially located at, under or above
their surface locations. We thus can assume that synchronic spatial relations
obtain between entities located at given spatial regions at given times in reflection
of the relations which obtain between the regions themselves. Some of these
relations have a mereotopological character, some are distance and orientation
relations. For example an airplane is above the city of Carlin, NV, Carlin is
situated along Interstate 80, Carlin is west of New York. An airplane flying over
Carlin, while spatially disconnected from the city, is such that its surface location
is part of (or overlaps) that of the city it is flying over.

Geotemporal and Geospatiotemporal Regions
Geotemporal Regions are the regions of time during which the Earth exists:

\[(\text{GeoTimeRegion}(x) \equiv_{\text{def}} \text{TimeRegion}(x) \land \forall y \ (\text{TimeSlice}(y, x) \rightarrow \text{ExistsAt}(\text{earth}, y)))\]

Since the Earth is a SNAP entity, it has a life, which is a SPAN entity. The
Earth exists at any time at which the process we call its life is occurring, and
conversely:

\[(\text{ExistsAt}(\text{earth}, t) \leftrightarrow \text{OccursAt}(\text{life}(\text{earth}), t))\]

The geotemporal regions are then precisely the parts of the region of time
occupied by the life of the Earth:

\[(\text{GeoTimeRegion}(x) \leftrightarrow (\text{TemporalLocation}(\text{life}(\text{earth}), y) \land \text{Part}(x, y)))\]

Geospatiotemporal Regions are the regions of spacetime which are parts of
the spatiotemporal location of the life of the Earth:

\[(\text{GeoSpacetimeRegion}(x) \equiv_{\text{def}} (\text{SpatiotemporalLocation}(\text{life}(\text{earth}), y) \land \text{Part}(x, y)))\]

Geo-Ontologies
In a full account of the framework, we would need to introduce predicate symbols
for each type of geographical ontology, including those specializations of
SnapΩ to which, for instance, maps correspond. SNAP geographical entities are
categorized, inter alia, by the fact that they have a geospatial location (A43)
and also a surface geospatial location (which ought to follow as a corollary of
A43 in a complete axiomatization of the theory). Further, they exist at geotem-
poral regions (A44). SPAN geographical entities are characterized by the fact
that they have a geospatiotemporal location (A45) and also a geotemporal
location (which ought to follow as a corollary of A45):

\[(A_{43}) \ (\text{SnapGeoEntity}(x) \land \text{Constituent}(x, \omega)) \rightarrow \exists y \ (\text{GeoSpatialRegion}(y) \land \text{SpatialLocation}(x, y, \omega))\]

\[(A_{44}) \ (\text{SnapGeoEntity}(x) \land \text{Constituent}(x, \omega)) \rightarrow \exists \omega' \exists y \ (\text{Span}_\Omega(\omega') \land \text{Constituent}(x, \omega') \land (\text{GeoTimeRegion}(y) \land \text{ExistsAt}(x, y, \omega)))\]

\[(A_{45}) \ (\text{SpanGeoEntity}(x) \land \text{Constituent}(x, \omega)) \rightarrow \exists y \ (\text{Constituent}(x, \omega) \land \text{GeoSpacetimeRegion}(y) \land \text{SpatiotemporalLocation}(x, y))\]

### 6.3 The SNAP Geographical Object Ontology

We discuss five major subcategories of geographical SNAP entities: features, artifacts, agents, places, and qualities.

(i) **Geographical Features.** The entities one first encounters in interacting with the world at geographic scale are geophysical features such as hills, mountains, lakes, and rivers. Such substantial entities are the most salient geographical entities for human subjects (Smith and Mark, 1999). In addition, there is a number of fiat geographical features defined by human geography and various branches of geographically oriented social sciences such as geopolitics, demography, etc. These include states, counties, postal districts, and a wide variety of other fiat objects whose demarcation is due to the activities of human beings.

(ii) **Geographical Artifacts** include buildings, roads, bridges, reservoirs, cities, towns, etc. These too are substantial entities.

(iii) **Geographical Agents.** A geographical artifact such as a city has a government, which is a geographical agent. Both the city and its government are SNAP geographical entities. Geopolitical agents are tied to certain geopolitical objects, which are tied in turn to certain geographical sites or places. There are also mobile geographical agents, such as an army on the march or the racing pool of the Tour de France. Mobile agents, like other mobile geographical objects, have the capacity to produce derived geographical objects: armies produce front lines; aggregates of viral agents produce quarantine zones. These too are substantial geographical entities.

(iv) **Geographical Places.** Objects take up space, they occupy some position, and they thereby help to carve out sites of various sorts, including those special kinds of sites we call places. Places – *inter alia*, the places whose names are listed in gazetteers – are organized into hierarchies of increasing size, from the small portions of territory (a street corner, a field), to countries and continents. There are also three-dimensional sites such as mine shafts and fiat volumes of the Earth’s crust demarcated when mining rights are assigned to underground resources. The family of places includes also cities and towns conceived as geopolitically demarcated sites or locations which one can inhabit (rather than as aggregates of material substances). The term ‘London’ is accordingly ambiguous as between London-as-site (‘John lives in London’) and London-as-geoartifact (‘John admired London from the air’).
(v) **Boundaries and Geographical Regions.** The boundaries of geographical substantial entities are of two sorts: fiat and bona fide. The former reflect human (for example administrative) decisions; the latter are real physical discontinuities. Often some extended substantial entity will be taken as an indicator of a boundary (a fence, for example, will indicate the delimitation of a property, a river will participate in the physical delineation of the border between two countries).

(vi) **Geographical Qualities.** In the geographical realm we are interested in qualities (and other SNAP dependent entities) of geographical substantial entities of geographical scale. These include qualities of features (for example the altitude of a mountain peak) and of agents (an army’s morale, a government’s level of respect for human rights).

### 6.4 The SPAN Geographical Process Ontology

A primary division among SPAN geographical entities is that according to the kinds of participants they involve. On the one hand we have **physical processes** such as erosion, water run-off, forest fires, whose participants are exclusively physical objects. On the other hand we have **social processes** such as demographic changes, epidemics, wars, which are processes involving aggregates of human beings. We can distinguish also various families of geographical actions such as army manoeuvres or the passing of a re-zoning ordinance.

Geographical changes reflect in the main spatial changes. Thus, there is the category of **substantial changes**, comprising different subcategories of events in which geographic objects are created and destroyed, together with related forms of changes for example of the sort discussed in (Hornsby and Egenhofer, 2000). Substantial changes are always instantaneous, though they are associated in every case with extended processes which precede or follow them.

Some geographical objects are tied essentially to their sites; this holds especially of geographical features such as canyons and mountains. However, a number of geographical objects endure through **changes of site**, either through movement (e.g., thunderstorms) or through shifts over time (e.g., front lines in a war, city limits). Other sorts of geographical changes include qualitative, structural and morphological changes: for example a change in vegetation coverage or in temperature across a given or site.

Note that a geographical change of temperature is at another level of granularity a non-geographical matter of the movements of molecules. This shows again the need for a formal apparatus for treating granularity in a way which can allow macroscopic geographical phenomena to be related to the microscopic processes by which they are constituted (Bittner and Smith, 2002).

**Patterns and Features of Processes**

In all regions of the Earth there are processes which occur more or less regularly, defining for example seasonal weather patterns. Patterns may be correlated with one another via complex relations of causality and dependence – for
example in the way in which ocean currents (patterns of water displacement) are correlated with patterns formed by winds blowing over the surface of the water (patterns of displacement of masses of air). In order to describe such SPAN patterns we have to build a framework for describing the characteristic features of processes, including in particular:

(i) Changes of intensity or rate of a process which give rise to characteristics such as: accelerating, alternating (e.g. tidal variations), gradual, instantaneous, continuous, discrete, regular, zigzagging, cyclical, seasonal, transient, and so on.

(ii) Changes in the geographical extent of a process, such as the increase in prevalence of a disease, expansion of commercial penetration, routing of an army.

In each case, these features are characterizable in our framework in terms of projections of processes onto the corresponding characteristic shapes of the spatiotemporal regions they occupy.

6.5 The SNAP Geographical Fields Ontology

Work on the ontology of geography has recognized two distinct perspectives, called the object and field perspectives, respectively (Couclelis, 1992; Peuquet et al., 1999; Galton, 2001; Smith and Mark, 2003). The object-perspective is precisely the SNAP geographical framework presented here. The field-perspective involves apprehending reality in terms of distributions of attributes such as temperature, population density or tree-coverage over a given spatial location. SNAP Fields are enduring entities which are located at or defined in terms of geospatial regions with which they coincide spatially. Field attributes are dependent SNAP entities which are associated with given locations at given times.

Here, too, we must distinguish between attributes as determinables and attributes as determinates. The former are universals such as temperature or elevation. The latter are specific values of universals such as −30°C or 4810 meters. Values of attributes form scales, sometimes demarcated in terms of real numbers, sometimes in terms of other cardinal or ordinal measures. Some attributes, for example soil types, have qualitative determinables and determinates.

The SNAP field- and object-perspectives in the geographical domain can be directed towards identical portions of reality. It follows that there are a number of trans-ontological relations between the entities recognized by each, echoing Broegaard’s discussion in (Peuquet et al., 1999).

A SNAP field ontology recognizes only one basic geospatial region (as base field) together with the associated distributions of attributes (SNAP dependent entities); thus it masks all of those entities in the geographic domain which are recognized by SNAP as substances and their substantial parts. There is no planet Earth visible from the field perspective, neither are there mountains and lakes. There are however fields (or parts of fields) which comprehend any relevant
aggregation of the features associated with the corresponding objects. These correspondences fall within the purview of yet another sort of trans-ontology.

Relations in SNAP Field Ontologies.
Fields are related to their attributes in a way that is analogous to the inherence relation between SNAP dependent entities and the substantial entities which are their bearers. Attributes in a field are necessarily bound to a given part of the field, and thus to a given spatial location. We use the relation of attribution between an attribute in a field and the corresponding field location, abbreviated in the symbol ‘AttributedTo’. The SNAP field and object perspectives are then linked as follows. Whenever $a$ is attributed to $b$ in a SNAP field ontology and $b$ is located at the position $c$, there is, in some SNAP object ontology (SnapObj$\Omega$), a substance located at $c$ in which $a'$, a proxy of $a$, inheres. For instance, there is a portion of the elevation field of the Earth corresponding to Mont Blanc.

\[(A46) \text{AttributedTo}(x, y, \omega) \land \text{TemporalIndex}(\omega, t) \rightarrow \exists \omega' \exists x' \exists y' (\text{SnapObj}\Omega(\omega') \land \text{TemporalIndex}(\omega', t) \land \text{InheresIn}(x, z, \omega'))\]

There has to be additional constraints on attribution, e.g., we may only allocate an attribute which depends on a field at a region which is part of that of the field.

\[(A47) (\text{AttributedTo}(x, y, \omega) \land \text{SpatialLocation}(x, x', \omega) \land \text{SpatialLocation}(y, y', \omega)) \rightarrow \text{Part}(x', y', \omega)\]

It is primarily SNAP geographical fields which are of importance in the literature. Reflection shows however that the field perspective can be combined also with the SPAN framework when the substratum of location in relation to which attributes are allocated is spatiotemporal. (Galton, 2003)

Conclusion

We have presented a framework which allows the formulation of relations between different ontologies, and we have applied this framework to the treatment of dynamic features of what exists in space and in spacetime. We outlined a system of spatial and spatiotemporal ontologies and a meta-ontological framework within which we can understand the relations obtaining between them.

Our theory cannot, of course, solve all the problems pertaining to dynamic spatial and geographical ontology. Nevertheless, we believe that we have advanced the foundations for a framework which can be extended and applied to the treatment of families of ontologies of other types, including ontologies at different levels of granularity as well as ontologies generated by the object- and field-perspectives. It can provide a tool also for dealing with the relations between distinct culturally-generated perspectives on the geographical domain, of the sort which are studied by linguists and anthropologists. Standardly, of course, such culturally-generated perspectives have been dealt with in relativistic fashion – so that, as it is sometimes said, the members of different cultural
groups ‘live in different worlds’. Perhaps the most characteristic feature of our framework is that it offers a way out of this relativistic trap. It shows how we can deal with pluralities of ontologies which are distinct yet nonetheless directed towards (because they are ontologies of) one and the same reality.

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