Topological Foundations of Cognitive Science

Barry Smith

This is a revised version of the introductory essay in C. Eschenbach, C. Habel and B. Smith (eds.), *Topological Foundations of Cognitive Science,* Hamburg: Graduiertenkolleg Kognitionswissenschaft, 1994, the text of a talk delivered at the First International Summer Institute in Cognitive Science in Buffalo in July 1994.

I shall begin by introducing the concepts at the heart of topology in an informal and intuitive fashion. Two well-known alternatives present themselves to this end. These prove to be equivalent from the mathematical point of view, but they point to distinct sorts of extensions and applications from the perspective of cognitive science.

1. The Concept of Transformation

A first introduction to the basic concepts of topology takes as its starting point the notion of *transformation.* We note, familiarly, that we can transform a spatial body such as a sheet of rubber in various ways which do not involve cutting or tearing. We can invert it, stretch or compress it, move it, bend it, twist it, or otherwise knead it out of shape. Certain properties of the body will in general be invariant under such transformations - which is to say under transformations which are neutral as to shape, size, motion and orientation. The transformations in question can be defined also as being those which do not affect the possibility of our connecting two points on the surface or in the interior of the body by means of a continuous line. Let us provisionally use the term 'topological spatial properties' to refer to those spatial properties of bodies which are invariant under such transformations (broadly: transformations which do not affect the *integrity* of the body - or other sort of spatial structure - with which we begin). Topological spatial properties will then in general fail to be invariant under more radical transformations, not only those which involve cutting or tearing, but also those which involve the gluing together of parts, or the drilling of holes through a body, or the decomposition of a body into separate constituent parts.

The property of being a (single, connected) body is a topological spatial property, as also are certain properties relating to the possession of holes (more specifically: properties relating to the possession of tunnels and internal cavities). The property of being a *collection* of bodies and that of being an *undetached part* of a body, too, are topological spatial properties. It is a topological spatial property of a pack of playing cards that it consists of this or that number of *separate* cards, and it is a topological spatial property of my arm that it is *connected* to my body.

This concept of topological property can of course be generalized beyond the spatial case. The class of phenomena structured by topological spatial properties is indeed wider than the class of phenomena to which, for example, Euclidean geometry, with its determinate Euclidean metric, can be applied. Thus topological spatial properties are possessed also by mental images of spatially extended bodies. Topological properties are discernible also in the temporal realm: they are those properties of temporal structures which are invariant under transformations of (for example) stretching (slowing down, speeding up) and temporal translocation. Intervals of time, melodies, simple and complex events, actions and processes can be seen to possess topological properties in this temporal sense. The motion of a bouncing ball can be said to be topologically isomorphic to another, slower or faster, motion of, for example, a trout in a lake or a child on a pogo-stick.

2. The Concept of Boundary

A second introduction to the basic concepts of topology takes as its starting point the intuitive notion of *boundary*. We begin, once again, with spatial examples. Imagine a solid and homogeneous metal sphere. We can distinguish, with some intellectual effort, two parts of the sphere which do not overlap (they have no parts in common): on the one hand is its *boundary*, its *exterior surface*; on the other hand is its *interior*, the *difference* between the sphere and this exterior surface (that which would result if, *per impossibile,* the latter could be subtracted from the former). Similarly in the temporal realm we can imagine an interval as being composed of its initial and its final points together with the *interior* which results when these points are removed from the interval as a whole.

Define, now, the *complement* x' of an entity x as that entity which results when we imagine x as having been deleted from the universe as a whole. The boundary of an entity x is from the point of view of classical mathematical topology also the boundary of the complement of x. We can imagine, however, a variant topology which would recognize *asymmetric* boundaries, such as we find, for example, in the figure-ground structure as this is manifested in visual perception. As Rubin (1921) first pointed out, the boundary of a figure is experienced as a part of the figure, and not simultaneously as boundary of the ground, which is experienced as running on behind the figure. Something similar applies also in the temporal sphere: the beginning and ending of a race, for example, are not in the same sense boundaries of any complement-entities (of all time prior to the race, and of all time subsequent to the race) as they are boundaries of the race itself.

The ideas of boundary and complement can be extended in a natural way into the conceptual realm. Imagine the instances of a concept arranged in a quasi-spatial way, as happens for example in familiar accounts of colour- or tone-space. Suppose that each concept is associated with some extended region in which its instances are contained, and suppose further that this is done in such a fashion that the *prototypes*, the most typical instances, are located in the centre of the relevant region and the less typical instances located at distances from this centre in proportion to their degree of non-typicality. Boundary or fringe cases can now be defined as those cases which are so untypical that even the slightest further deviation from the norm would imply that they are no longer instances of the given concept at all. The notion of *similarity* can be understood in this light as a topological notion.[(1)] In the realm of colours, for example, a is similar to b might be taken to mean that the colours of a and b lie so close together in colour-space that they cannot be discriminated with the naked eye. A similarity relation is in general symmetric and reflexive, but it falls short of transitivity, and is thus not an equivalence relation. This means that it partitions the space of instances not into tidily disjoint and exhaustive equivalence classes, but rather into overlapping *circles of similars*. This falling short of the discreteness and exhaustiveness of partitions of the type which are generated by equivalence relations is characteristic of topological structures. In some cases there is a continuous transition from one concept to its neighbouring concepts in concept-space, as for example in the transition from *red* to *yellow* which results from a continuous variation of hue. In other cases circles of similars are separated by gaps. This is so in regard to the transition from, say, *dog* to *cat* or from *cyclo-octatetrene* to *cyclobutadiene.*

3. The Concept of Closure

The two approaches briefly sketched above may be unified into a single system by means of the notion of *closure*, which we can think of as an operation of such a sort that, when applied to an entity x it results in a whole which comprehends both x and its boundaries. We employ as basis of our definition of closure the notions of mereology.[(2)] Some of the reasons why we shun the set-theoretical instruments employed in standard presentations of the foundations of topology will be set out below.

We shall employ '$x \leq y$' to signify 'x is a proper or improper part of y', and '$x \cup y$' to signify the mereological sum of two objects of x and y. The range of our variables is *regions* of one or other type, including boundaries and punctiform regions. An operation of *closure* (c) is defined in such a way as to satisfy the following axioms:

(AC1) $x \leq \mathrm{c}(x)$ (expansiveness)

(each object is a part of its closure)

(AC2) $\mathrm{c}(\mathrm{c}(x)) \leq \mathrm{c}(x)$ (idempotence)

(the closure of the closure adds nothing to the closure of an object)

(AC3) $\mathrm{c}(x \cup y) = \mathrm{c}(x) \cup \mathrm{c}(y)$ (additivity)

(the closure of the sum of two objects is equal to the sum of their closures)

These axioms were first set out in informal terms by the Hungarian topologist Friedrich Riesz in 1906, and independently by the Pole Kazimierz Kuratowski in 1922. The axioms define a well-known kind of structure, that of a *closure algebra*, which is the algebraic equivalent of the simplest kind of topological space. Kuratowski's list includes in addition the following axiom, where '0' would designate a null element (the empty set, in a set-theoretical formulation):

(AC0) $\mathrm{c}(0) = 0$ (zero)

Here, however, we are interested in a mereological formulation of topology, and because there is no mereological analogue of the empty set this further axiom has no meaning.

Various modifications and weakenings of these axioms are possible which preserve the possibility of defining analogues of the standard topological notions of boundary, interior, etc.[(3)] Thus we may drop the axiom of additivity, which, as Hammer puts it, might most properly 'be called the *sterility* axiom. ... it requires that two sets cannot produce anything (a limit point) by union that one of them alone cannot produce.'[(4)] If we apply our notion of closure to the conceptual sphere, then the closure of an object can be defined as the smallest circle of similars including this object. That additivity then fails is seen by considering the case where x is an instance of red and y an instance of green. The closure of x is the circle of similars including all instances of red; the closure of y is the circle of similars including all instances of green. The sum of these closures is then strictly smaller than the closure of $x \cup y$, which is the circle of similars including all instances of colour in general.

4. The Concept of Connectedness

Let 'x' ' stand for the mereological complement of x and '$\cap$' for mereological intersection. On the basis of the notion of closure we can now define the standard topological notion of (symmetrical) *boundary*, b(x), as follows:

(DB) $\mathrm{b}(x) := \mathrm{c}(x) \cap \mathrm{c}(x')$ (boundary)

Note that it is a trivial consequence of the definition of boundary here supplied that the boundary of an entity is in every case also the boundary of the complement of that entity.

It is indeed possible to define in standard topological terms an asymmetrical notion of 'border', as the intersection of an object with the closure of its complement:

(DB*) $\mathrm{b}^*(x) := x \cap \mathrm{c}(x')$ (border)

Moreover, where Kuratowski's axioms were formulated in terms of the single topological primitive of closure, Zarycki showed (1927) that a set of axioms equivalent to those of Kuratowski can be formulated also in terms of the single primitive notion of border, and the same applies, too, in regard to the notions of interior and boundary.

The notion of interior is defined as follows. We first of all introduce 'x - y' to signify the result of subtracting from x those parts of x which overlap with y. We then set:

(DI) $i(x) := x - b(x)$ (interior)

We may define a *closed object* as an object which *is identical with its closure.* An *open object,* similarly, is an object which is identical with its interior. The complement of a closed object is thus open, that of an open object closed. Some objects will be partly open and partly closed. (Consider for example the semi-open interval (0,1], which consists of all real numbers x which are greater than 0 and less than or equal to 1.) These notions can be used to relate the two approaches to topology distinguished above: topological *transformations* are those transformations which map open objects onto open objects.

A closed object is, intuitively, an independent constituent - it is an object which exists on its own, without the need for any other object which would serve as its host. But a closed object need not be *connected* in the sense that we can proceed from any one point in the object to any other and remain within the confines of the object itself. The notion of *connectedness*, too, is a topological notion, which we can define as follows:

(DCn) $\mathrm{Cn}(x) := \forall yz(x = y \cup z \rightarrow \exists w(w \leq (c(y) \cap c(z))))$ (connectedness)

(a connected object is such that all ways of splitting the object into two parts yield parts whose closures overlap)

The following yields an alternative concept of connectedness which is useful for certain purposes:

(DCn*) $\mathrm{Cn}^*(x) := \forall yz(x = y \cup z \rightarrow (\exists w(w \leq x \wedge w \leq c(y)) \vee \exists w(w \leq c(x) \wedge w \leq y)))$

(connectedness*)

(a connected object is such that, given any way of splitting the object into two parts x and y, either x overlaps with the closure of y or y overlaps with the closure of x)

Neither of these notions is quite satisfactory however. Thus examination reveals that a whole made up of two adjacent spheres which are momentarily in contact with each other will satisfy either condition of connectedness as thus defined. For certain purposes, therefore, it is useful to operate in terms of a notion of *strong connectedness* which rules out cases such as this. This latter notion may be defined as follows:

(DSCn) $\mathrm{Scn}(x) := \mathrm{Cn}^*(\mathrm{i}(x))$

(an object is strongly connected if its interior is connected*)

5. Mereotopology vs. Set Theory

The rationale for insisting on a mereological rather than a set-theoretic foundation for the axioms and definitions of topology for our present purposes can be stated as follows. Imagine that we are seeking a theory of the boundary-continuum structure as this makes itself manifest in the realm of everyday human experience. The standard set-theoretic account of the continuum, initiated by Cantor and Dedekind and contained in all standard textbooks of the theory of sets, will be inadequate to this purpose for at least the following reasons:

1. The application of set theory to a subject-matter presupposes the isolation of some basic level of *Urelemente* in such a way as to make possible a simulation of all structures appearing on higher levels by means of sets of successively higher types. If, however, as holds in the case of investigations of the ontology of the experienced world, we are dealing with mesoscopic entities and with their mesoscopic constituents (the latter the products of more or less arbitrary real or imagined divisions along a variety of distinct axes), then there are no *Urelemente* to serve as our starting-point. [(5)] This idea is, incidentally, at the heart also of Gestalt-theoretical criticisms of psychological atomism, which in many respects parallel criticisms of set-theory-induced atomism of the sort presented here.

2. The experienced continuum is in ever cases a concrete, changing phenomenon, a phenomenon existing in time, a whole which can gain and lose parts. Sets, in

contrast, are abstract entities, entities defined entirely via the specification of their members.

3. In the absence of points or elements, the experienced continuum does not sustain the sorts of cardinal number constructions imposed by the Dedekindian approach. The experienced continuum is not isomorphic to any real-number structure; indeed standard mathematical oppositions, such as that between a dense and a continuous series, here find no application.

4. Even if points or elements were capable of being isolated in the experienced continuum, the set-theoretical construction would still be predicated on the highly questionable thesis that out of unextended building blocks an extended whole can somehow be constructed.[(6)] The experienced continuum, in contrast, is organized not in such a way that it would be built up out of particles or atoms, but rather in such a way that the wholes, including the medium of space, come before the parts which these wholes might contain and which might be distinguished on various levels within them.

Of course, set theory is a mathematical theory of tremendous power, and none of the above precludes the possibility of reconstructing topological theories adequate for cognitive-science purposes also on a set-theoretic basis. Standard representation theorems indeed imply that for any precisely formulated topological theory formulated in non-set-theoretic terms we can find an isomorphic set-theoretic counterpart. Even so, however, the reservations stated above imply that the resultant set-theoretic framework could yield at best a *model* of the experienced continuum and similar structures, not a theory of these structures themselves (for the latter are after all *not sets,* in light of the categorial distinction mentioned under 2. above).

Our suggestion, then, is that mereotopology will yield more interesting research hypotheses, and in a more direct and straightforward fashion, than would be the case should we be constrained to work with set-theoretic instruments.

6. Foundations of Cognitive Science

On the one hand, then, there is topology as a branch of mathematics. Topology in this mathematical sense has been used by cognitive scientists in work on the mathematical properties of connectionist networks and elsewhere. On the other hand there is mereotopology, a treatment of concepts such as 'region', 'connectedness', 'boundary', 'surface', 'point', 'neighbourhood', 'nearness', and so on, that is inspired by standard mathematical treatments but which involves departures from standard mathematical topology in ways designed to meet the requirements of the subject-matters encountered within specific domains of cognitive science. Mereotopology as here conceived is not, however, a matter of loose analogies; rather it is a matter of deviations from standard topology which can be rigorously defined.

7. Husserl's Mereotopology

The idea of using topology as a foundation for cognitive science is not without precursors. It is above all in the tradition established by Brentano, a tradition which extends through Carl Stumpf to the Berlin school of Gestalt psychology, that the most important early contributions are to be found. Two such contributions will be dealt with here, those of Edmund Husserl and Kurt Lewin. Husserl's *Logical Investigations* (1900/01) contain a formal theory of part, whole and dependence that is used by Husserl to provide a framework for the analysis of mind and language of just the sort that is presupposed in the idea of a topological foundation for cognitive science.(7) The title of the third of Husserl's Logical Investigations is "On the Theory of Wholes and Parts" and it divides into two chapters: "The Difference between Independent and Dependent Objects" and "Thoughts Towards a Theory of the Pure Forms of Wholes and Parts". Unlike more familiar theories of wholes and parts, such as those prounded by Lesniewski, and before him by Bolzano, Husserl's theory does not concern itself merely with what we might think of as the vertical relations between parts and the wholes which comprehend them on successive levels as we move upwards towards ever larger wholes. Rather, Husserl's theory is concerned also with the horizontal relations between the different parts within a single whole, relations which serve to give unity or integrity to the wholes in question. To put the matter simply: some parts of a whole exist merely side by side, they can be destroyed or removed from the whole without detriment to the residue. A whole all of whose parts manifest exclusively such side-by-sideness relations with each other is called a heap or aggregate or, more technically, a purely summative whole (what we referred to above as an *Und-Verbindung*). In many wholes, however, and one might say in *all* wholes manifesting any kind of unity, certain parts stand to each other in relations of what Husserl called *necessary dependence* (which is sometimes, but not always, necessary *interdependence*). Such parts, for example the individual instances of hue, saturation and brightness involved in a given instance of colour, cannot, as a matter of necessity, exist, except in association with their complementary parts in a whole of the given type. There is a huge variety of such lateral dependence relations giving rise to correspondingly huge variety of different types of whole which more standard approaches of 'extensional mereology'[(8)] are unable to distinguish.

The connection between part and whole on the one hand and dependence on the other may be seen in the fact that every whole can be regarded as being dependent on its own constituent parts. This thesis may amount to no more than the trivial claim that every object is such that it cannot exist unless all the objects which are, at different times, its parts, also exist at those times. Or it may consist in a non-trivial thesis to the effect that certain special sorts of objects are such as to contain special 'integral parts' which must exist at all the times when these objects exist: their loss (for example the loss of brain or heart in a mammal) is sufficient to bring about the destruction of the whole. Or, finally, it may be transformed into the metaphysical thesis of mereological essentialism, i.e. into the assertion that *every* spatio-temporal object is dependent in the non-trivial sense upon *all* its parts, so that the ship ceases to exist (becomes another thing) with the removal of the first splinter of wood. It is one not inconsiderable advantage of Husserl's theory that it allows a precise formulation of these and a range of related theses within a single framework, a framework, furthermore, that is rooted on ideas concerning part, whole and dependence which are consistent with our common intuitions. Both Stanislaw Lesniewski, the founder of mereology, and the linguist Roman Jakobson applied Husserl's ideas on parts, wholes and categories from the *Logical Investigations* in different branches of linguistics, in the early development of categorial grammar and of phonology, respectively.[(9)] Thus Jakobson's account of distinctive features is as he himself admits an application of Husserl's idea of dependent moments from the third Investigation.

The topological background of Husserl's work makes itself felt already in his theory of dependence.[(10)] It comes to the fore above all however in his treatment of the notion of *phenomenal fusion*:[(11)] the relation which holds between two adjacent parts of an extended totality when there is no qualitative discontinuity between the two. Adjacent squares on a chess-board array are not fused together in this sense; but if we imagine a band of colour that is subject to a gradual transition from red through orange to yellow, then each region of this band is fused with its immediately adjacent regions. After distinguishing dependent and independent contents (for example, in the visual field, between a colour- or brightness-content on the one hand, and a content corresponding to the image of a moving projectile, on the other), Husserl goes on to note that there is in the field of intuitive data an additional distinction,

> between *intuitively* separated contents, contents *set in relief from* or *separated off from* adjoining contents, on the one hand, and contents which are *fused* with adjoining contents, or which *flow over* into them without separation, on the other (Investigation III, §8, 449).

He points out that independent contents

> which are what they are no matter what goes on in their neighbourhood, need not have this quite different independence of separateness. The parts of an intuitive surface of a uniform or continuously shaded white are independent, but not separated. *(Loc. cit.*)

Such content Husserl calls 'fused'; they form an 'undifferentiated whole' in the sense that the moments of the one pass 'continuously' ['*stetig*'] into corresponding moments of the other. (§9, 450)

That Husserl was at least implicitly aware of the topological aspect of his ideas, even if not under this name, is unsurprising given that he was a student of the mathematician Weierstrass in Berlin, and that it was Cantor, Husserl's friend and colleague in Halle during the period when the *Logical Investigations* were being written, who first defined the fundamental topological notions of open, closed, dense, perfect set, boundary of a set, accumulation point, and so on. Husserl consciously employed Cantor's topological ideas, not least in his writings on the general theory of (extensive and intensive) magnitudes which make up one preliminary stage on the road to the third Investigation.[(12)]

More generally, it is worth pointing out that the early development of topology on the part of Cantor and others was part of a wider project on the part of both mathematicians and philosophers in the nineteenth century to produce a *general theory of space* - to find ways of constructing fruitful generalizations of such notions as extension, dimension, separation, neighbourhood, distance, proximity, continuity, and boundary. Husserl participated in this project with Stumpf and other students of Brentano such as Meinong.[(13)] Significantly, the 1906 paper on "The Origins of the Concept of Space", in which Riesz first formulated the closure axioms at the heart of topology is in fact a contribution to formal phenomenology, a study of the structures of *spatial presentations*, in which the attempt is made to specify the additional topological properties which must be possessed by a mathematical continuum if it is adequately to characterize the continuity and order properties of our experience of space.

8. Lewin's Topological Psychology

Of all the precursors of contemporary applications of topology in cognitive science, the most notorious is the work on "topological and vector psychology" of the German Gestalt psychologist Kurt Lewin. It will suffice for our present purposes merely to illustrate some of the ways in which Lewin uses topological notions in his *Principles of Topological Psychology* of 1936.

Lewin begins with the opposition *thing* (intuitively: a closed connected unity) and *region* (intuitively: a space within which things are free to move). As Lewin points out, what is a thing from one psychological perspective may be a region from another: 'A hut in the mountain has the character of a thing as long as one is trying to reach it from a distance. As soon as one goes in, it serves as a region in which one can move about.' (1936, p. 116) He then defines the notion of a *boundary zone* z between two disconnected but proximate regions m and n, as the region, foreign to m and n, which has to be crossed in passing from one to the other. The whole $m + n + z$ is then *connected* in the topological sense. (1936, p. 121)

The concept of a *barrier* he defines as a boundary zone which offers resistance to passage of things between one region and another. Such resistance may be asymmetric; thus it may be greater in one dimension than in the opposite direction. Barriers effect the degree of communication between one region and another, or in other words the *degree of influence* of the *state* of one region on that of another region. Hence the notion of degree of influence, too, need not be symmetric: the fact that a is in a certain degree of communication with b does not imply that b is in equally close communication with a.

Two regions a and b are said to be parts of a *dynamically connected region* if a change of state of a results in a change of state of b. The notion of *dynamic connectedness*, too, is by what was said earlier a matter of degree. In fact we can distinguish a hierarchy of degrees of interlinkage between regions, and here Lewin echoes discussions in the Gestalt-theoretical literature of the notions of 'strong' and 'weak' Gestalten (1936, pp. 173f.). A strong Gestalt may be defined as a complex with a high degree of dynamic connectedness between its parts. Examples are: an organism, an electromagnetic field. A weak Gestalt, for example a chess-club or a crowd of onlookers, has a lesser but still non-zero dynamic connectedness between its parts, while a purely summative whole (an '*Und-Verbindung*' in Gestaltist terminology) is such that its separate units manifest a zero degree of dynamic connectedness. Interestingly, in light of our discussions of the two alternative motivations underlying topological theory above, the notions central to Gestalt theory can be defined not merely on the basis of the notion of dynamic connectedness, but also in terms of structure-preserving transformations.[(14)]

We have deliberately introduced the basic concepts of Lewin's topological psychology in a rather general way, abstaining from any specific applications to psychological matters. The notions are, as Lewin himself sometimes recognizes, *formal* in the sense that they can be applied indiscriminately to a wide range of different sorts of material regions. The general tendency of Lewin's writings, however, is to switch too unthinkingly to psychological applications, whereby his use of the concepts in question - concepts of 'barrier', 'path', '(psychological) locomotion', 'dynamic interdependence (as the main determinant of the topology of the person)', 'tension', 'resistance', 'inhibition', etc. - seems often to remain on the level of metaphor. Some cognitive scientists may be content never to pass beyond this level in their investigations. Lewin's critics, however, rightly drew attention to a certain crucial shortfall in his use of mathematical notions in his writings. As was correctly pointed out, Lewin rarely makes the mathematical theory of notions such as connectedness, boundary, separateness, and so on, do any substantial work within the framework of his investigations. This criticism was put forward in an influential article by London (1944), an article which did much to thwart the further development of topological psychology (or of a topologically founded cognitive science) as Lewin had conceived it. Yet London's critique is in some respects exaggerated, as is shown by the fact that certain aspects of Lewin's generalizations of standard topology have since shown themselves to be highly fruitful. These generalizations include:

1. the recognition that it is possible to construct topology on a non-atomistic, mereological basis which works in terms of *wholes (regions)* as well as parts - where London, a defender of the analytic method at the heart of physical science, holds that for scientific purposes 'all experience must be dealt with in bits' (p. 279);

2. the systematic employment of the notion of asymmetric boundary, a notion which turns out to be crucial in many cognitive spheres;

3. the employment of topological ideas and methods also in relation to finite domains of objects. London argues (pp. 288f.) that topology makes sense only in infinite domains; as Latecki (1992) and others have shown, however, it is possible to construct in rigorous fashion finitistic systems in which analogues of topological notions can be defined.

More recent developments have demonstrated that it is possible to go beyond the merely metaphorical employment of topological concepts in cognitive science and to exploit the formal-ontological properties of these concepts for theoretical purposes in a genuinely fruitful way. Topology can serve as a theoretical basis for a unification of diverse types of psychological facts.[(15)] Thus many of Lewin's ideas recall principles of 'force dynamics' worked out in greater sophistication in the linguistic sphere by Talmy (1988), and Talmy, along with Petitot and others, has demonstrated the importance of topology for the understanding of a variety of different sorts of linguistic structuring.[(16)] As Talmy notes, the conceptual structuring effected by language is illustrated most easily in the case of prepositions. A preposition such as 'in' is magnitude neutral (in a thimble, in a volcano), shape neutral (in a well, in a trench), closure-neutral (in a bowl, in a ball); it is not however discontinuity neutral (in a bell-jar, in a bird cage). Work on verb-aspect and the mass-count distinction, too, has profited from a topological orientation.[(17)]

Topological structures play a central role also in studies of naive physics, not least in virtue of the fact that even well-attested departures from true physics on the part of common sense leave the topology and vectorial orientation of the underlying physical phenomena invariant:[(18)] our common sense would thus seem to have a veridical grasp of the topology and broad general orientation of physical phenomena even where it illegitimately modifies the relevant shape and metric properties.

In talking somewhat grandly of 'topological foundations for cognitive science', now, we are contending that the topological approach yields not simply a collection of insights and methods in selected fields, but a unifying framework for a range of different types of research across the breadth of cognitive science and a common language for the formulation of hypotheses drawn from a variety of seemingly disparate fields. Initial evidence for the correctness of this view is provided not just by the scope of the inquiries referred to above, but also by the degree to which in different ways they overlap amongst themselves and support each other mutually.

One rationale behind the idea that the inventory of topological concepts can yield a unifying framework for cognitive science turns on the fact that, as has often been pointed out (see e.g. Gibson 1986), boundaries are centres of salience not only in the spatial but also in the temporal world (the beginnings and endings of events, the boundaries of qualitative changes for example in the unfolding of speech events: cf. Petitot 1989). Moreover, topological properties are more widely applicable than are those properties (for example of a geometrical sort) with which metric notions are associated. Metric features have certainly proved highly useful for the purposes of natural science. Given the pervasiveness of qualitative elements in every cognitive dimension, however, and also the similar pervasiveness of notions like *continuity, integrity, boundary*, *prototypicality*, etc., we can conjecture that topology will be not merely sufficiently general to encompass a broad range of cognitive science subject-matters, but also that it will have the tools to do justice to these subject-matters without imposing alien features thereon.

References

Ajdukiewicz, Kazimierz 1935 "Die syntaktische Konnexität, *Studia Philosophica*, 1, 1-27, English translation in S. McCall, ed., *Polish Logic, 1920-39*, Oxford: Clarendon Press, 1967, 207-301.

Asenjo, F. G. 1993 "Continua without Sets", *Logic and Logical Philosophy*, 1, 95-128.

Back, Kurt W. 1992 "This Business of Topology", *Journal of Social Issues*, 48 (special issue on: The Heritage of Kurt Lewin: Theory, Research, and Practice), 51-66.

Bochman, Alexander 1990 "Mereology as a Theory of Part-Whole", *Logique et Analyse*, 129-130, 75-101.

Bozzi, P. 1958 "Analisi fenomenologica del moto pendolare armonico", *Rivista di Psicologia*, 52, 281-302.

Bozzi, P. 1959 "Le condizioni del movimento `naturale' lungo i piani inclinati", *Rivista di Psicologia*, 53, 337-352.

Brandt, Per Aage 1989 "Agonistique et analyse dynamique catastophiste du modal et de l'aspectuel: Quelques remarques sur la linguistique cognitive de L. Talmy", *Semiotica*, 77, 151-162.

Brentano, Franz 1988 *Philosophical Investigations on Space, Time and the Continuum*, translated by Barry Smith, London/New York/Sydney: Croom Helm.

Casati, Roberto 1991 "Fusion", in H. Burkhardt and B. Smith (eds.), *Handbook of Metaphysics and Ontology*, Munich/Hamden/Vienna: Philosophia, 287-289.

Casati, Roberto and Varzi, Achille 1994 *Holes and Other Superficialities*, Cambridge, Mass. and London: MIT Press.

Desclès, Jean-P. 1989 "State, Event, Process, and Topology", *General Linguistics,* 29, 159-200.

Fine, K. 1995 "Part-Whole", in B. Smith and D.W. Smith (eds.), *The Cambridge Companion to Husserl*, Cambridge: Cambridge University Press, 463-485.

Galton, Antony 1984 *The Logic of Aspect. An Axiomatic Approach,* Oxford: Clarendon Press.

Gibson, J. J. 1986 *The Ecological Approach to Visual Perception*, Hillsdale, NJ: Lawrence Erlbaum.

Hammer, P. C. 1960 "Kuratowski's Closure Theorem", *Nieuw Archief voor Wiskunde*, 8, 74-80.

Hammer, P. C. 1962 "Extended Topology: Set-Valued Set-Functions", *Nieuw Archief voor Wiskunde*, 10, 55-77.

Hammer, P. C. 1962a "Extended Topology: Additive and Sub-Additive Subfunctions", *Rendiconto del rcolo Matematico di Palermo*, Ser. 2, 11, 262-70.

Hammer, P. C. 1967 "Language, Approximation, and Extended Topologies", in I. Rauch and C. T. Scott (eds.), *Approaches in Linguistic Methodology*, Madison: University of Wisconsin Press, 33-40.

Hammer, P. C. 1969 "Continuity", in P. C. Hammer (ed.), *Advances in Mathematical Systems Theory*, University Park and London: The Pennsylvania State University Press, 93-105.

Hoeksema, J. 1985 *Categorial Morphology*, New York/London: Garland Publishing.

Holenstein, Elmar 1975 *Roman Jakobson's Approach to Language,* Bloomington and London: Indiana University Press.

Husserl, Edmund 1900/01 *Logische Untersuchungen*, 1st ed., Halle: Niemeyer, 2nd ed., 1913/21 (both now available in a comparative edition as *Husserliana* XVIII-XIX, The Hague: Nijhoff, 1975, 1984). Page references unless otherwise indicated are to Findlay's English translation of the 2nd ed. (*Logical Investigations*, London: Routledge, 1970), translation amended where necessary.

Husserl, Edmund 1983 *Studien zur Arithmetik und Geometrie. Texte aus dem Nachlass, 1886-1901,* The Hague: Nijhoff (*Husserliana* XXI).

Jackendoff, Ray 1991 "Parts and Boundaries", *Cognition*, 41, 9-45.

Kuratowski, Kazimierz 1922 "Sur l'opération A^- d'analysis situs", *Fundamenta Mathematica*, 3, 182-99.

Lakoff, George 1989 "Some Empirical Results about the Nature of Concepts", *Mind and Language*, 4, 103-129.

Latecki, Longin 1992 *Digitale und allgemeine Topologie in der bildhaften Wissensrepräsentation*, St. Augustin: infix.

Leeper, Robert W. 1943 *Lewin's Topological and Vector Psychology. A Digest and a Critique*, Eugene: University of Oregon Press.

Lewin, Kurt 1936 *Principles of Topological Psychology*, New York and London: McGraw-Hill.

London, Ivan D. "Psychologists' Misuse of the Auxiliary Concepts of Physics and Mathematics", *Psychological Review*, 51, 266-91.

McCloskey, M. 1983 "Intuitive Physics", *Scientific American*, 248(4), 122-130.

Meinong, Alexius von 1903 "Bermerkungen über die Farbenkörper und das Mischungsgesetz", *Zeitschrift für Psychologie und Physiologie der Sinnesorgane*, 33, 1-80, and in Meinong, *Gesamtausgabe*, vol. I, 497- 576.

Mostowski, Marcin 1983 "Similarities and Topology", *Studies in Logic, Grammar and Rhetoric*, 3, 106-119.

Mourelatos, A. P. D. 1981 "Events, Processes and States", in P. J. Tedeschi and A. Zaenen, eds., *Tense and Aspect* (*Syntax and Semantics*, Vol. 14), New York: Academic Press, 191-211.

Netzer, N. 1977 "Konvergenz in verallgemeinerten topologischen Räumen", *Mathematische Nachrichten*, 77, 245-62.

Netzer, N. 1978 "Verallgemeinerte topologische Strukturen", *Jahrbuch Überblicke Mathematik*, 1978, 87-106.

Nöbeling, G. 1954 *Grundlagen der analytischen Topologie,* Berlin: Springer.

Ore, O. 1943 "Some Studies on Closure Relations", *Duke Mathematical Journal*, 10, 761-85.

Petitot, Jean 1989 "Morphodynamics and the Categorial Perception of Phonological Units", *Theoretical Linguistics*, 15, 1989, 25-71.

Petitot, Jean 1992 "Natural Dynamical Models for Natural Cognitive Grammar", *International Journal of Communication*, 2, 81-104.

Petitot, Jean 1992a *Physique du Sens*, Paris: Editions du CNRS.

Petitot, Jean 1994 "Phenomenology of Perception, Qualitative Physics and Sheaf Mereology", in R. Casati, B. Smith and G. White (eds.), *Philosophy and the Cognitive Sciences*, Vienna: Hölder-Pichler-Tempsky, 387-408..

Riesz, F. 1906 "Die Genesis des Raumbegriffs", *Mathematische und naturwissenschaftliche Berichte aus Ungarn*, 24, 309-53.

Riesz, F. 1909 "Stetigkeitsbegriff und abstrakte Mengenlehre", *Atti del IV Cong. Int. dei Mat.*, II, Rome, 18-24.

Rubin, E. 1921 *Visuell wahrgenommene Figuren. Studien in psychologischer Analyse,* Copenhagen: Gyldenalske.

Simons, Peter M. 1987 *Parts. A Study in Ontology*, Oxford: Clarendon Press.

Simons, Peter M. 1988 "Gestalt and Functional Dependence", in Smith (ed.), 158-190.

Smith, Barry 1997 "Boundaries: An Essay in Mereotopology", in L. H. Hahn (ed.), *The Philosophy of Roderick Chisholm* (Library of Living Philosophers), Chicago and LaSalle: Open Court, 1997, 534-561.

Smith, Barry (ed.) 1982 *Parts and Moments. Studies in Logic and Formal Ontology*, Munich: Philosophia.

Smith, Barry (ed.) 1988 *Foundations of Gestalt Theory*, Munich and Vienna: Philosophia.

Smith, Barry and Casati, Roberto 1994 "Naive physics", *Philosophical Psychology*, 7/2, 225-244.

Stumpf, C. 1873 *Über den psychologischen Ursprung der Raumvorstellung,* Leipzig: Hirzel.

Talmy, Leonard 1977 "Rubber-Sheet Cognition in Language." In: *Papers from the 13th Regional Meeting*, *Chicago Linguistic Society*. W. Beach, *et al.* (eds.), University of Chicago.

Talmy, Leonard 1983 "How Language Structures Space", in H. Pick and L. Acredolo, eds., *Spatial Orientation: Theory, Research, and Application*, New York: Plenum Press, 225-82.

Talmy, Leonard 1988 "Force Dynamics in Language and Cognition", *Cognitive Science*, 12, 49-100.

Varzi, Achille C. 1994 "On the boundary between mereology and topology", in R. Casati, B. Smith and G. White (eds.), *Philosophy and Cognitive Science. Proceedings of the 16th International Wittgenstein Symposium*, Vienna: Hölder-Pichler-Tempsky, 419-438.

Wildgen, W. 1982 *Catastrophe Theoretic Semantics. An Elaboration and Application of René Thom's Theory*, Amsterdam: John Benjamins.

Zarycki, Miron 1927 "Quelques notions fondamentales de l'Analysis Situs aux point du vue de l'Algèbre de la Logique", *Fundamenta Mathematica*, 9, 3-15.

Endnotes

1. Mostowski 1983.

2. Simons 1987, Varzi 1994.

3. See the works by Ore, Hammer, Nöbeling, Netzer in the list below.

4. Hammer 1962, p. 65.

5. See Bochman 1990.

6. Brentano 1988, Asenjo 1993, Smith 1987.

7. See Smith (ed.), 1982.

8. Simons 1987, Ch. 1.

9. Ajdukiewicz 1935, Holenstein 1975.

10. See Fine 1995.

11. Casati 1991, Petitot 1994.

12. See Husserl 1983, pp. 83f, 95, 413, etc. and compare §§22 and 70 of the Prolegomena to the *Logical Investigations*.

13. See Husserl 1983, pp. 275-300, 402-410; Stumpf 1873; Meinong 1903, esp. §2: "Farbengeometrie und Farbenpsychologie", and §5: "Der Farbenraum und seine Dimensionen".

14. Simons 1988.

15. Cf. Back 1992, p. 52.

16. See for example Jackendoff 1991, Lakoff 1989, Petitot 1992, 1992a, Wildgen 1982

17. See e.g. Mourelatos 1981, Galton 1984, Hoeksema 1985, Desclès 1989, Brandt 1989.

18. Bozzi 1958, 1959, McCloskey 1983, Smith and Casati 1994.