

Barry Smith, Editor

Parts and Moments
Studies in Logic and
Formal Ontology

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Preface

The present volume had its origin in seminars on phenomenology organised by Wolfe Mays in Manchester between 1973 and 1976. A constantly recurring theme of these seminars, propounded initially by Kevin Mulligan, was the importance to phenomenology, and to philosophy in general, of the theory of part and whole sketched by Husserl in the 3rd Logical Investigation. The Manchester seminars led in 1977 to the formation of the Seminar for Austro-German Philosophy. Our ideas on whole and part were much clarified, in particular, by a meeting of the Seminar held in Sheffield in May 1978 on *Whole-Part Theory and the History of Logic*, at which papers were read by Peter Simons, Prof. C. Lejewski, Ivor Grattan-Guinness, Prof. F. G. Asenjo and Wolfgang Degen. We are grateful to Degen, in particular, for invaluable inspiration, and to Prof. R. M. Chisholm, Hans Burkhardt, Ignacio Angelelli and Herman Philipse for their criticism and encouragement.

The Seminar for Austro-German Philosophy owes its existence to the kind support of Prof. P. H. Nidditch of the Department of Philosophy of the University of Sheffield. I should like to express my thanks also to the Faculty of Arts of that University for the grant of a research fellowship from 1976 to 1979, during which period my own contributions to this volume were written.

B. S.

Manchester, February 1981.

Pieces of a Theory

§ 1 From Aristotle to Brentano

The whole, that which is compounded out of something, is one, not like a heap, but like a syllable. Now the syllable is not its elements; *ba* is not the same as *b* and *a*, nor is flesh the same as fire and earth (for when these are separated the wholes, i. e. the flesh and the syllable, no longer exist, but the elements of the syllable exist, and so do fire and earth); the syllable, then, is something – not only its elements (the vowel and the consonant) but also something else, and the flesh is not only fire and earth, or the hot and the cold, but also something else.

Metaphysics, 1041^b 12, trans. Ross, slightly amended.

1.1 There is an Averroist saying to the effect that all sciences are perfect insofar as Aristotle treated of them. This applies not least to the formal ontological theory of wholes and parts developed by logicians, psychologists, phenomenologists and others in the last hundred years, and it is the contrast Aristotle has in mind in the passage above, between a heap and a whole or unity in the strict sense, which will be our principal concern in the pages to follow. We shall find that the opposition is by no means a simple one; that even the notion of heap, of a merely additive or summative whole, comprehends a number of quite different notions, and that the specification of the various kinds of unified whole and of the relations between them is by no means easy.

Consider, for example, the relations of part and whole encountered in the discipline of chemistry. It will be readily accepted that sub-atomic particles (*Teilchen*, in German) are in some sense constituents of atoms,

which are in turn constituents of the molecules which constitute the various things and stuffs making up the furniture of the material world. Similarly, in the field of linguistics, it will be acknowledged that individual phonemes are in some sense constituents of spoken words and phrases, that these are in turn constituents of sentences in conversations, narratives, arguments, and so on. The question of the nature of physical and chemical bonds played an important role in the work of philosophers from the 17th to the 19th centuries. Contemporary philosophers, in contrast, have tended to focus their attention on linguistic bonds, on the question how words and phrases are joined together to make a sentence.¹ It is however interesting to note that the answer to this question proffered by both Wittgenstein and Frege involves appeal to a theory of entities conceived as standing in need of completion (or *saturation*) in ways which call to mind the valency-rules of atomic theory.

In the present paper we wish to consider the possibility of a purely general theory of part-whole relations. It will be objected that the expectation of producing a formal ontological theory of such relations is a spurious one. For even if it is admitted that scientists (chemists, linguists, literary theorists, . . .) do in fact investigate such relational structures, still, the kinds of relations treated by each such discipline will surely be materially different, in drastic and unforeseeable ways, from those treated by every other discipline. Thus it would seem that we could hope for very little from a merely formal analysis of superficial similarities between individual cases, selected more or less at random.

Certainly much of the literature on wholes and parts – the writings of those scholastics, for example, who attempted to systematise Aristotle's remarks in this area, or the works of 19th century holistic biologists and social theorists – has failed to achieve more than a flaccid, analogical character. One is reminded of Ryle's criticism of Plato's discussion of the 'Greatest kinds' in the *Sophist* as consisting of little more than

puddings of verbal and other abstract nouns, together with opaque metaphors like 'participate', 'merge', 'blend', and 'pervade', with nothing to indicate whether we are to translate these culinary metaphors (1) in terms of 'ingredient' and 'compound' or (2) in terms of 'if' and 'therefore', that is, whether they stand for (1) relations between what can be named or (2) relations between what can be said (1960, p. 69 f).²

It could be objected further that many apparent ingredient/component/constituency relations in the object-realms of particular scientific

disciplines are in themselves nothing of the kind. In the case of the relation between atoms and molecules, for example, it might be held that the first term of the relation exists only as a theoretical construct within the framework of some physico-chemical theory. Similarly (and indeed *a fortiori*) sub-atomic particles are not *parts* of atoms, except in some highly generalised sense of 'part' whose credentials might seem dubious.

It is hoped that these and similar objections may be put to rest in the remainder of this work. Briefly, we hope to show that the case for introducing precisely such a generalised sense of the term 'part' was rigorously made by a group of philosophers working in Austria and Germany in the later 19th and early 20th centuries. They argued that the term should be recognised as comprehending not merely *extensive parts* – pieces, fragments, phases, portions, constituents – but also non-extensive *aspects, features or moments*. And they presented a system of *a priori* formal laws governing the relation of part to whole as thereby determined, the implications of which have still fully to be appreciated.

1.2 The thinkers who made the most important contributions to this theory centred around Franz Brentano, and from him they inherited a system of philosophical and psychological insights rooted in Aristotelian and scholastic ideas. Hence it will be useful, before moving on to give an account of the theory, if we provide a brief survey of the relevant treatments of part-whole relations in this tradition.

There are a number of different senses in which one thing can be said to be 'in' another. Aristotle himself (*Physics* IV 3 210–24 a 14) distinguished eight such *modes of being in*. They are formulated by Peter of Spain as follows:

According to the first mode of being in, something is said to be in something, as an integral part is in its whole, e. g. a finger is in a hand, a wall is in a house.

According to the second mode, an integral whole is in its parts, e. g. a house is in its wall, roof and foundations.

According to the third mode, a species is in a genus, e. g. man is in animal.

According to the fourth mode, a genus is in a species, e. g. the animal is in the man.

According to the fifth mode, a form is in a matter. [*But this fifth mode was subdivided by Boethius into two modes.*] because there are both (1) substantial forms, e. g. the soul is the substantial form to man, and on the other hand also (2) accidental forms, e. g. the white[ness] of [a] man. Now the former is properly said to be in, as the form is in the matter, e. g. the soul in the body; the latter is said

to be in, as the accident is in the subject, e. g. the whiteness in a wall, colour in a body.

According to the sixth mode, something is in a prime cause, e. g. a reign in a regent.

According to the seventh mode, something is in an end or goal, e. g. virtue is in happiness.

According to the eighth mode, something is in a container, and generally what is placed is in a place (*Tractatus* III 2, trans. W. Degen).

Not all of these modes correspond strictly to part-whole relations in the sense propounded here, though in any fully adequate treatment of such relations it would be necessary to give some account of all of them, and of the associated issues in Aristotelian ontology (above all the theories of species and genus, matter and form).³ The importance for our present purposes of Aristotle's first mode, the relation of an integral part to its whole, will become clear only later. Let it suffice here to point out that the combinatorial approach to logic, brought to successive degrees of perfection by Leibniz and Boole and later by Husserl, Leśniewski and Ajdukiewicz (see § 5 below), stems from insights of scholastic grammarians concerning the nature of integral wholes in language and the fitting together of sentences and their parts.⁴

Of much more immediate importance is Aristotle's fifth mode – or the relation of an individual accident to the substance in which it inheres. To fix our ideas we shall present one specific example of such inherence in some detail. Imagine a sheet of glass, *s*, which is uniformly red in colour. '*s* is red' is true, according to the theory of individual accidents, in virtue of the possession by *s* of a certain individual redness, *r*, the relation of inherence between *r* and *s* being understood as a specific type of relation of part to whole. As was stressed already by Aristotle (*Cat* I a 24–25) *r* is not a part of *s* in any of the familiar senses of 'part' (as 'piece', 'component', 'phase', etc.). The thesis that *r* is part of *s* involves an appeal to precisely that generalised sense of 'part' referred to above.

The term 'accident' is correlative with the Aristotelian 'substance'. The latter designates both mental substances (souls) and material substances: spatio-temporally extended, unified wholes which endure, are self-subsistent, and can admit contrary properties (can be, alternately, red and green, in virtue of the inherence of contrary accidents). Thus a material substance is an entity such as a sheet of glass, a human being, the head of a human being, a living cell, a molecule, perhaps also – to depart even further from Aristotle's original range of examples – a batalli-

on or division in an army, a university, a nation state, a planetary system.

Each substance is structured internally in some specific way (has what, in classical terms, would be called an essence or nature). The structures characteristic of liquid or solid consignments of inorganic matter are clearly radically distinct from, say, the structures of orbital systems, which differ in turn from the structures of, say, living organisms. All substances however are, according to their specific natures, able to acquire and lose accidents without detriment to their continued existence. Just as a human being may suffer from a headache, so an army platoon may suffer from low morale. A human being may acquire, and later lose, a knowledge of some local dialect; an army platoon may acquire, and later lose, a knowledge of some local terrain.

An accident (of a material substance) is e. g. a redness, a fall, a blush, a whistle, a salute, a specific waving of a flag; the specific taste of this piece of cheese at present on my tongue; the specific shape existing in this particular chair for the interval of time that I am presently sitting upon it; a splitting of an amoeba; a specific, concretely existing acquired knowledge, habit or skill; a specific, concretely existing disease or packet of diseases. These are all cases of accidents inhering in what might be called substances of lower order (non-institutional, non-collective substances). Examples of accidents inhering in higher order substances would be a symphony performance (inhering in a complex whole consisting of an orchestra together with a certain consignment of air-molecules and perhaps other associated objects); a state of being at war (inhering in a country); the light of Venice; the pace of Manhattan; a depression over the Atlantic.

A number of distinctions can be made amongst accidents, along various axes. They are first of all to varying degrees *measurable*. A university examination, for example, may be conceived as a device for measuring an individual knowledge of Greek. Those accidents which form one-dimensional manifolds (Gilman, 1892) may be subject either to a cardinal metric, when they exhibit what, in the tradition, were called extensive magnitudes, or to an ordinal metric (e. g. on an order of preferences), when they exhibit intensive magnitudes. We postpone further discussion of these issues to a future paper.

A distinction can be drawn, secondly, between relatively simple and relatively complex accidents. A battle, for example, consists of a number of relatively simple constituents (individual missile-movements, falls, deaths, whistles, etc.) interwoven together in complex ways; a mili-

tary procession consists similarly of a number of salutes, marchings, bangs, flyings of flags, etc. Some accidents (reddenings, falls, salutes) manifest positional and/or qualitative changes, are what we can term *dynamic* accidents. A redness, a shape, a being seated, in contrast, are all *static* accidents. A static accident is sometimes called a (concrete or individual) *property, state or condition*; a dynamic accident is sometimes called a *process or event*.⁵

Accidents may be further classified into relational and nonrelational. Substances are structured in specific ways, not only internally (in virtue of their essence or nature and of the accidents inhering in them and the internal relations among these accidents), but also externally: i. e. there are external relations (of causality, for example) between one substance and another. These external relations, too, insofar as they inhere in concrete individual substances for specific intervals of time, are individual accidents, but accidents inhering in more than one substance simultaneously. Examples of dynamic relational accidents are: a kiss, a hit, a throw; of static accidents: a specific relation of ownership or of being married, a specific magnetic or gravitational attraction or contractual tie existing within a given interval of time between two bodies. (Here the laws of physics and the laws of contract are seen to relate to entities which are, ontologically speaking, of the same form.) Relational accidents clearly raise difficult problems for the part-whole account of inherence presented above.

1.3 Historically the theory of individual accidents can be recognised as forming one important root of Leibniz's monadology – which rests on the denial of the existence of relational accidents (a denial which had been accepted as orthodoxy by the majority of scholastic philosophers up to Leibniz's day).⁶ The theory of substance and accident formed a constantly recurring element in the ontological theories developed by subsequent generations of German philosophers outside the immediate orbit of Kant and Hegel (one thinks particularly of Trendelenburg, Beneke, Drobisch and Julius Bergmann). It can be recognised also in the semantics underlying Bolzano's *Wissenschaftslehre* where the form of the elementary Bolzanian *Satz an sich* is that of 's has r', in the sense in which Socrates has, say, a specific individual whiteness, intelligence, or headache.⁷

What distinguishes Bolzano from his predecessors, however, is that his treatment of the substance-accident relation forms part of a more

general treatment of relations of part and whole. Thus §§ 80 and 81 of the *Wissenschaftslehre*, on 'Ideas of Attributes and Relations' and 'Ideas of Matter and Form', are followed by a considerable discussion (§§ 82–87) of the ideas of *class* (and in particular the idea of a *class of individually listed objects*), of *set* (as a class where the manner of connection between elements is not specified), of *sum* (as a set the parts of whose parts are themselves parts of the whole), of *sequence*, *unity*, *manifold*, *totality*, and *finite and infinite quantity*.⁸

This account of different types of whole influenced Twardowski, and through him had an impact also on the Polish school of logic.⁹ The style and content of Bolzano's account suggests that there was an influence also upon Husserl, whose *Logical Investigations* will play a prominent role in the discussions to follow.¹⁰

A second connecting link between classical substance-accident theory and 20th century philosophy is Brentano's work in logic and ontology, especially as collected in the *Kategorienlehre* (1933). The influence of classical metaphysics makes itself felt in the work of Brentano's students in the fields of logic and theoretical psychology, and it is in these works that almost all of the more recent interest in and investigation of the relations of part and whole had its mediate or immediate origin.¹¹

A recurrent theme of scholastic metaphysics was the idea that the network of concepts and theses of a scientific discipline should approximate to the ideal of being a representationally adequate mirror of the universe. This idea finds its clearest expression in Leibniz's project of a *lingua characteristica*, but it can be recognised also in Bolzano's and Husserl's accounts of logic,¹² and even in the theory of integral wholes, of the *sich zueinander Verhalten* of simple objects in states of affairs and of terms in propositions, sketched by Wittgenstein in the *Tractatus*. There is, in each case, an appeal to a parallelism between the way in which words, ideas or concepts are conceived as being integral to the sentence, thought or scientific theory in the nexus of which they acquire their meaning, and the way in which corresponding objects and attributes are linked together in the realm of reference. In the *Tractatus*, in particular, an analogy is suggested between the interweaving of syntactic units and the manner in which machine or bodily parts are integral to the machine or body for which they have been tooled.¹³

Ontological ideas of this kind were however kept alive in German universities also by another route. To an extent as yet insufficiently appre-

ciated, the study of *Jura*, and particularly of the Roman law, served to carry forward into the 19th and early 20th centuries many crucial elements of scholastic realism, including the ontology of substance and accident. The German civil law codes were framed, quite explicitly, as standing in a relation of projection to an independently existing legal order, the individual sentences of the code mirroring corresponding legal complexes in the world of human behaviour.¹⁴

Just as a sentence, if it is to be a sentence, must as a matter of *a priori* necessity contain a verb, so there are *a priori* relationships of necessitation in the world of human behaviour, determining that, for example, a mental process of premeditation is integral to an act of murder. This is true not only in the sense that the former is seamlessly reticulated with the residue of the latter (and with events, such as the death of the victim, ensuing upon it), but also in the sense that it is indispensable to it: the existence of the act (*qua* act of murder) is dependent, as a matter of necessity, upon the existence of the process of premeditation.

It is not only *a priori* relationships of *necessitation* that can be distinguished amongst actual and possible elements of legal states of affairs however. We can distinguish also relations of *exclusion* (an act of theft, for example, excludes of necessity the obtaining of a prior relation of ownership between the thief and his booty; an act of kindness excludes, of necessity, a moment of dishonest intent); of *compatibility* (an act of contracting for gain to do Φ is of necessity compatible with a prior intention to do Φ); of *indifference* (that *a* has been murdered is of necessity indifferent to the fact that he would in any case have died of cancer at the moment of his death); of *overlapping* (of, say, burglary, and trespass); and perhaps also relationships of a kind whereby one element establishes an *a priori tendency* toward the realisation of another element (an act of promising, for example, brings about a tendency toward the realisation of its content).¹⁵

The goal of providing a completely adequate catalogue of the components of legally relevant states of affairs, of their essential possibilities of combination, and of the types of *Rechtsverhältnisse* thereby constituted, has been held in common by many of the classic works of German legal theory.¹⁶ For our present purposes it is Jhering's *Geist des römischen Rechts*¹⁷ which is of greatest relevance. Jhering propounds in this work – in complete conformity with the scholastic idea of language as a mirror of the world – what he calls an 'alphabet' of possible elements of *Rechtsverhältnisse*. These elements are to be conceived as being reticulated

with entities which are not intrinsically legal (e. g. movements, and other basic actions) in complex ways, corresponding to our intuitions of the juridical world as a conceptually though not materially isolable stratum of reality. A juridical *lingua characteristic*a would make possible the direct depiction of these elements as standing in combination with each other and with extra-legal entities in such a way as to constitute integral wholes of varying types.

The most important task for any theory of integral wholes is that of providing an account of how the relevant elements are fitted together (the problem associated in the British idealist tradition with the concept of internal relation, and in the writings of Frege with the concept of saturatedness). In his treatment of this issue Jhering comes surprisingly close to the solution which was, as we shall see, developed within the Brentano school as part of a general formal ontological theory of part-whole relations. He distinguishes two sorts of component parts of legally relevant integral wholes, which he designates as elements of *local* and of *abstract applicability* respectively:

The former can be designated as self-subsistent or concrete legal substances [*Rechtskörper*], for as concrete entities they can occur in our experience without any additional supplement. Examples would be a bill of sale, a [*deed specifying a*] right of way, a will. The abstract constituents of the alphabet, in contrast, never come into our experience of themselves alone, just as little as does a property [*Eigenschaft*], but always in and of specific independent items. Examples are provided by error, invalidity, delay. An error as such, i. e. independently of any concrete *Rechtsverhältnis*, a delay as such, i. e. without any relation to an actually existing obligation, is a practical absurdity; both can occur only as bound up with self-subsistent substances. The abstract elements have an incomparably wider applicability, since they are not bound up with a specific type of relationship; error, for example, can occur in relation to a contract, a custom, a payment, a legacy, and so on. The self-subsistent elements, in contrast, relate always to quite specific types of relationships.¹⁸

The *elements* of this alphabet come together, in the world, to make legal complexes, sequences of states of affairs within the orbit of the law (as when, say, an *error* in a *contract* causes *delay* in the *execution* of some *commission*, which gives rise in turn to some specific *action* in the civil courts). Interrelationships of this kind between elements are then mirrored, in the court, by a parallel intermeshing of linguistic proxies of these same elements in the utterances of the lawyers. The legal drama in which the lawyers are engaged thereby constitutes a *picture* of the underlying substantive issue.¹⁹

1.4 That strand in the history of the theory of wholes and parts which has been most adequately treated in the literature is the theory of extensive wholes developed initially by Boole, Peirce, Schröder and their followers and further refined by Leśniewski and his school and by American logicians such as Leonard and Goodman. This work grew out of attempts by 18th century logicians and mathematicians to subject to algebraic treatment the relations amongst concepts or concept-extensions (species-genus relations) and of work on the general theory of magnitudes by, for example, the Grassmanns, Riemann and Helmholtz. Work on the algebra of logic evolved in tandem with Cantor's and Dedekind's early investigations in the theory of sets, as well as with work by Frege and Russell on concepts, concept-extensions, functions and relations.²⁰

Unfortunately it is the latter, set-theoretical experiments which have formed the almost exclusive concern of modern philosophical logic. This entrenched position of set theory has been established partly as a result of the fact that the early growth of set/class/*Wertverlauf* theory was associated with the development of many of the most crucial advances in modern mathematical logic. It reflects also the fact that set theory is considerably more interesting from the point of view of the mathematician than the theory of extensive wholes which was, in Schröder's day, its principal competitor. Set theory has indeed subsequently come to be associated with purely mathematical advances, for example in topology.

This mathematical superiority of set theory unfortunately gave rise to the tacit assumption in the minds of philosophers that set theory is also *ontologically* superior (is possessed of greater descriptive adequacy). And this assumption was reinforced by the fact that the language of sets does indeed possess greater expressive power, even in extra-mathematical contexts, than does the language of extensive wholes. Thus it is impossible, in Schröderian, to express the distinction between *inclusion* of, say, the totality of Stagirites in the totality of Greeks, and *membership* of, say, Socrates, in the same totality, a distinction which is very economically expressed in the language of sets by the distinction between ' \subseteq ' and ' \in '.²¹ Nor is it possible to capture in Schröderian the distinction between arbitrary aggregates (say the totality of left hands of soldiers in the German army in 1900) and unified wholes in the proper sense (e.g. the German army itself); this distinction can be expressed in set-theoretical terms, though only by appeal to assumptions which are *prima facie* counterintuitive (e.g. that soldiers are sets of their bodily parts).

What is certainly not the case, however, in contradiction to the tacit assumption of modern philosophical logic, is that *all* ontologically important distinctions can be captured – whether naturally or unnaturally – within the set-theoretical framework: the language of sets is *not* an adequate basis for a *lingua characteristic* in the sense demanded of a realistic, descriptively adequate, formal ontology. The claim advanced here²² is that it is possible to develop an ontologically more adequate formal language, including as one fundamental component a theory of (extensive and non-extensive) part-whole relations, the underlying logic of which would have a degree of mathematical interest and sophistication at least comparable to those parts of orthodox set theory which are today employed by analytic philosophers, e. g. in work in the semantics of natural languages.

We shall have occasion in the sequel to return to this critique of the descriptive or representational adequacy of formal-ontological theories founded on a set-theoretical basis. Note that early philosophical proponents of set theory such as Russell at the time of the *Principles of Mathematics* were still, in almost all cases, concerned to develop realistic (descriptively adequate) formal ontological theories: the \in - and \subseteq -relations were held to correspond to distinct relations amongst entities existing independently of the theory. Only subsequently, particularly with the gradual recognition of an apparently irremovable arbitrariness in the proposed resolutions of the Russell paradox, did there occur a retreat to an essentially pragmatic conception of the formal logical machinery of sets, as a device for *simulating* important (or sometimes merely readily simulable) ontological relations in the world.²³ Philosophical proponents of a theory of wholes and parts, in contrast, even of the comparatively weak theory of extensive wholes, have retained the classical idea of formal ontology as a representationally adequate mirror of the world.

§ 2 Stumpf's Theory of Psychological Parts

2.1 The most common objections to the project of a general theory of part-whole relations rest on an illegitimate restriction to some one sense of the term 'part' – usually the extensive sense in which, e. g., one surface or mass is said to be part of a second, more inclusive surface or mass. It is then pointed out that the resultant theory is too weak to capture, for ex-

ample, the distinction between the sense in which Styria is part of Austria and the sense in which the land-area of Austria to the east of Vienna is part of Austria; or the sense in which 'th' or 'tf' are parts of the word 'thoughtful' and that in which 'thought' is a part of this word. Our claim is that a general theory of part and whole can indeed capture such distinctions. To this end however – as already intimated – it is necessary to recognise the extreme generality of the term 'part', a generality within the compass of which the requisite distinctions will be capable of being made.

To be more specific, we shall take the term to comprehend not only what we shall call *pieces*, i. e. detachable or separable parts (in a sense of these terms to be made clear below), but also (non-detachable or inseparable) *moments*. The notion of moment, of an existentially dependent entity (an entity which, as a matter of necessity, cannot exist except as a constituent part of some more inclusive whole) is an outgrowth of the Aristotelian *accident* discussed above. It can be distinguished in various forms in the works of many philosophers, even of those who on no occasion concerned themselves explicitly with the classical theory of substance and accident. In almost all cases, however, talk of recognising moments of a thing has been regarded as pertaining not to any structural features on the side of the thing itself, but only to features of our cognitive access to it.

The notion occurs, in particular, in the psychological writings of the British empiricists, especially in their treatments of the (epistemological) problem of abstraction.²⁴ Consider, for example, the following passage from Berkeley:

They who assert that figure, motion, and the rest of the primary or original qualities do exist without the mind in unthinking substances, do at the same time acknowledge that colours sounds, heat, cold, and suchlike secondary qualities, do not – which they tell us are sensations existing in the mind alone, that depend on and are occasioned by the different size, texture, and motion of the minute particles of matter. This they take for an undoubted truth, which they can demonstrate beyond all exception. Now if it be certain that those original qualities are inseparably united with the other sensible qualities, and not, even in thought, capable of being abstracted from them, it plainly follows that they exist only in the mind. But I desire any one to reflect and try whether he can, by any abstraction of thought, conceive the extension and motion of a body without all other sensible qualities. For my own part, I see evidently that it is not in my power to frame an idea of a body extended and moving, but I must withal give it some colour or other sensible quality which is acknowledged to exist only in the mind. In short, extension, figure, and motion, abstracted from all other qualities, are in-

conceivable. Where therefore the other sensible qualities are, there must these be also, to wit, in the mind and nowhere else. (*Treatise*, § 10).

It occurs also in various treatments of the problem of abstraction by continental thinkers influenced by the British psychological tradition (for instance Lotze, Lipps and Külpe). At the centre of all such treatments is a view of the concrete/abstract opposition resting on the idea that concrete entities are in some sense thinkable or presentable *in isolation*, abstract entities, in contrast, thinkable or presentable only as bound up with or in association with other entities.

It was under the combined influence of such psychological views and of the scholastic ideas preserved in classical German legal theory and in the works of philosophers such as Trendelenburg, Herbart, Beneke, Drobisch, Lotze and above all Brentano,²⁵ that there issued the beginnings of a systematic clarification of the notions abstract/concrete, separable/inseparable, etc. This crystallised in 1873 in a work whose significance for the subsequent investigation of the part-whole relation can hardly be underestimated. The work, entitled *On the Psychological Origin of the Presentation of Space*, was by Carl Stumpf, a student of Brentano and Lotze who became one of the pioneers of the discipline of experimental psychology in Germany, making fundamental contributions particularly to the psychology of sound.²⁶ Stumpf's monograph, which grew out of an unfinished history of the concept of substance, consists of a thorough historical survey and systematic criticism of previous treatments of the problem of abstraction in German and British psychological writings on the theory of visual perception (particularly on the perception of space and of spatial relationships), culminating in Stumpf's own proposed resolution of the problem. This appears in § 5 of the work, significantly entitled "The Theory of Psychological Parts". Here Stumpf considers the nature of the relation, in our presentations, between *space* and what he calls *quality* (i. e. the qualitative data of visual perception, specifically of colour). That is, he considers "*wie sich Raum und Qualität in der Vorstellung zueinander verhalten*" (Stumpf, 1873, p. 107). To answer this question, he argues, it is necessary to recognise that mental contents fall into two classes, which he calls *self-subsistent* and *partial contents* (*selbständige Inhalte und Teilinhalte*) respectively:

Independent contents are present there, where the elements of a complex of presentations could also in virtue of their nature be presented separately; partial contents where this is not the case (p. 109).

Thus for example

one cannot present to oneself a colour hue without some brightness or other, or a motion without some velocity or other . . . and it is evident to anyone who makes the attempt that it is impossible that we could either present to ourselves extension without colour or colour without extension (loc. cit.).²⁷

The intensity of a tone is not something indifferent to its quality: the intensity cannot be held as it is while the quality varies at will, or is allowed to vanish. The two are “in their nature inseparable, they in some manner compose a total content of which they are merely partial contents” (p. 113).

Thus Stumpf’s answer to the question how space and quality *sich zueinander verhalten* is that they are mutually correlated partial contents, i. e. that they are such that according to their nature they cannot exist in presentations in separation from each other: “some space or other is given immediately with and in every presentation of quality” (p. 115). Moments of colour and extension, or the constituent moments of hue, brightness and saturation in a colour-datum,²⁸ or the constituent moments of pitch, timbre and loudness in a musical sound, are not self-subsistent atoms which somehow become (or as a matter of fact already are) glued or bonded together by association,²⁹ but entities of a quite new type, perhaps not hitherto investigated for their own sake.

This view is not without problems however. For the impossibility of separate presentation, insofar as this can be determined by a given subject or group of subjects, does not of itself suffice to demarcate an objective character of ‘partiality’ or non-self-subsistence of contents. Whilst, as Stumpf himself recognised, if we succeed in presenting to ourselves two contents in separation, then these are indeed self-subsistent contents, “nothing is decided by a lack of success in this regard” (p. 110). A pair of apparently partial contents may in fact be independent contents bound together by deeply rooted associations which could be overcome – if at all – only by protracted mental exercises of a kind not specifiable in advance.

This problem is not a trivial one since it was, after all, the impossibility of separate presentation that was used by Stumpf to introduce the notion of partial content into his theory.

One solution may be suggested by the recognition that

however consistently we might present to ourselves, say, Schiller in association with Goethe, or Beethoven in association with a sheet of music, still it never occurs to us to *predicate* the one of the other (p. 114).

That is, we should acknowledge partiality of contents only where we should find it natural (in some sense) to predicate one content of another – as is the case where we perceive an accident inhering in a substance. To adopt such a criterion would be to assume, however, that the notion of predication is itself unproblematic: would we, presented with a piece of metal, predicate the character of heaviness of the character of being made of iron, or would we not rather predicate both characters of the underlying substantial object? In the end it is clear that Stumpf does not succeed in resolving this problem. A number of interesting issues are nevertheless raised by this first, halting development of a theory of dependent and independent parts:

The isolation of the concept of mutually dependent, partial contents is, first of all, a fundamental advance over the Humean/Herbartian atomistic assumptions which had hitherto predominated amongst theoretical psychologists. It implies that

in a certain sense neither sensations of place nor sensations of quality are basic. What is properly basic and really perceived are those unified and unnameable contents which continually change and to which, with these changes in mind, we give names such as red, blue, etc. (op. cit., p. 136)

– a view which would indeed subsequently give rise amongst students of Stumpf to a highly successful research programme in the experimental psychology of perception.³⁰

Secondly, Stumpf's analyses immediately suggest a thesis – to which we shall return below – that relations of mutual correlation or interwovenness amongst *contents* may correspond, at least in some cases, to relations of mutual correlation amongst corresponding external objects.

And thirdly, whilst all examples considered by Stumpf seem to be a matter of *mutual* or *reciprocal* correlation amongst contents,³¹ for example the two-sided reciprocal correlation between colour and extension, or the three-sided reciprocal correlation between hue, brightness and saturation amongst the constituent partial contents in the presentation of a colour, – his work nevertheless leaves open the possibility of distinguishing between such mutual correlation and the *one-sided* correlation

typified by the relation of an accident to the substance in which it inheres. Thus for example there exists a relationship of one-sided dependence between the presentation of a motion, on the one hand, and of the moving body on the other (a body which can of course in principle be presented in a state of rest).

2.2 This distinction between one-sided and mutual dependence was first drawn explicitly by Stumpf's teacher Brentano, and the question here arises as to the possibility of influences between Brentano and Stumpf in this regard.³² We know that Stumpf discussed his work on presentations of space with his teacher, who was to advance his own views on these issues in works published only later. Unfortunately it is at this stage almost impossible to determine the extent to which Brentano contributed to the account of partial contents developed by Stumpf. Brentano's work on *The Psychology of Aristotle*, published in 1867,³³ contains passages which have a superficial terminological similarity with Stumpf's 1873; for example II,2 "Von den Seelenteilen . . ." and II,4c "Von der bewußten Einwirkung des geistigen Teiles auf den sinnlichen", but the work offers no evidence that Brentano had at that stage grasped the notions of partial content and of mutual dependence in any systematic way.

In Brentano's later works however the notion of one-sided dependence can be distinguished as having played a crucial role.

Consider, first of all, his theory of inner perception, put forward in Book 2 of the *Psychologie vom empirischen Standpunkt*. This rests on a distinction between what Brentano calls *physical phenomena* (colours, shapes, warmth, cold, odours, and similar formations appearing in the imagination), and *psychical phenomena* (the seeing of a coloured object, feeling warmth or cold, every judgment, recollection, expectation, inference, anger, love, hate, desire, act of will, and so on). An inner perception is defined as an act having a psychical phenomenon as its object, an outer perception as one whose object is a physical phenomenon. Brentano's thesis that every consciousness is bound up with a self-consciousness can now be expressed in the form: every act of outer perception is bound up with an inner perception of the act in question. A mental state or event which did not meet this condition would not be a 'consciousness' at all. But it is not as if – as the terminology of 'inner' and 'outer' perception may unfortunately suggest, – this element of self-consciousness is conceived as an additional act which would exist in the mind

somehow alongside the original consciousness. It is, rather, a merely abstractly distinguishable moment of the original consciousness, a (one-sidedly dependent) partial act of a type that can of necessity exist only as embedded within such a larger, circumcluding act-whole.³⁴

The second point at which the notion of one-sided dependence can be discerned in Brentano's work is in his theory of evidence.³⁵ Brentano claims that the only objects of which we can have an absolutely certain apprehension are psychical phenomena, i. e. the acts and states of our own consciousness. Of these alone can we assert with an absolutely evident knowledge that they are in reality as they appear to be in our perceptions of them. Thus whilst, when we are perceiving something given as outside us, we cannot have absolute evidence that the object of our perception is as it seems – we could e. g. be hallucinating, – we *can* know with absolute evidence that this and this particular state of consciousness exists, and that it is structured in such and such a way. Illusions and hallucinations of a kind with which outer perception is indelibly plagued are, Brentano claimed, alien to the world of inner perception.

What he means by this can be explained as follows: absolute evidence is obtained only if a judgment (i. e. the psychical phenomenon) and that which is judged (i. e., for Brentano, that object whose existence is acknowledged or denied), are somehow united in a single whole which is available to consciousness in such a way that the correctness of the judgment can be grasped directly. Experience tells us that such a unity is impossible for judgments of outer perception. Consider, however, judgments of reflexive self-awareness such as 'I am thinking' or 'I have a visual image of a red surface'. Judgmental contents of this sort are, in Brentano's view, merely abstractly isolable moments of more inclusive act-wholes (of thinking, having such and such a visual image, etc.). Thus the desired kind of unity between judgment and that which is judged is here so to speak *already* to hand: our experiences of psychical phenomena are already of themselves experiences having the character of immediate evidence.

The notion of one-sided dependence is discernible further in Brentano's account of the three classes of mental act (of presentation, judgment, and of love or hate/preference or disapproval).³⁶ Here the structure of the realm of acts is determined by relations of one-sided dependence between the three levels: an act of love or hate is one-sidedly dependent upon an act of judgment (of the existence of the thing loved or hated), which is in turn one-sidedly dependent upon an act of presenta-

tion of that thing. It is as if the act of judgment could be *detached* from, could exist separately from, a moment of love (though not vice versa), and as if the act of presentation could be detached from the moment of judgment (though not vice versa), love and judgment thereby ceasing, in these successive detachments, to exist.

The notion is discernible also in Brentano's treatment of syncategorematic or synsemantic terms (terms whose meaning is one-sidedly dependent upon that of other, categorematic terms).³⁷ Finally Brentano's theory of double judgments (*Doppelurteile*) rests on the same basis. For Brentano the judgments 'this man is mortal' and 'all men are mortal' contain, respectively, the independent parts 'this man exists' and 'men exist', upon which the remainder of the judgments, which carries the relevant predicative content, is in each case (one-sidedly) dependent. This remainder, not being capable of existing in isolation, is seen as one-sidedly enmeshed, *einseitig verflochten*, in the given constituent judgments.³⁸ One-sided rather than reciprocal dependence obtains here in virtue of the fact that predication is seen by Brentano as demanding a logically prior judgment (acknowledgment) of the existence of the predicated object, but not vice versa.³⁹

2.3 Brentano's approach, in all of the above, reveals the same basic ontological preference towards a world of (separable or detachable, *ablösbare*) things and their dependent (inseparable or non-detachable) parts.⁴⁰ His ontology rests on the distinction between what he calls primary and secondary entities. The former are either things (material things: men, musicians, stones, works of sculpture) or souls; the latter – what Husserl would call dependent parts – Brentano would rather conceive as fictions to be dissolved by analysis.⁴¹ The single important exception to this dominance of one-sided dependence in Brentano's published writings is his analysis of continua. Brentano views continua as things made up of parts, distinguished from collectives by the fact that the parts which constitute the latter are independent things, where the parts which make up a continuum become independent only when separated from the continuum. Both collectives and continua are to be distinguished from things which are not made up of parts in either of these two senses. Brentano's only clear-cut defence of a case of mutual (in fact two-sided) dependence is to be found in his treatment of *boundaries*: the existence of the boundary of a continuum depends upon the existence of the continuum and vice versa.⁴²

It can thus be said that where Stumpf's ontology over-emphasises the role of mutual at the expense of one-sided dependence, Brentano errs by over-emphasis in the opposite direction. Only in Husserl's work, to be treated in more detail below, do we find a recognition of the pervasiveness of both one-sided and mutual dependence relations. The difference between Brentano and Husserl expresses itself first of all terminologically: Brentano is interested in detachable (*ablösbare*), i. e. in thing-like parts, Husserl in dependence or inseparability *relations* between parts.⁴³ In contrast to Brentano, whose ideas here recall the scholastic onion-model of the relation between a substance and its accidents (the successive skins of the onion conceived as surrounding an ultimate, unitary substance not capable of further analysis), – this is seen for example in his discussion of the various types of mental act, in his theory of judgments, and elsewhere – Husserl recognises that acts, sentences, judgments, exhibit internal structures involving ramified relations of mutual dependence, where no element can be picked out as a thing-like core. Thus whilst in his treatment of judgments Husserl agrees with Brentano that the presence of the logical or objectifying quality of positing (the assertive force) is the indispensable criterion for something's being a judgment (see his 5th Logical Investigation), he modifies Brentano's views to the extent of recognising logical or objectifying acts as consisting of four mutually dependent moments (i. e. moments some of which require the existence of others). It is perhaps because Husserl freed himself only gradually from the influence of Brentano's notion of presentation that it was only after the first edition of the *Investigations* that he rejected the view that sentences contain independent (sub-sentential) meanings, i. e. meanings that might in principle make up the content of a complete act.⁴⁴

Brentano's most explicit employment of the ontology of part and whole occurs in his *Kategorienlehre* (1933), a compilation of writings on Aristotelian ideas dating from 1907. Since an admirable discussion of this work exists already in the literature (Chisholm, 1978), we shall content ourselves here with only a brief presentation.

Consider a man standing before a table on which he perceives or imagines he perceives an orange. The man may or may not at any given instant have before his mind an image of an orange. However the man *qua* bearer of such an image is, according to Brentano, a *whole* standing in a relation of one-sided dependence upon the man himself as *part*. The man himself may exist in separation from (*abgelöst von*) the man *qua*

orange-presenter – i. e., in more normal terms, the man would not cease to exist should he cease to perceive the orange – though not conversely. Hence, according to Brentano, we have here a case of one-sided dependence of the orange-presenter upon the man. Note however that the dependent whole is not the result of adding to the independent part some additional, conceptually isolable entity, as is the case within the framework of the classical theory of substance and accident. Indeed Brentano proposes a terminological revision, with respect to this theory, according to which the dependent whole should itself be designated 'accident', accidents in this sense therefore being recognised as including their substances as parts.⁴⁵

This way of conceiving accidents is not without foundation in Aristotle's theory of categories. Aristotle distinguishes not only a (concrete or abstract) *quality*, ποιότης, but also a (concrete or abstract) thus-and-thus *qualified thing* (or substance), ποιόν, both the ποιότης and the ποιόν being listed as categories different from the category of (unqualified) substance (οὐσία). In post-Aristotelian writings, e. g. in Porphyrius, all categories different from that of substance are classified as accidents.

The disadvantage of Brentano's terminology, however, which restricts the term 'accident' *exclusively* to qualified substances, will be obvious. The conceptual pressure exerted by Brentano's view that there exist only what he calls 'things' (men, oranges, perceivers of oranges, etc.) makes it impossible for him to refer, – except in a contrived and round-about way, – to moments, aspects or qualities of things, and thus for him the realm of entities isolated e. g. in scholastic writings on individual accidents, in Stumpf's theory of partial contents, or indeed in the whole corpus of Husserlian phenomenology, becomes inaccessible to scientific investigation.⁴⁶ Further Brentano's use of the term 'ultimate unitary substance' (*letzte einheitliche Substanz*), to designate an entity which is contained in all of its accidents as a part, involves, surely, a metaphysical presumption of some considerable magnitude. How could the existence of such a common part be demonstrated? (Cf. Stumpf, 1939/40, vol. I, p. 41).

Despite these problems however it is undeniable that Brentano's use of the part-whole relation, and above all his application of the theory of part and whole in his psychological writings, is of lasting importance. The influence of Brentano's work on dependence relations is particularly discernible in the writings of Anton Marty (another student of Brentano, who had enjoyed a close association with Stumpf in both Würzburg

and Halle), specifically in Marty's development of the opposition between categorematic and syncategorematic terms.⁴⁷ Brentano's influence is discernible also in the works of Meinong, Höfler, Twardowski and Ehrenfels. It is however in the early works of Husserl that the truly decisive generalisations on the basis of Brentano's (and Stumpf's) work are effected. It is possible to follow the development of Husserl's thought from his first scattered discussions of various types of *Verbindung* in the *Philosophy of Arithmetic*, to criticism and expansion of Stumpf's insights in the "Psychological Studies on Elementary Logic", culminating in the fully systematic exposition of a completely general theory of wholes and parts in the *Logical Investigations*.

§ 3 Husserl's 3rd Logical Investigation: The Formal Ontology of the Part-Whole Relation

3.1 Stumpf, as we have already seen, drew his examples of dependence relations exclusively from the psychological domain. It was initially in his investigations of the psychology of number that Husserl conceived the idea that such relations, alongside other formal-ontological relations, could be applied completely generally, to all entities and systems of entities whatsoever. Here Husserl's work on functional dependence with his teacher Weierstrass, his discussions on set theory with Cantor in Halle, his study of Schröder's logic and of the Riemannian theory of manifolds, are of importance.⁴⁸

Already in the *Philosophie der Arithmetik* (PdA) of 1891, Husserl had recognised at least four types of relation amongst parts:

(i) the merely *kollektive Verbindungen* (collective combinations) between the separate elements of a group of things tied together only in the weakest possible sense that we think of them as such (e. g. objects on a tray, or on a list);⁴⁹

(ii) purely 'extensive' relations of contiguity, succession, etc., amongst regions of physical space, amongst phases of time, and amongst extensive manifolds and sub-manifolds in other spheres;

(iii) relations of the type which exist between Stumpf's psychological parts (partial contents), but which are now recognised as of considerably wider scope;

(iv) the so-called '*figurale Momente*' (individual relations connecting geese into flocks, trees into avenues, dots into patterns, and so on) simultaneously discussed by Ehrenfels in his 1890.

Brentano, in his lectures of the period, referred to the third type of relation as a *metaphysische Verbindung*⁵⁰ (the first type of relation was referred to by Husserl in PdA and in the first edition of the *Logical Investigations* as a 'psychical relation', the second as a 'physical relation'). Metaphysical connections include the relations between the properties of an object, which may exhibit one-sided detachability (as when, for example, the colour of an object loses its lustre) but which are typically a matter of reciprocal interdependence. As Husserl put it (PdA, p. 159), the properties of an object 'constitute a whole of parts which are bound together substantially (which reciprocally interpenetrate)'.⁵¹

We can point out in passing how recognition of the new type of whole which is generated by such reciprocal interpenetration allowed convincing arguments to be marshalled against Herbart's atomistic concept of substance which, in the course of the 19th century, had achieved a position of some dominance as the official ontology of German psychologists. In defending his view – which has distinctly Tractarian overtones – that the world consists of a plurality of absolutely simple, atomic substances, configured together into complexes, Herbart appealed to the identity theory of the copula, i. e. to the view that the 'is' in '*S* is *p*' is an 'is' of identity. If we interpret this view in what seems to be the most intuitively acceptable way, then we may say that '*S* is *p*' is to be given a canonical expression as:

Some part (or accident) of *S* is identical with *p*,

where '*p*' plays the role of a proper name of some accident, feature or determination. 'Socrates is red', for example, expresses the identity of some part of Socrates with (some individual accident) red. Now, Herbart argues, what is *real*, the substance of the world, cannot have a multiplicity of determinations; for let us suppose that '*S*' denotes such an ultimate substance having the distinct determinations *p*₁ and *p*₂, i. e. in canonical form:

part of *S*, say *s*₁, is identical with *p*₁ and part of *S*, say *s*₂, is identical with *p*₂.

Now suppose *s*₁ and *s*₂ are non-identical parts of *S*. From this it would follow, according to Herbart, that *S* can be decomposed into a number of different parts. But then these parts are more ultimate, ontologically, than *S*, which contradicts our hypothesis. Hence *s*₁ and *s*₂ are identical, from which follows also the identity of *p*₁ and *p*₂, again contradicting the initial hypothesis. Hence we must conclude that what is real is absolutely simple.⁵² The weak link in Herbart's argument: the as-

sumption that an object *S* having non-identical parts *s*, can be decomposed or disassembled into separate individual *s*, is precisely what is called into question by the recognition of *metaphysische Verbindung* amongst parts and by the denial of the thesis that every whole is a mere sum of pieces.

Brentano acknowledged also a further type of part-whole relation in his early lectures, which he called a *logische Teilungsverhältnis*. In echo of Peter of Spain's third and fourth *modes of being in* (see § 1.2 above), this is conceived as a relation between moments corresponding to the logical relation of species to genus. A particular instance of red, for example, is, even abstracting from its spatial extent and leaving aside its constituent hue, brightness, etc., non-simple from the point of view of its logical parts. For in the abstractum *red* there lies the moment *colour*. But it is not as if colour is somehow filled out to become red by the adjoining of some further moment. Colour rather specifies itself in red. The latter *is* colour, and yet is not identical with colour.⁵³

3.2 Whilst at the time of PdA Husserl had no more than a rudimentary theory of part-whole relations as such, he moved progressively toward such a theory in his writings between 1891 and 1900, the year of publication of the 1st volume of the *Logical Investigations*. By this time he was in a position to put forward a perfectly general '*a priori* theory of wholes and parts, i. e. of forms of connection (*Verbindung*) and unity' (Husserl, 1913, p. 131), laws valid not only in the field of descriptive psychology (i. e. purely amongst mental contents), but for all objects whatsoever.

These laws are presented in a highly compressed form in the 3rd Logical Investigation, a work which is, for all its inadequacies, the single most important contribution to realist (Aristotelian) ontology in the modern period. Its significance was understood by Husserl's most immediate followers – particularly the members of the Munich-Göttingen school of phenomenology (see the work by Reinach translated below) – but this understanding waned amongst subsequent generations of thinkers around Husserl, despite the fact that he himself recommended it to his students as central to the understanding of both the remaining Investigations and of his later phenomenology.⁵⁴ It has nonetheless, as we shall see in the sections which follow, exerted a not inconsiderable influence on the thought of the twentieth century.

The Investigation begins by quoting with approval Stumpf's⁵⁵ definition of partial contents as contents whose nature forbids them to have an

isolated and mutually independent existence in our presentations (Stumpf, loc. cit., p. 113). Husserl states however that he intends to use this definition only as a starting point for a more precise definition of the concept of dependence, one which will be at one and the same time free of the drawback of relativisation to subjective capacities for presentation (as in Stumpf's original theory) and allow a generalisation of the concept of dependence beyond the purely psychological sphere.

To say that a content can be presented 'in isolation' clearly cannot mean that it 'can be freed from *all* fusion with coexistent contents, can therefore ultimately be torn out of the unity of consciousness altogether' (LU III, § 5) – for *all* mental contents are inseparable in this sense; all presentations are presentations against some co-presented background or other. Isolability, Husserl concludes, can only mean something like: capable of being held constant in presentation under conditions of absolutely free variation,⁵⁶ within the limits set by the nature of the content in question, of all contents associated with it, so that it should indeed in the end, but only in principle, remain unaffected by the very elimination of such contents.

This self-evidently entails that the existence of this content in presentation and in consciousness generally is not at all conditioned by the existence of other contents, that it could exist, just as it is, even if there were in consciousness nothing at all beside it, or even if everything about it should vary arbitrarily, i. e. without principle (loc. cit.).

But this implies that the content is in itself such that there is rooted in its nature no necessary interwovenness with other contents.

Now a content which in itself is such that its ideally graspable essence, its intrinsic structure, that which makes it what it is, also 'leaves it unconcerned with' all other contents is called by Husserl independent (*selbständig*). A content which in its ideally graspable essence or nature is bound to other contents, which cannot be if other contents are not there together with it, he calls dependent (*unselbständig*).⁵⁷

With this shift from talk of 'possibilities of separate presentation' to talk of intrinsic essences or natures (intrinsic structures) of the contents involved, Husserl has eliminated from his definition of dependency all reference to the conscious subject, except incidentally – the conscious subject is someone who may potentially grasp by a process of imaginative variation the essences in question.⁵⁸ And all references to 'differences in mode of presentation' have also been eliminated. Husserl has,

in other words, departed considerably from Stumpf's (and the traditional, e. g. Berkeleyan) account of independence as signifying that a content is capable of being presented in isolation from other contents. He has moved much more closely to the position of the scholastic realists, to the concept of independence as an objective character of contents capable of existing in isolation from other contents.

Simply by substituting the word 'object' for 'content' in this account it now becomes possible to effect an immediate generalisation of the dependence/independence opposition beyond the purely psychological sphere to apply directly, in reflection of what Husserl calls a 'universal ontological difference' (§ 9), *to all entities whatsoever*. Just as a presentation of colour cannot exist in isolation from a presentation of space, so a *reconciliation*, say, cannot exist in isolation from a prior disagreement, an *answer* cannot exist in isolation from a prior question, a *husband* cannot exist without a wife, a *sales representative* cannot exist without the company he represents and goods to be sold. Such dependence relations amongst parts correspond, we shall argue, to systems of *a priori* truths.⁵⁹

3.3 Husserl's theory of whole and part is a theory which makes room for both independent and dependent parts. We are already familiar with many examples of the former (the items of furniture which make up the everyday world are all of them independent parts – in the sense that each could continue to exist even though all the others should go out of existence). The independent parts of a whole such as, for example, a sheet of glass, coincide with the extensive parts of such a whole as these are determined, e. g. by Leśniewskian mereology or by the calculus of individuals (or by breaking the sheet of glass into smaller pieces). But it is not as though Husserl, with his distinction between dependent and independent parts, is presenting what is in the end merely a theory of extensive parts of the familiar (mereological) sort, to which is conjoined an additional, in principle dispensable, capacity of recognising particular esoteric marginal cases of ('non-extensive') parts enjoying logically peculiar properties.

To make this clear consider, again, the sheet of glass *s*, which is uniformly red (has a uniform red-moment – dependent part – *r*, inhering in it), and is recognisable as being decomposable, e. g. by careful slicing, into two specific smaller sheets, *s*₁ and *s*₂. Clearly in such circumstances *r*, too, must be recognised as being decomposable into two correspond-

ing constituent red-moments r_1 and r_2 , inhering in s_1 and s_2 respectively. The same considerations clearly apply equally to non-homogeneously coloured bodies. And they apply further to moments extended not through space, but through time: knightships, for example, or diseases, or bonds of wedlock. Thus consider a man m and a woman w joined together by the specific bond (two-place relational accident) b of holy matrimony existing between the two of them across the specific interval of time t . Here again, the bond (unifying moment) b_t existing in some given proper extensive part t_i of t is surely an extensive part or *phase* of the moment b .

According to Husserl, extensive moments, whether spatial or temporal, and moments founded on them have the property which no other moment has of being pieceable. Thus the examples given show that even in the superficially non-extensive sphere of dependent parts we are able to establish – effectively by drawing (or recognising) *boundaries* – extensive relations of part to whole, i. e. that *there are moments which have pieces* (cf. § 17, where Husserl also gives a precise sense to what we have here loosely designated ‘extensive part’). Pieces of moments may in turn possess moments of their own, which may in their turn be recognisable as decomposable into further pieces or as the bearers of further systems of moments, and so on, without any generally establishable limit.

Husserl’s theory of extensive and non-extensive wholes differs from a two-sorted theory (mereology supplemented by the facility of recognising moment-whole relationships) in acknowledging the existence, as it were below the surface level of everyday reality, of a hierarchically organised sequence of banks of moments and pieces reticulated amongst each other in complex ways.⁶⁰ But that is not all. There is also a certain sense in which moment-whole relations can be established also in strata (in universes of discourse) as it were *above* the level of everyday reality. To these belong all ‘syntactic unities’⁶¹ (words, sentences, scientific theories, and perhaps sets, classes, . . .), all of those entities designated by Meinong (1891; 1899) as *objects of higher order*, all institutional entities, and so on. The head of corporation c , for example, *qua* head of c , is not a mere *piece* (independent part), since of course should the remainder of the corporation cease to exist then he too (in his capacity as its head) will also pass out of existence. This is not to deny that the moment of c which is its head is not – in the relevant interval of time – coincident with the independent whole which is the corresponding human being. But coincidence is not identity, as the proponents of an exclusive-

ly extensionalist ontology – for whom all strata collapse onto a single stratum (the isolation of which is presumed to be somehow unproblematic) – would have us believe.

If we say, with Meinong, that a higher order object such as a melody is *founded* upon its lower order *fundamenta* (in this case the individual tones), then this is just to say that these fundamenta constitute a whole of parts which do not merely exist together side by side.⁶² They are, rather, ravelled together in virtue of the tendency of each tone to set up expectations as to the order of its successors and to consolidate an order in our memory of the preceding tones. The higher order object which is a session of the Austrian Imperial Council is a whole of parts: statements, questions, answers, orders, standings up and sittings down, ravelled together both by the complex of interrelations between these elements themselves, and by the relations between these elements and events outside the Council. From this conception it follows, trivially, that two different higher order objects (a football team and a submarine crew) may have identical constituents (ravelled together in different ways). And it follows also that to dispose of a higher order object such as a family, a philosophical movement, a nation or a culture, it need not be sufficient merely to separate its parts: these parts must also (if this is possible) be unravelled.

3.4 Husserl's discussion of the upwards and downwards piece/moment hierarchy is in the very important § 13 of the 3rd Investigation, on "Relative Dependence and Independence". A given visual moment of extension e is, as we have already seen, dependent, within the sphere of mental contents, on a co-perceived moment of colour (or configuration of such moments). The fact that we can distinguish, extensively, constituent e_i (by drawing arbitrary boundaries), that we can imagine the remainder of e disappearing whilst any given e_i is held fixed in presentation, signifies that the e_i are independent, – but only relatively to the whole e . They are not, of course, absolutely independent, since each e_i is, like e itself, dependent upon moments outside the realm of visual extent: we cannot imagine a given visual extent remaining fixed in presentation whilst *all* visual filling (i. e. all colour-data) should disappear.

Within, and relatively to, the concrete totality of a momentary visual intuition, each portion of our visual field, each concretely filled section of it, is independent; each colour of such a portion, the colour-pattern of the whole, etc., is de-

pendent. And again, in, and relatively to, the whole of our momentary total sense-intuition, the visual field with its contents, the tactual field with its contents, etc., are independent, whereas the qualities, forms, etc., whether attaching to whole fields, or to their individual members, are dependent (§ 13).

Relative to the summative whole *words of the English language*, the words of this sentence are independent pieces; relative to the sentence itself they are dependent moments. Relative to the whole which is the object-world depicted in a given novel, the characters of the novel are independent pieces; relative to any whole which includes the totality of experiences of readers of the novel, these characters are merely abstractly distinguishable dependent moments. Relative to the world as a whole (that is to say *absolutely*), the Hamburg representative ν of a São Paulo coffee trading company is independent (the company could cease to exist without this bringing about the annihilation of its representative as an item of the furniture of the world); relative, however, to any whole in which ν functions essentially *as* a representative – e. g. to a complex of events in which ν signs contracts, makes commitments, fulfils obligations in the name of his principal, etc., – ν is a dependent moment – (should the company cease to exist then this would, of necessity, bring about the consequence that ν too, *qua* agent of the company, would cease to exist).⁶³

This last example suggests a sense in which dependence-relations may occur between wholes which are spatially or temporally disparate, or between wholes from distinct ontological regions.⁶⁴ A husband *qua* husband is dependent upon a certain other human being (his wife), in that should she cease to exist then he too, of necessity and of his very nature, would thereby also cease to exist: a husband (master, king, employer, slave-owner . . .) as such cannot exist except in a more comprehensive unity which associates him with a wife (servant, subject, employee, slave . . .).

3.5 The types of dependency relations which have been recognised so far include:

that type of dependency which holds between a husband and a wife (or equally, for our present purposes, between a claim and a mutually correlated obligation, or between the north and south poles of a magnet);

that type of dependency which holds between a higher order whole and its lower order *fundamenta* (for example, between a sentence and

its words, between the *natio hungarica* and the totality of Magyars, or between the institution of holy matrimony and the totality of wedlock bonds);

that type of dependency which holds between an answer and a question, between a reconciliation and a disagreement, between the fulfilment of a promise and an act of promising, and so on.

Without here attempting more than a rudimentary classification,⁶⁵ we might instance also the following additional examples of dependency relations:

the mutual dependence of the purely (abstractly isolated) psychological parts of the whole which is a thinking human being, and the purely physical (biochemical) parts of the same whole;

the one-sided dependence of a 10 Mark note *qua* sum of money upon the relevant *readiness to accept* distributed across a given population; of the human race *qua* extant biological species upon a specific system of climatic and other environmental conditions:⁶⁶ of the scientific character (originality, rigour, etc.) of the products of a given academic community upon a specific system of economic and institutional rules, customs and conventions, to which its members are subjected; and so on.

In order to be able to express in a completely general way the character which is shared by all possible dependence relations, Husserl reintroduced Meinong's terminology of foundation, though he endowed it with a different sense: a husband, as Husserl would now express the matter, is *founded upon or through or requires foundation by or in* a wife (§ 14).

Definition: If there is a law of essence that an α as such cannot exist except in a more comprehensive unity which connects it with a β , then we say that an α requires foundation by a β (loc. cit.).⁶⁷

He then elaborates a series of highly general laws in which insights deriving from the work of Brentano and Stumpf and from the Aristotelian doctrine of substance and accident, from the Boole-Peirce-Schröder algebra of logic and the theory of formal manifolds initiated by Herbart and Riemann, and from the discussions of part-whole relations in Bolzano's *Wissenschaftslehre* and in Twardowski's *Zur Lehre vom Inhalt und Gegenstand der Vorstellung*, are welded together in a single formal ontological system.

These laws are expressed in terms of the concept of foundation as defined above. We shall quote them in full as a basis for discussions in later papers in this volume.⁶⁸

Theorem I: If an α as such requires foundation through a β , then every whole having an α but not a β as part requires a similar foundation.

Theorem II: A whole which includes a dependent moment without including as its part the supplement which that moment demands, is likewise dependent, and is so relative to every superordinate independent whole in which that dependent moment is contained.

Theorem III: If a is an independent part of (and thus also relative to) b , then every independent part c of a is also an independent part of b .⁶⁹

Theorem IV: If a is a dependent part of a whole b , it is also a dependent part of every other whole of which b is a part.

Theorem V: A relatively dependent object is also absolutely dependent, whereas a relatively independent object may be dependent in an absolute sense.

Theorem VI: If a and b are independent parts of some whole c , they are also independent relatively to one another.

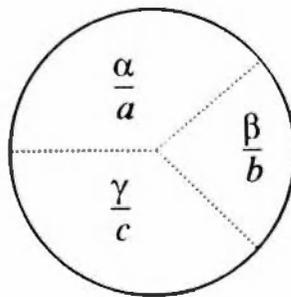
The reader is referred to the first of Simons' three papers below for a discussion of Husserl's proofs of these theorems and for a detailed indication of their interrelationships and consequences. In presenting these theorems Husserl is concerned to stress that, despite the vast number and complexity of materially different types of part-whole relation, there exists nonetheless a system of formal *a priori* relationships, both between parts and their circumcluding wholes and amongst the various levels of systems of parts of a single whole, and that these relationships generate *a priori* laws relating, for example, to the relative nearness and remoteness of parts from each other (§§ 18–20), to the possible structures of temporally ordered wholes (§§ 14, 25), or to the processes of decomposition or *piecing* of wholes (§§ 17, 25). And he is concerned to stress also that continual tacit appeal is made to such *a priori* relationships in our everyday and scientific talk of different sorts of wholes and

parts. The remainder of his Investigation is devoted to the development of the formal ontological theory of these *a priori* relationships and to the provision of sketches of applications of the theory to specific material regions (above all, still in the shadow of Stumpf, to the region of mental acts and their contents and, in the 4th Logical Investigation – see § 5 below – to the region of grammatical part-whole relationships).

§ 4 The Theory of Material *A priori* Structures; Phenomenology and Formal Ontology

4.1 ‘It is evident to anyone who makes the attempt that it is impossible that we could present to ourselves either extension without colour or colour without extension’ (Stumpf, 1873, p. 109). This passage, together with the similar passages quoted in § 2 above, points to the existence of an *a priori* order in the domain of perceptual contents. In marked contrast to the associationistic elementarism still dominant amongst psychologists in Stumpf’s day, according to which it should be possible, at least in principle, to establish or disestablish connections between *any* mental contents whatever, the arguments advanced by Stumpf suggest that there is a (perhaps highly complex) system of *a priori* structural conditions of possibility amongst such contents.⁷⁰

This *a priori* order shows itself most straightforwardly in the relationship of three-sided foundation between the specific hue, brightness and saturation of an individual colour-datum, between the specific constituent moments of pitch, timbre and loudness of an individual tone, or between the distinctive features of an individual phoneme.⁷¹ Such relationships may be illustrated, in simple cases, as follows:⁷²



The validity of these abstract decompositions, which were originally discovered by *a priori* analyses (cf. above all the work of E. Hering),⁷³

was overwhelmingly established, with amendments for specific variant cases, by a vast amount of empirical work undertaken in the early, seminal decades of experimental psychology. It was demonstrated, from a number of distinct perspectives, that each of the constituent moments that had been distinguished reflected an axis of independent variation in the nature of (a distinct dimension in the geometry of⁷⁴) perceptual contents of each specific type.

The question immediately arises as to whether it would be possible to give corresponding resolutions, i. e. corresponding systems of foundation relations amongst abstractly distinguishable moments, for mental contents of other types, e. g. for the contents – if this is here still an appropriate term – associated with acts of volition, with emotions, with mental processes of thinking, judging, inferring, remembering and so on. Husserl's answer to this question was of course an affirmative one, and indeed Husserlian phenomenology can be most illuminatingly characterised as the working out of the thesis that *a priori* laws of foundation,  – type analyses, can be provided for *all* mental contents, however complex.

There are in addition passages in the *Logical Investigations* which demonstrate that – at that stage at least – Husserl believed that *a priori* laws of foundation can ultimately be provided not only for mental contents but for objects in general, including events, actions and processes in the material world. This view was extended to apply to the objects of scientific disciplines, of history, of literary theory, and so on, by the members of the Munich-Göttingen circle of phenomenologists in their now sadly neglected contributions to applied phenomenology (see e. g. Schmücker, 1958). Husserl's claims also provoked Stumpf, in his Berlin Academy Lecture "On the Demarcation of Scientific Disciplines" (1906a), to advance a compromise position according to which there are indeed a limited number of what he called *Vorwissenschaften*, relating to domains – including the domain of sense-contents – which are characterised by subjection to *a priori* foundation relationships, but that sciences proper, except insofar as they rest on principles derived from the given *Vorwissenschaften*, have to deal with non-*a priori* features of the world which can be determined only empirically.⁷⁵

It is our business in this essay only to point out in broad terms the nature and influence of the theory of dependent and independent parts developed in particular by Husserl. We shall therefore seek neither to estab-

lish the rights and wrongs in this specific methodological issue, nor to examine in detail Husserl's claim to have established, in his investigations of the phenomenological structure of conscious experience, a vast new realm of *a priori* relationships.⁷⁶ It will be interesting, however, to survey some of the ways in which the theory made itself felt in the development of Husserl's thought, taking as our basis the hypothesis that, as a matter of fact and of principle, all propositions of phenomenology are expressions of what we shall call material *a priori* connections between moments,⁷⁷ are capable of being perspicuously represented within the framework of the theory of part and whole.

This applies first and foremost to the individual analyses of phenomenology, i. e. to the analyses of acts of perception, of memory, of predication, and so on. It can however be extended also to some of Husserl's metaphysical claims, for example to the claim that all regions of being are dependent (i. e. are one-sidedly founded) on the region of transcendental consciousness.⁷⁸ Consider also two important criticisms Husserl made of Kant: that he conceived of the faculties as pieces (*Stücke*); and that he failed to see that every genuine *a priori* proposition, whether analytic or synthetic, becomes counter-sensical (yields *Widersinn*) when negated.⁷⁹ In the first of these criticisms Husserl is drawing attention to the fact, missed by Kant, that despite the apparent transience and plasticity of the phenomena of consciousness, there is nevertheless an *a priori* system of intrinsic interrelationships amongst these phenomena which is capable of being disclosed. An act of joy, for example, presupposes an act of grasping a state of affairs of a specific type; an act of memory presupposes a temporally ordered sequence of acts in which it is rooted in a quite specific way, and so on.⁸⁰

The phenomenologist, independently of his metaphysical position, recognises that acts of consciousness do not form a self-contained region (*Weltstück*) isolated from the region of external objects (including living bodies), from the region of human action, or from the region of linguistic structures, but are, rather, reticulated with these in myriad ways. Hence his analyses will typically be directed towards more than one single region; the material *a priori* *Sachverhalte* that he discloses will often straddle the boundaries between regions.⁸¹ They are, nevertheless, objective constituents of reality.⁸² Their aprioricity (or intelligibility) is not the result of any conceptual, linguistic or grammatical convention. This implies an *ontological* approach to the problem of the *a priori*, – an approach which stands in opposition to the logico-linguistic

approach, inspired above all by Frege, which has come to be accepted as orthodoxy by Anglo-Saxon philosophers.

Whilst for Husserl the *a priori* of judgment and the *a priori* of *Sachverhalte* are viewed as two sides of a single coin,⁸³ his disciple Reinach viewed the linguistic *a priori* as derivative of the material *a priori*. That 'orange lies between red and yellow in the order of similarity' is an *a priori* proposition, results from the fact that orange, red and yellow are themselves thus ordered in the region of colours.⁸⁴ Reinach argued indeed, particularly in his *On the Theory of the Negative Judgment* (translated below) that this priority of the *Sachverhalt* over correlated sentence, judgment or proposition implies the need for a new, ontological foundation of logic. Mutually contradictory judgments or propositions, for example, *are* mutually contradictory, according to Reinach, in virtue of the ontological incompatibility of the corresponding *Sachverhalte*.⁸⁵

The reconciliation of Hans and Erna is founded upon their (temporally prior) conflict of opinion or conduct; the later event is, as a reconciliation, of necessity bound up with (its existence is dependent upon) the earlier.⁸⁶ The three moments founded in Fritz which consist in (1) his *refuting Popper*, (2) his *writing a book*, and (3) his *going to the library every day* form, with Fritz himself, a whole whose structure is such that (3) is the *means* for (2) as end, where in its turn (2) is the means for (1) as end. The complex temporally extended moment which is the daily visit to the library is such that it is essentially possible for it to found the writing of a book. It is, in contrast impossible for the writing of a book to found the visit to the library, as is reflected in the absurdity of *Fritz goes to the library every day by writing a book*.⁸⁷

4.2 The fact that the world, in all its strata, is thus criss-crossed with a multifarious system of material *a priori* foundation relations has an important consequence for the theory of individuation. Traditionally the assumption has been that it is the spatio-temporal coordinates of an object, event or process which are to be taken as yielding its principle of individuation.⁸⁸ If, however, every object is, independently of any relation it may bear to the co-ordinate systems of space and time, to be recognised as traversed also by networks of essential interrelationships of other kinds (depending on its own specific nature), then it may prove that space and time fail to provide sufficient conditions for individua-

tion: Wolfgang's *jumping up and down in a frenzy* and his *getting warm* may occupy an identical spatio-temporal extent, but in such a way as to be embedded in distinct systems of surrounding *Sachverhalte*.⁸⁹ Such objects are, in the terminology introduced above, *coincident* but not *identical*. The way in which the concept of spatio-temporal location applies to the objects of scientific disciplines (phonemes, molecular structures, animal species, . . .) differs from the way in which the concept applies to the medium-sized durables in a speaker's perceptual environment, and this differs in turn from the way in which the concept applies to institutional and cultural artefacts such as legal persons, joint stock companies, brands of coffee.⁹⁰ These ideas, which Husserl regarded as 'a first decisive step in the division of *a priori* ontologies'⁹¹ are taken up again in his later writings on regional ontologies.⁹²

There are many different types of unifying relations, both within a single object – relations which constitute that object into a relatively isolated, integrated whole,⁹³ – and between objects – relations of causality,⁹⁴ for example, or of submission or respect. Systems of unifying relations within an object correspond to predicatively formed *Sachverhalt*-complexes; systems of interobjectual unifying relations generate networks of relational *Sachverhalte* which rest, in the end, upon purely formal (content-less) relations of foundation. As Husserl puts it: "Alles wahrhaft Einigende . . . sind die Verhältnisse der Fundierung".⁹⁵ That is, all that is truly unifying are relations of foundation. Such a content-less foundation is necessary, since otherwise the classical infinite regress – familiar from the Third Man argument, from the work of Bradley, Russell, Stout, *et al.*, on internal relations,⁹⁶ or from Bergmann's defence of his nexus of instantiation⁹⁷ – is threatened. Suppose *a* and *b* stand to each other in the external relation *r* (not a relation of foundation). Then further relations, *r'* and *r''*, must exist to connect *a* to *r* and *r* to *b* respectively. Now either *r'* and *r''* are ultimate relations of foundation or again further relations *r'''*, etc., are needed to colligate these with their respective relata. Foundation relations put a stop to this regress, since only contents can have a foundation, not foundation relations themselves.⁹⁸

The logic of foundation is used by Husserl not only in the *Investigations* but also in his later works, for example as an indispensable means of displaying the structure of inner time,⁹⁹ of material objects,¹⁰⁰ of the sensory plenum,¹⁰¹ and in his theories of pure grammar (discussed in § 5 below). Here we shall concentrate on his employment of the theory in

giving an account of the structure of mental acts, including (linguistically borne) acts of predication.

As for Brentano, so also for Husserl, mental acts may exhibit a peculiar many-layered or stratified structure. Thus:

The subject-member of a categorical asserting is an underlying act, a positing of a subject, on which is built the positing of a predicate, its attribution or denial. Just so the antecedent of a hypothetical assertion constitutes itself in a clearly demarcated part-act, on which is built the conditioned positing of the consequent . . . On such a composite act (whose members may in turn themselves be composite) a new act may be built, e. g. an act of joy built on the taking in of a state of affairs, a joy about that state of affairs. The joy is not a concrete act in its own right, and the judgment an additional act alongside it: the judgment is rather the founding act for the joy, it determines its content, brings to realisation its abstract possibility – for without some such foundation (in the strict sense of our 3rd Investigation) there can be no joy at all (LU V, § 18).¹⁰²

The concept of stratification was most fully exploited, within the phenomenological movement, by Roman Ingarden, particularly in his theory of the literary work of art (1931).¹⁰³

An act, according to Husserl, possesses a *matter* and a *quality*. The act-quality stamps the act as a judgment – and so involving positing – or as a mere presentation, as an emotional act, an act of doubt, etc. The act-matter is simply the content of the act which determines it as a presenting of *this*, as a judging of *that*, etc. (LU V, § 20). According to Husserl,

the act-quality is undoubtedly *an abstract moment of an act*, something that would be utterly unthinkable detached from all matter. Or should we perhaps hold an experience possible which would be a judgment-quality but not the judgment of some definite matter? The judgment would after all thereby lose its character as an intentional experience, a character which evidently belongs to it essentially.

The same will hold of matter. A matter that was not matter for presentation, nor for judgment, etc., will be deemed to be unthinkable (loc. cit.).

Thus the distinction of matter and quality is a distinction amongst mutually founding act-moments, and that *every* act has both matter and quality can be asserted as a material *a priori* truth.

Part-whole relations enter into Husserl's account of the coherence of series of acts and partial acts and into his account of the way these coincide or conflict with the objects or states of affairs toward which they are directed.¹⁰⁴ They enter, in particular, in his account of the 'mutual

belongingness' of signitive acts of empty intention and the intuitively filled acts in which they find their fulfilment (LU VI, § 8), in which, that is to say, 'the intentional essence of the act of intuition gets more or less perfectly fitted into the semantic essence of the act of expression' (loc. cit.).¹⁰⁵ We shall return again to this logic of fitting below.

Every act, as a matter of *a priori* necessity, has a moment of fulfilment of some specific degree. It is clearly important to note that fulfilment may be 'more or less perfect' – may, indeed, be totally imperfect, when an act of signitive intention is wholly frustrated by a conflicting intuitive content. Typically it is *partial fulfilment* (and therefore sometimes also partial frustration) with which we have to deal. An intention can be fulfilled in an act which contains either *more* or *less* than its fulfilment (in the sense of total agreement) would demand. The fulfilling act may offer merely a part or moment of that total content which would be required to achieve a perfect fitting, or it may offer an object or state of affairs which itself properly includes as part or moment the object of the original intention (cf. LU VI, § 12 which contains a valuable discussion of the syntheses involved in partial fulfilment).

Perfect fulfilment is possible in virtue of the fact that wherever simple acts (of, say, perception or judgment) are interwoven into more complex acts (of joy, surmise, doubt, questioning, wishing, etc.), corresponding interweavings are established also on the object-side amongst the intended objects and states of affairs of the corresponding acts and part-acts. That total state of affairs the intuitive grasp of which fulfils, a hypothetical judgment¹⁰⁶ has an objectual antecedent-consequent structure corresponding to the logical antecedent-consequent structure of the judgment itself. A wishful intention finds its fulfilment only when a mere presentation of the thing wished for becomes transformed into a corresponding perception (LU VI, § 13, and cf. Duncker, 1941).

There are, therefore, manifold ways in which acts may be combined into other acts (and correspondingly manifold ways in which objects may be combined into higher-order act-correlates). Yet whilst

the briefest consideration shows that in the ways in which acts are interwoven with each other or are founded upon underlying acts which open up the possibility of their realisation there are striking differences, the systematic investigation of these differences . . . is as yet hardly in its beginnings (LU V § 18).

In particular the formal ontological *theory of fitting* (of part-acts into total acts, or of part-objects into total objects, or of isolated cognitive

endeavours into the totality which is a scientific theory: see the essay by Willard below) has been denied attention by mathematicians and formal ontologists to the benefit of other, related yet distinct formal-ontological disciplines.¹⁰⁷

4.3 The context in which whole-part theory is treated in *Experience and Judgment* differs from that of the *Investigations*.¹⁰⁸ Husserl here takes up again the problem of the relations between acts (or series of acts) and their objects (*qua* act-correlates), and sets out a detailed account of the interrelations between

- i) the phenomenological structures of series of acts and partial acts in which something is made thematic,
- ii) the logical form of propositions in which this being-made-thematic is expressed,
- iii) the ontological form of the objects and object-parts involved therewith.

But it is for the sake of the theory of pre-predicative (perceptual) and predicative explication that this account is provided. *Wie diese Sachen sich zueinander verhalten*, that is, how the phenomenological, logical and ontological features of our experience complement and constrain each other, is elucidated as part of a general theory of the basis and role of predication in experience.

Objects are thematised for the sake of predication: the objects serve as substrates for the determinations brought into play through acts of judgment.

The distinction between substrate and determination shows itself at first as purely relative. Everything that affects and is objective can just as well play the role of object-substrate as that of object-determination or explicate. And just as we can, continuously and at ever higher levels, make explicates independent and thus make them into substrates . . . in the same way we can also colligate every object, every autonomous substrate, with other objects, and then make the collection as a whole into a theme, enter into its members by explication, in this way exhibiting the whole by determining it, so that each of the formerly independent object-substrates henceforth acquires the character of explicate (E & U, § 9).

It is however possible, by appeal to the concept of mediate and immediate part elaborated in LU III, to recognise not only a relative distinction between substrate and determination, but also *absolute substrates* (im-

mediately experienceable independent wholes) and *absolute determinations* (dependent moments which appear only as such).¹⁰⁹

Husserl also discusses the distinction between the 'is' of predication ('*a* is triangular'; '*a* is an instance of the species *triangularity*') and the 'has' of judgments such as '*a* has triangularity'. A predicate may correspond to either a moment or a species.¹¹⁰ Not every moment, however, corresponds to a predicate. The scope of 'moment' is, in other words, much greater than that of 'property' understood in the sense familiar to analytic philosophers as the sort of entity to which a predicate expression can correspond. Examples of moments which are not properties in this sense include first of all the dynamic moments discussed in § 1.2 above. But they include also certain static moments; for example, the edge of a material thing, or its total surface. In general, every boundary is a dependent part of the object it bounds (in virtue of the fact that it cannot be removed from the object in such a way that two separate pieces are thereby created). Yet a boundary is not a property of the object it bounds. One possible view suggested by Husserl (E&U, § 32a), would be to regard the boundary as in some sense a non-immediate property of its object. Thus the specific individual extendedness or spatial *Gestalt* (or extendedness-distribution over time) of an object is an immediate property of that object. If, now, we can regard this specific individual extendedness as having as its immediate properties the relevant boundaries (edges, surfaces, etc.), then the latter become recognisable as mediate properties of the original object.

4.4 We conclude this section with some remarks concerning the innovations of Husserl's work in whole-part theory. These innovations centre around the recognition on Husserl's part of *ontological structure*; relations of foundation are seen not, as in Stumpf or Twardowski, as exclusively a matter of relations amongst mental contents, nor, as in Schlick and Wittgenstein, as a matter of grammar. They are, rather, necessarily all-pervasive, extending through *all* material ontological regions, including both the linguistic and the psychological.

More specifically, we can identify the following advances made by Husserl over the Aristotelian theories of parts and wholes, including those developed by Stumpf and Brentano, that had preceded him:

1) the replacement of conceivability restrictions on mental contents by ontological relations holding independently of conceivability (independently of all cognition);

2) the recognition of whole-part theory as a formal ontology (applying to all matters and thus not e. g. to mental contents only), a formal ontology distinct from and more inclusive than formal logic (cf. LU IV, § 14; cf. also Smith, 1981 a);

3) an account of the piecing of moments of extension and of the radical differences between moments of extension and other moments (Brentano comes close to making the same distinctions in his 1978);

4) an account of what a formal, as opposed to a material concept is; recognition that foundation is a formal concept;

5) an elaboration of example-domains for whole-part theory to include both bi- and multilateral foundation relations. Where, in traditional discussions of the synthetic *a priori*, the focus has been almost exclusively on propositions of the form 'If x is P then x is Q ' (that is, on propositions expressing relations between predicates holding of a single object), or universalisations thereof, Husserl recognised the ubiquity of synthetic *a priori* relations amongst object-pluralities (for example, amongst the specific hue, brightness and saturation of a given colour); such relations pose crucial problems for the traditional approach, even in its modern semantic formulation, since they may involve objects falling under no common determinable (cf. Husserl on *disjunct parts*, LU III, § I, esp. 1st ed.).

6) the emancipation of philosophical ontology from the metaphysical dichotomy of atomism/holism (as propounded by, respectively, Herbart and Wittgenstein, and Spinoza and Bradley);

7) the development of a formal ontology of meanings: where, in the *Tractatus*, meaning (sentence sense) and *Sachverhalt* have an identical formal multiplicity, in the *Logical Investigations* meaning and *Sachverhalt* merely have the same sort of multiplicity, i. e. both involve foundation relations;

8) the extension of the theory of wholes and parts by means of a logic of fit, to describe exemplification and verification (fulfilment, frustration) of sentences (sentence-using acts): epistemology becomes a descriptive science (LU VI):¹¹¹

9) a description of what it is for an object to be a simple object within some cognitive/theoretical frame, avoiding the pitfalls mentioned in (6), (E&U, § 29).

The ontological structure, both formal and material, uncovered by Husserl has been obscured to philosophers working within the analytic tradition primarily in virtue of the unargued identification of the *formal*

with the formal *logical*. Once the distinction between formal logic (i. e. formal theory of meaning-connections) and formal ontology (formal object-theory) is clearly drawn, then it becomes possible to recognise also material connections both amongst meanings and amongst objects.¹¹² It has been part of our purpose here to demonstrate that there is nothing intrinsic to the analytic method that should impede the recognition of such structure and therewith the adoption of Husserl's view that the formal (matter-independent) relations of part to whole are capable of founding a general and non-trivial ontological theory of the structures exhibited not only by mental acts and their contents, by observables and by linguistic complexes, but also, in principle, by the objects of every scientific discipline.

As a partial illustration of this claim we now turn to an area where Husserl's theory has already exercised considerable influence, and where there is good reason to think that the theory will sustain further exploitation: the formal study of meaning in logic and linguistics.

§ 5 The Influence of the *Logical Investigations* on Logical Grammar and Linguistics. Husserl and Leśniewski

Qu'en la cort Grammaire a plus d' angles qu'il
n'a en Logique de jangles
Quar en toute science est gars mestres qui n'en-
tent bien ses pars.

from Henri d'Andeli, *La Bataille des Sept
Ars*.

5.1¹¹³ Husserl's theory of wholes and parts, as we have seen, embodies many insights to be found also in work in contemporary psychology and legal theory. In very few cases however can we talk of a substantial influence of Husserl's theory on the main stream of either psychology or jurisprudence. We have to deal, rather, with shared concerns and tendencies deriving from common roots in 18th and 19th century Austro-German thought. The case is different with respect to logical grammar and linguistics. Husserl's application of the theory of wholes and parts to the problem of independent and dependent meanings in the 4th Investigation decisively influenced Leśniewski's seminal work in the field of what is today called categorial grammar; and it was taken up also by Aj-

dukiewicz and other Polish logicians, whose work has in turn influenced Anglo-Saxon logicians of the last three decades.¹¹⁴ It is perhaps worth noting however that the theory of mereology, which represents Leśniewski's own attempt to formalise the relationship of part to whole,¹¹⁵ reveals little influence of the *Logical Investigations*, being essentially a logically sophisticated variant of the Schröderian calculus of domains. It does not however derive from Schröder,¹¹⁶ but from Leśniewski's own attempts to develop an alternative to the approach to the foundations of mathematics based on classes or propositional functions propounded in Whitehead and Russell's *Principia Mathematica*.

Leśniewski studied philosophy in various German universities before taking his Ph. D. under Twardowski in Lwów in 1912, having developed in particular a deep interest in Marty's philosophy of language. (Leśniewski at one point conceived the project of translating and writing a commentary on Marty's 1902; see Surma, 1977.) He discovered modern symbolic logic through his reading in 1911 of Łukasiewicz's early monograph on the principle of contradiction in Aristotle.¹¹⁷ According to Leśniewski, Russell's paradox rests on an equivocation in the concept of *class*. We can express his point as follows: we distinguish, in a manner reminiscent of Frege in the latter's review of Schröder, 1890/1905, the *collective* from the *distributive* concept of class.¹¹⁸ Something is a member of the distributive class of α 's if and only if it is an α : thus ' a is a member of the distributive class of α 's' is, for Leśniewski, just a long-winded way of saying ' a is an α '. By contrast, a member of the collective class of α 's need not be an α . A collective class is what Russell had earlier called a *class-as-one* (Russell, 1903; see also § 1 of the 3rd essay by Simons below), and what Leonard and Goodman were later to call a *fusion*. To take an example used by Leśniewski, the line AB is divided into segments by the points C and D in the diagram below:



In the distributive sense, the segments AD and CB form a class which has neither the whole AB nor the segment CD as members, whereas in the collective sense the class consisting of AD and CB is identical with the whole line AB and also has the segment CD, among others, as member. The collective sense of 'member', then, is simply that of '(proper or

improper) part'. In this sense, as Frege had pointed out earlier, and as Russell held for classes-as-one, there is no question of the existence of such a thing as an empty class. Leśniewski also incidentally rejected the concept of an existing empty class even for the distributive notion of class, as indeed did Russell: the class of α 's exists, according to Leśniewski, if and only if there is at least one α .

It is clear now why Leśniewski rejected the formulation of the Russell paradox. According to Leśniewski's collective conception of class, every class is a member (i. e. a part, but not of course a proper part) of itself. Hence no class is not a member of itself. Hence there is no such object as the class of classes which are not members of themselves, by the principle ruling out empty classes, and since Russell's argument depended on the assumption that there is such a class, then, according to Leśniewski, the appearance of paradox vanishes. If, on the other hand, by 'class' is meant the distributive concept, then the only classes which are members of themselves are singletons, i. e. $a \in a$ is true if and only if there is exactly one a . So the class of classes which are not members of themselves will not exist unless there are at least two objects, in which case it is identical with the class of all the (two or more) objects there are, and consequently not a member of itself.

It is clear that in adopting a view of classes as concrete entities, Leśniewski is subscribing to a view wholly alien to the later, Frege-Peano concept of a class as an abstract unit. He is here closer in his attitude to that of Schröder or the early Russell. However, Leśniewski admired Frege's formal work more than he admired that of Russell and Whitehead, which he considered sloppy in its use of definitions.¹¹⁹ He consequently developed his own formal theories to explicate the concept of collective class, and these he used in his analysis of the Russell paradox. The first axiomatic treatment of mereology, initially called by Leśniewski the theory of manifolds, appears in Leśniewski, 1916, where, although the theory is not expressed in a wholly formal way, the treatment is nevertheless rigorous. It is based on the notion of *proper part*, taken (correctly) by Leśniewski as the most intuitive idea of the theory, and contains four axioms, defining on the way the concept of class or complete collection.

This early, slightly inelegant, treatment was replaced later by axiomatic systems based on single notions. Leśniewski himself produced systems based on 'ingredient' (i. e. proper or improper part) and the binary functor '— is outside . . .'; while later workers have produced sys-

tems based, e. g. on '— overlaps . . .' (Lejewski, 1954), or indeed on the monadic predicate '—'s are discrete'.¹²⁰

Although *mereology* was the first formal theory developed by Leśniewski, it is based on principles valid for propositional logic and the logic of noun-expressions, which were developed next. The logic of noun-expressions is called by Leśniewski *ontology*, as it is conceived as a logic of the copula *is*.¹²¹ Ontology in turn presupposes *protothetic*, Leśniewski's propositional logic, which contains variable functors and quantifiers binding both functorial and propositional variables. Thus Leśniewski's logical systems of protothetic, ontology and mereology were developed by him in the reverse order of their order of logical priority.

Each of these formal theories is couched in a language which conforms to rigorous grammatical preconditions on what is formally acceptable. These are stated in the extensive terminological explanations and directives which accompany Leśniewski's presentation of the systems. The expressions of each system have a structure readily and exhaustively describable in the terms of a categorial grammar having as its basic categories *sentence* and *name* (the latter applying only to ontology and mereology, not to protothetic). Derived or functor categories are defined in terms of these, expressions of functor categories being in each case one-sidedly dependent on the corresponding argument expressions.¹²²

This grammatical sensitivity on Leśniewski's part was at least to some extent a result of the impression made on him by Husserl's 4th Logical Investigation with its description of the ideal of a pure grammar. Leśniewski's principles were codified in Ajdukiewicz, 1935. Where both Husserl and Leśniewski spoke of meaning or semantic categories, subsequent developments of the same ideas have tended to concentrate on the more tractable issues of syntax.¹²³

Because of the differences between the underlying logical syntax of Leśniewski's mereology and of the calculus of individuals of Leonard and Goodman, the two systems are not directly comparable. It is however generally acknowledged that they cover the same subject-matter in much the same way. Both, for instance, are extensional, and both deliberately eschew any commitment to universals, sets or other abstract entities. One difference is that Leśniewski's three theories are very clearly demarcated from each other: the general logic of propositions preceding that of existence and identity, and this in turn preceding the general

theory of whole and part. This clear division is obscured in Leonard and Goodman by their definition of identity in mereological terms.

The Leśniewski system has proved to be a durable object of investigation. It can be interpreted in and is thus consistent relative to protothetic. The fact that it can be approached from many different directions suggests that it has an intuitive solidity shared by systems such as classical and intuitionistic logic and the S4 modal system. The question remains as to its interpretation, and the extent to which it can be considered *the* logic of part and whole. One problem is that it can be extended in more than one way. *Atomistic* mereology (Sobociński, 1971) assumes that every object contains at least one atom, i. e. an object whose only part is itself, whereas *atomless* mereology asserts that every object has a proper part.¹²⁴ Each of these systems can be developed on its own terms, and the atomistic hypothesis is known to be independent of the principles of general mereology. The question as to whether atomistic or atomless mereology represents the world more adequately appears on the face of it to be an empirical one, although it is far from clear as to how it could be empirically resolved.

It could be suggested however that the atomistic hypothesis is one which will have to come up for consideration by any formal theory of part and whole, so this is not a problem peculiar to Leśniewskian mereology. However, it is customary for logicians influenced by Leśniewski's thinking in logic to regard his logical theories as distinguishing themselves from their rivals in being true of the world in which we live. This means that the empirical question is especially pressing. A theory which is purely formal in the sense of Husserl, however, is one for which the question of its adequacy to the world does not as yet arise, for as a purely formal theory its theorems are valid irrespective of how the world should be. Given this attitude to formal logic, it is clear that neither atomistic nor atomless mereology can be a purely formal theory, but must contain an admixture of something empirical. As for the general mereology contained in both the atomistic and atomless variants, it appears to be an open question whether it is a logical theory.¹²⁵

A perhaps more difficult problem concerns the basic logical principles presupposed in Leśniewskian mereology, in particular its extensionalism. For Leśniewski, there cannot be two distinct propositions which could have the same truth value under all circumstances. There are thus, according to this point of view, only four possible functions of a single proposition, namely the four truth-functions. Similarly the the-

ory of existence and identity found in *ontology* has no room for necessity or essentialistic notions, such as are needed to explicate the idea of dependent and independent parts in Husserl's sense. While the laws of extensionality used within Leśniewski's formal systems do greatly facilitate inference, they are not ontologically neutral, since they in effect deny the existence of e. g. distinct but co-extensive properties, or distinct but equivalent propositions or states of affairs. Such an extreme form of extensionalism would today find few supporters. However, the undoubted solidity of the achievements of Leśniewski and his followers suggests that the extensional whole-part theory first elaborated by him is a true theory, even if it turns out to be insufficiently rich for all uses.¹²⁶

5.2 The core of Husserl's argument in the 4th Investigation is that

the -model of n-sided foundation can be applied to grammatical

wholes, and indeed to linguistic wholes in the widest sense, including meanings. This is held to yield, first of all, a precise explication of the distinction between categorematic and syncategorematic linguistic categories, i. e. between those linguistic units (noun-phrases, complete sentences, etc.) which can stand alone as meaningful utterances of various types (see Heirich, 1910), and those which 'stand in need of completion' by linguistic units of other specific types: which cannot exist in a meaningful utterance except in a more comprehensive unity which connects them with units of those given types. It is held ultimately to yield a system of purely formal distinctions amongst different types of linguistic units according to the types of foundation relation and supplement which they involve. The sentential negation functor *it is not the case that . . .* for example, becomes recognised as a moment standing in a relation of one-sided dependence upon the category *sentence*; the nominal connective *and* as a moment founded one-sidedly upon the category *pair of names*; the sentential prefix *if* as a second-order moment founded mutually upon a sentential *then*, the compound moment thereby constituted being in its turn one-sidedly dependent upon a pair of sentences; and so on.¹²⁷

Husserl argued that it would be possible, with the aid of mathematics, to develop a purely formal theory of all possible foundation relations amongst all possible categories of linguistic unit.¹²⁸ This need not imply that any actually existing language need possess examples of every possible mode of linguistic connection. We can imagine, for example, a lan-

guage in which there are no independently meaningful sub-sentential units (in which all sub-sentential foundation is reciprocal foundation). For such a language there could be no categorial grammar (function-argument grammar) in the usual sense, since the opposition between basic and functor category could not be made. It follows from this however that the criticism of Husserl's idea of a 'pure logical grammar', – a criticism encouraged, perhaps, by the parochiality of the examples Husserl chooses – that it in some sense represents an imposition of Indo-European categories upon other languages,¹²⁹ is surely misplaced. Indeed the universal generality of Husserl's pure grammar is shown by the fact that it can be applied even to the diagrammatic languages employed in chemistry, choreography, and elsewhere (as well as to the formal languages of mathematics and mathematical logic).

The distinctions and arguments presented in the 4th Investigation do not by any means exhaust those of Husserl's ideas which are of direct relevance to logical grammar. There is a wealth of material in his hitherto neglected early papers and reviews on logic,¹³⁰ in writings collected in the Husserliana edition of the *Philosophie der Arithmetik*, and in the appendix on syntactic forms and stuffs to *Formale und transzendente Logik*.¹³¹ Thus Husserl's early manuscripts¹³² contain a startling anticipation of the Tractarian account of the role of operations in logic and arithmetic and of formal concepts (cf. Mulligan, 1980 f.). The distinctions between *Unsinn* and *Widersinn* and between formation rules and laws of transformation are clearly and repeatedly expounded by Husserl in his writings from the *Logical Investigations* onward,¹³³ and these writings include also an account of modification used by Husserl to explain such phenomena as nominalisation and the use/mention opposition.¹³⁴ Whilst Husserl's work on modification and his distinction between syntactic forms and syntactic cores have been ignored by most contemporary logicians, close analogues to these distinctions have played an important role in linguistic accounts of different levels of meaning and linguistic structure, particularly in phonology and syntax.¹³⁵

5.3 Husserl's *Investigations*, including the arguments against psychologism presented in the Prolegomena to Pure Logic, had an influence not only in Poland but also in Russia and Bohemia, particularly amongst the members of the Moscow and Prague Linguistic Circles.¹³⁶ Thus the first ever translation of a work by Husserl was a translation of the Prole-

gomena into Russian which appeared in 1909.¹³⁷ Here it is Husserl's influence on Roman Jakobson which is of most importance,¹³⁸ particularly as manifested in Jakobson's work on phonology, which he established as a paradigm not only for other branches of linguistics but also for the human sciences in general.¹³⁹

At the centre of Jakobson's many contributions to phonology is on the one hand the idea that phonological systems contain first of all phonemes such as /p/, /b/, /z/, and on the other hand the concept of *distinctive features* (Husserlian moments) such as compact/diffuse, nasal/non-nasal, etc.

The phoneme is neither identical to the sound nor exterior to it, but is necessarily present in the sound, it remains as something which inheres in it and which is imposed on it: it is the invariant in the variations.¹⁴⁰

We have already seen the notion of an invariant in variations at work in the writings of Husserl and Stumpf. It is employed by Jakobson in his definition of phonemes as that which distinguish words with different meanings in a language.¹⁴¹ If the replacement of one sound by another in a word or morpheme has no effect on the meaning involved, then the two sounds count only as phonetic variants of a single phoneme.

Phonemes so defined are 'complex unities' of binary distinctive features.¹⁴² /p/, for example, in the English consonantal system, is + Labial, - Voiced, - Nasal. A number of questions have been raised as to whether distinctive features must in every case be *binary* oppositions.¹⁴³ More important in the present context however are other claims about distinctive features made by Jakobson which are independent of this issue. The first is the claim that the existence of one distinctive feature necessarily implies the existence of the opposed feature.¹⁴⁴ Jakobson's repeated emphasis on the inseparability, within a linguistic system, of the positive and negative poles of a distinctive feature, amounts to the view that this inseparability cannot be construed in terms of independent properties (pieces) of independent phonemes: each of the terms univocally, reversibly and necessarily calls for its opposite.¹⁴⁵

His second claim takes the form of an ontological objection to the conception of phonemes as classes of sounds, one of the most influential alternatives to the 'inner' approach to phonology in terms of systems of inseparable moments. Jakobson and Halle, 1956, argue against taking the relation between a phoneme and particular sounds in terms of

class-membership. A phoneme cannot be a family or class of sounds related, say, through an equivalence relation of phonetic resemblance, since this would fly in the face of the fact that when analysing phonetic data we deal directly with invariant properties:

When operating with a phoneme or distinctive feature we are primarily concerned with a constant which is present in the various particulars. If we state that in English the phoneme /k/ occurs before /n/ it is not at all the whole family of its various submembers, but only the bundle of distinctive features common to all of them that appears in this position.¹⁴⁶

In the same spirit, we can recognise relations of *partial coincidence* between all of the successive members of the following series: distinctive features, phonemes, syllables, morphemes, words, phrases, clauses, sentences, utterances, discourse. Each of these wholes can be seen as a context for the parts that constitute it:

the word is the context of the morphemes, just as the sentence is the verbal context of words . . . while a morpheme in its turn is the context of phonemes.¹⁴⁷

Philosophers of language have, by and large, not concerned themselves with structural and ontological connections of this sort.¹⁴⁸ They have been tempted, rather, by the possibilities of set-theoretic model-building, i. e. by the construction of analogues of these connections within a set-theoretical framework. Features are conceived as classes and bundles of features are conceived as classes of classes. Once these assumptions are made it is easy to interpret relations of foundation in terms of intersections and Cartesian products of appropriately chosen class-analogues.¹⁴⁹

In one of his most philosophically interesting papers, "Zur Struktur des Phonems" (1939), Jakobson – in the context of a discussion of the importance of Husserl's work for linguistics – considers the controversial question of the reality of phonemes. He points out that phonology is not required to take up a position on the existence of phonemes but that proponents of, for example, the view that phonemes are fictitious constructs should not overlook the fact that such a view commits them to the fictitious nature of *all* linguistic entities. Similarly a view of phonemes as (abstract) classes, or as classes of classes, etc., commits its defenders to a view of all linguistic entities as abstracta.

Between the elements of a phonological system there are relations of foundation of the types distinguished above. In his 1929 Jakobson

pointed out that laws of foundedness can be reformulated as laws of implication:¹⁵⁰

if *a* exists then *b* exists too (necessitation),

if *a* exists then *b* is absent (exclusion),

and in principle we can distinguish also, following Hohenstein, laws of compatibility:

if *a* exists then *b*, *c*, *d* are possible.

An example of necessitation would be: the acquisition of velar and palatal consonants presupposes acquisition of labials and dentals. Similarly the presence of velopalatals implies the simultaneous existence of labials and dentals. The foundation is not, however, reversible: the presence of labials and dentals does not imply the presence of velopalatals.¹⁵¹

Further examples of foundation relations are discussed in Jakobson's "Kindersprache, Aphasie und allgemeine Lautgesetze" (1940/42, §§ 14–17): the acquisition of fricatives presupposes the existence of stops, the existence of one is founded upon the existence of the other.

The laws of one-sided foundation determine the inventory of phonetic systems but also the relative degree of utilisation of particular phonemes in language . . . When both phonemes, the founding as well as the founded, are introduced into child language, the former element generally appears in speech more frequently than the latter.

Jakobson's demonstration that the foundation relations which are to be found in all phonological systems govern the temporal processes of acquisition of linguistic systems by communities of language-users as well as by the individual child, and that they govern the breakdown of such systems in linguistic change and aphasia,¹⁵² provides the most important piece of evidence for the ontological autonomy of phonological systems.

Every phonological system is a stratified structure, that is, forms superposed strata. The hierarchy of these strata is very nearly universal and constant. It appears both in the synchrony and in the diachrony of language; it is, therefore, a panchronic order. If there is a relation of irreversible solidarity [i. e. of one-sided foundation] between two phonological values, the secondary value cannot appear without the first value and the primary value cannot be eliminated without the secondary value. This order shows itself in the existing phonological system and it governs all its mutations; the same order determines, as we have shown,

the learning of language, systems in process of development and – let us add – it persists in language disturbances, systems in the process of breaking up.¹⁵³

Jakobson's seminal analysis of phonemes, their distinctive features and their relations, has been extended by him to syntax, morphology and semantics, and this work has thrown much light on the status of semantic and syntactic features and on their relations of dependence.¹⁵⁴ Holenstein describes Jakobson's analysis of the Russian case-system, for example, as a contribution to eidetic phenomenology which 'shows how single domains of objects can be described by a harmonious system of relational properties'.¹⁵⁵

The two cases of influence of Husserl described briefly above¹⁵⁶ suggest important substantive questions. To what extent has Husserl's theory, in its application to categorial grammar and to phonology, been modified? Are there good grounds for the modifications and is the potential of Husserl's theory for throwing light on grammar and linguistics yet exhausted?

§ 6 Further Developments: Köhler, Lewin, Rausch

6.1 The most interesting example of applied whole-part theory outside the field of linguistics is provided by Wolfgang Köhler's *Die physischen Gestalten in Ruhe und im stationären Zustand* (Physical Gestalten at Rest and in the Stationary State) of 1920. Köhler's principal thesis can be stated, somewhat crudely, as follows: that even the most prototypical cases of summative or additive wholes – a heap of resistors, for example, or a line of motor-cycles, or a sprinkling of iron filings – can be converted into wholes that are non-summative by immersion in an electromagnetic field, or by the wiring up of their elements to an electric current.

Köhler's work on physical Gestalten and his recognition of the scientific importance of non-summative wholes were initially sparked by experiments on the behaviour of apes in relation to their environment (op. cit., p. vii). Such experiments had implied that there are states and processes whose characteristic properties and effects are not compounded out of properties and effects of their parts.¹⁵⁷ Such states and processes had, since Ehrenfels' classic paper "Über Gestaltqualitäten" of 1890, come to be called 'Gestalten'. The two criteria for Gestalthood formulated by Ehrenfels were as follows:

A psychological whole (complex content) exhibits Gestalt structure only if

I. the n constituent stimuli (e. g. the n notes of a melody) are such that when experienced in order by a single subject then the total stimulus is greater than the sum of the separate stimuli as these would be experienced by n separate subjects; (the difference between the two is, in Ehrenfels' terms, the specific Gestalt-quality of the complex); and

II. this specific property is such that it remains unchanged when the complex of stimuli on which it rests suffers certain determinate kinds of displacement (e. g. the transposition of the melody into a different key).¹⁵⁸

Criterion I is satisfied, Kohler argues, only if the constituents of the complex satisfy the condition which he calls *functional proximity* (absent when, for example, the notes of a melody are sounded at one month intervals; cf. 1920, p. 35).¹⁵⁹ Further, both criteria apply just as well to continuous manifolds as to the finite wholes of discreta considered by Ehrenfels. Substituting the more general criterion of functional proximity for Ehrenfels' criterion I, it can then be claimed that not only psychological complexes but also certain physical wholes exhibit Gestalt structure in the modified sense. The two resultant criteria are satisfied, for example, by electrostatic structures, invariant with respect to transpositions of the material make-up of the conductor (which can be made of lead, of silver, etc.), of the spatial position and dimensions (though not the form) of the conductor, and with respect to variation in the total charge. Similarly, the electromotive force at the boundary between two electrolytes is invariant, given constant relative difference, with respect to changes in absolute concentration.¹⁶⁰

Köhler put forward the hypothesis that, physics being a much further advanced discipline than psychology, it would be possible to clarify psychological Gestalten by investigating their physical analogues. The hypothesis gains additional strength from the fact that the central nervous system itself, and each specific stimulus field within the human organism, can be conceived as a physical Gestalt-structure, nerve reactions as Köhler conceived them being in some respects analogous to electrochemical reactions in weak, partially ionised solutions (1920, p. 5 f.).¹⁶¹

Köhler's own definition of 'summativity' was as follows:

A collection (*Zusammen*) is a pure sum of parts or pieces [is a purely summative whole] if and only if it can be assembled from its parts one after the other without

any of the parts suffering any alteration as a consequence of the process of assembly (op. cit., p. 42).¹⁶²

He conceives this definition as equivalent to the following:

A collection is a pure sum if and only if, through separation of parts or pieces, neither the residue partial collection . . . nor the part that has been separated should suffer any alteration (loc. cit.).

These definitions are, as we shall see in 6.3 below, too crude to demarcate a single homogeneous category of cases. Not only is their equivalence highly questionable, but they are too crude also in virtue of the fact that a given whole may exhibit summativity in respect to certain properties, but Gestalt-structure¹⁶³ in respect to others. Three electrostatically charged conductors in proximity to each other, for example, constitute a pure sum from the point of view of weight, but not from the point of view of distribution of charge: differences in electrostatic potential across a system are not reducible to electrostatic properties of the parts of the system.

Examples of absolutely summative wholes are difficult to come by. Even a heap of stones fails to satisfy the definition in an absolute sense, in virtue of the gravitational relations between the individual stones and the earth, as a result of which any relative motion of the stones will normally cause (at least) changes in position of the residue of the heap. We can however agree that

with certain exceptions those objects readily designated as 'things'¹⁶⁴ constitute purely summative wholes with one another so long as they do not come into contact . . . or, if they do come into contact, so long as the contact is weak and lies perpendicular to the direction of the earth's gravity (op. cit., p. 48).

If, in contrast, we look at the charge-structure of a conductor: any change in the physical form, any partitioning of the conductor, any alteration or removal of charge at any point, of necessity brings about a change in the charge-structure of the whole. It is as if the parts of the conductor interpenetrate reciprocally; they do not exist merely side by side with each other, as is the case in a purely summative whole.

In attempting to develop a logic of non-summative wholes Köhler introduces the concept of an *Eigenstruktur* (inherent or intrinsic structure), i. e. of that kind of spontaneous order which affects given physical

materials in reflection of certain types of surrounding conditions. An *Eigenstruktur* is not decomposable into pieces (it is not the *sum* of any constituent microstructures). It is however possible to distinguish abstractly within it *moments* of the structure: the *charge at a given point* in a complex conductor is a moment of the total charge structure.¹⁶⁵ This implies that it is impossible to create an *Eigenstruktur* by injecting an appropriate charge at every point: the moments of the structure *verhalten sich gegenseitig nicht wie 'Dinge'* (cf. op. cit., p. 66).

Gestalten as conceived by Ehrenfels are produced by mental activity on the basis of discrete, pre-existent contents, merely externally related to each other and in themselves undergoing no change as a result of becoming combined into complexes. Physical Gestalten are not produced in this sense. And, Köhler argued, even with regard to mental formations it is not the case that they are built up out of elementary sensations: we cannot explain perception as a summation of externally related micro-stimuli. To account adequately for either physical or psychological phenomena a more general theory of part-whole relations is required. It has been argued above that such a theory was outlined by Husserl in the 3rd Logical Investigation; hence it will be interesting to examine Köhler's reactions to Husserl's views. We have already pointed out that Husserl borrowed from Meinong the terminology of foundation in advancing his theory of part and whole. Meinong had merely substituted for Ehrenfels' 'Gestalt' the term 'founded content', accepting Ehrenfels' underlying theory with only minor hesitations (see Meinong, 1891 and 1899 and compare Ehrenfels, 1937). Unfortunately Köhler, in his discussion of Husserl's work, assumes that Husserlian foundation is to be understood exactly as in the Ehrenfels-Meinong theory.

Consider, once again, a system Σ of conductors, *a*, *b*, *c*, in an electrostatic field. Köhler argues that *a*, *b* and *c* are not dependent parts of the system in Husserl's sense (see § 3 above), since each might well occur in isolation from the others (p. 32 f.). This is to ignore Husserl's distinction between *a qua* constituent of Σ (or *a qua* bearer of the given charge) and *a qua* consignment of conductive material, the former a moment, the latter a mere piece of the system Σ . That is, it is to ignore Husserl's thesis to the effect that foundation relations hold between individuals only in reflection of the essential structures of those individuals.

Köhler also criticises Husserl for ignoring the question as to how and under what conditions entities come together to form a whole (p. 58). For Köhler, what is interesting is precisely the real, physical possibility

of an object's remaining in existence whilst at the same time suffering a determinate and spontaneous transformation of its intrinsic properties: "Husserl, as far as I can see, asks to what extent pieces somehow placed in relation to each other as if they were things can allow a whole to arise above them" (loc. cit.).

Such remarks reveal that Köhler, again through confusion with Meinong's specifically psychological concept of foundation, misunderstood the purely formal nature of Husserl's work. This was to a certain extent encouraged by Husserl's continued use of the psychological terminology of 'contents' and by his lack of consideration of non-psychological examples of foundation relations. The deficit is made up, to some extent, by vol. III of Ingarden's *Der Streit um die Existenz der Welt* (1974).¹⁶⁶

Köhler contrasts his own ontological views with *absolute holism*, on the one hand, and *atomism*, on the other (pp. 153 ff.).¹⁶⁷ According to the position of absolute holism, nature contains no independent parts; all states and processes are real only in the nexus of the world as a whole, all parts are products of abstraction. This view, which Köhler castigates as a form of romanticism, implies the impossibility of natural science. The atomist position, in contrast, regards nature as built up out of *Und-Verbindungen* (mere sums) of independent parts.¹⁶⁸ Köhler takes a position according to which holism is of only limited or local validity. It is, he claims, the fundamental experience of all experimenters, that – in contrast to the absolute holist view – leaving aside the interdependencies to be found within finite neighbourhoods of certain determinate types, the interconnections between physical processes in different regions of the world are relatively trivial (p. 156). Unfortunately the unacceptability of absolute holism, combined with the fact that the overwhelming bulk of the furniture of the world of ordinary everyday experience exhibits relations of a merely additive character, has led to the acceptance of atomistic or micro-reductivist research programmes in all developed sciences¹⁶⁹ at the expense of a lack of understanding of those types of structures crucial to the understanding of the phenomena of psychology.

6.2 The second development of the theory of part and whole considered here was made by another student of Stumpf, the psychologist Kurt Lewin in his book *Der Begriff der Genese in Physik, Biologie und Entwicklungsgeschichte. Eine Untersuchung zur vergleichenden Wissen-*

schaftslehre (The Concept of Genesis in Physics, Biology and Evolutionary History. An Investigation in Comparative Theory of Science).¹⁷⁰ We tend to conceive a physical object such as a stone as the same object from moment to moment. Lewin however sees temporally extended objects as multiplicities of successive entities. Consider, for example, a perfectly isolated bell-jar in which chemical reactions are taking place across a temporal interval (t_1, t_2). Define G_{t_1} as the totality of simple or complex chemical formations existing in the bell-jar at t_1 . Then G_{t_2} stands to G_{t_1} in the relation of existential being-such-as-to-have-come-forth-from (*existentiellen Auseinanderhervorgegangenseins*) to G_{t_1} , a relation which is independent of the specific properties of the constituents of the G_{t_1} . It is this same relation of existential antecedency that one has in mind when one considers e. g. the phenomenon of expansion of a metal in physics. It was Lewin who introduced the term 'genidentity' to designate the given relation. However, the concept of genidentity that is relevant to physical formations is, as we shall see, distinct from that which is relevant to biology: the transition from one discipline to another implies a corresponding change in the manner of dividing up reality into units. Hence we shall find it necessary to distinguish a number of distinct (though interrelated) concepts of existential being-such-as-to-have-come-forth-from.

It is necessary, first of all, to distinguish simple from complete genidentity. An amputated limb is simply genidentical with the whole body from which it had been amputated. The relation of complete genidentity, in contrast, holds only between the whole body on the one hand, and the totality consisting of mutilated body, limb and residue, on the other. Complete genidentity is thus simply an expression of the physicist's attempt to isolate his experiments from extraneous disturbances.

The concept of complete (physical) genidentity is required if we are to formulate, for example, the law of conservation of mass. This should properly read (cf. Lewin, op. cit., p. 12):

if two or more physical formations are completely genidentical with each other, then they are of identical mass.

This law expresses a relation among objects. The law of conservation of energy expresses a relation not among objects, but among events, which also exhibit relations of genidentity (for example between a dying and a death).¹⁷¹

Simple (physical) genidentity may be symbolised by means of ' \equiv^p '; complete (physical) genidentity by ' \equiv^c '; absolute identity by ' \equiv '. Writing ' \leq ' for 'is a proper or improper part of' and ' \prime ' for 'is discrete from'¹⁷², ' \equiv^p ' can be defined in terms of ' \equiv ' as follows:

$$a^p \equiv b := \neg \exists x (x/a \ \& \ x^p = b) \ \& \ \neg \exists x (x/b \ \& \ x^p = a)$$

(cf. Lewin, op. cit., p. 27). Then clearly:

$$a^p \equiv b \rightarrow \forall x (x/b \rightarrow a^p \neq x)$$

$$a^p = b \rightarrow \forall x (x/a \rightarrow b^p \neq x)$$

$$a^p = b \rightarrow \exists a' \exists b' (a' \leq a \ \& \ b' \leq b \ \& \ a'^p \equiv b')$$

Thus if a piece of metal falls into an acid, then we know that this substance must be present in some form in the liquid, even though it may have entered into chemical combination with it (op. cit., p. 29).

If we define $t(x)$ as the temporal position of the object x , then we may formulate the following principle of temporal density of genidentity:

$$a^p \equiv b \rightarrow \exists x (t(a) < t(x) < t(b) \ \& \ a^p = x \ \& \ x^p = b)$$

(and similarly for ' \equiv^c ').

We may also formulate principles of continuity:

$$a^p \equiv b \rightarrow \forall t (t(a) < t < t(b) \rightarrow \exists c (t(c) = t \ \& \ a^p = c \ \& \ c^p = b))$$

and of transitivity:

$$(a^p \equiv b \ \& \ b^p \equiv c) \rightarrow a^p \equiv c$$

(this principle does not hold for ' \equiv^c ').

Sequences of physical formations exhibiting complete genidentity extend indefinitely, both into the past and into the future:

$$\forall a \exists b \exists c (a^p = b^p = c \ \& \ t(b) < t(a) < t(c)).$$

Further, such sequences possess no singularity points. Every arbitrary section through a sequence of genidentical formations unequivocally determines the whole sequence in both directions.

In Lewin's view, physical formations may be demarcated into parts and wholes at will. "Every real part of a physical formation and every

complex of physical formations can be considered in its turn, insofar as genidentity relations come into question, as a physical formation in its own right" (p. 39). We can now assert the following principles concerning the decomposition of physical complexes into their constituents (e. g. into constituent molecules):

$$(a \equiv [a_1, a_2, \dots, a_n] \& a^p \equiv b) \rightarrow \exists_{i=1}^n b_i \{ b \equiv [b_1, b_2, \dots, b_n] \& \forall j (b_i / b_j \rightarrow i \neq j) \& a_1^p \equiv b_1 \& \dots \& a_n^p \equiv b_n \};^{173}$$

$$(a \equiv [a_1, a_2, \dots, a_n] \& b \equiv [b_1, b_2, \dots, b_n] \& a^p \equiv b_1 \& \dots \& a_n^p \equiv b_n) \rightarrow a^p \equiv b.$$

Biological genidentity relations hold wherever roots, sprouts, eggs, embryos, develop into something which they are not.

Embryology, evolutionary theory, in short the whole of biology insofar as it is concerned with phylogenetic or ontogenetic processes of development, whether of a morphological or a physiological kind, consists above all in the investigation of biological formations which stand in relations of existential being-such-as-to-have-come-forth-from (p. 53).

Sequences of biologically genidentical formations are to be distinguished from physical genidentity sequences:

The physical genidentity sequences flowing forwards from an egg lead to the widest variety of possible formations, and if any kind of physical genidentity relation obtains between the egg and the hen, then there belongs to the adult hen at most formations which are physically genidentical with a fraction of the egg (p. 56).

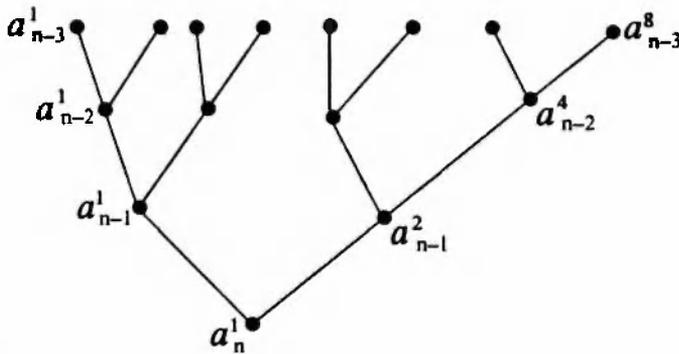
Lewin distinguishes two relations of biological genidentity, the relation of genidentity between successive sections of a single individual, (discussed below) and the relation of (simple and complete) genidentity between an individual and his descendents, whether this obtains between metazoa, protozoa, animals, plants, between complete organisms or individual cells, whether through vegetative or sexual reproduction, through sprouting or division. He calls the relation between an individual and his descendents (or, more generally, between successive members of different generations) *Avalgenidentität*.

If we write ' $a^a = b$ ' for ' a stands to b in the relation of descendent to

forbear', then typically we have to face systems of (simple) avalgenidentity relations such as the following:¹⁷⁴

$$\begin{array}{ccc}
 a_{n-1} & a \cong & \\
 a'_{n-1} & a \cong & \\
 a''_{n-1} & a \cong &
 \end{array}
 \begin{array}{c}
 \vdots \\
 \\
 \\
 \\
 \vdots
 \end{array}
 \begin{array}{ccc}
 & a \cong & a_{n+1} \\
 & a \cong & a'_{n+1} \\
 & a \cong & a''_{n+1}
 \end{array}$$

Complete avalgenidentity, symbolised by $a \equiv$, gives rise to systems such as the following:



where $a_n^1 a \equiv [a_{n-1}^1, a_{n-1}^2] a \equiv [a_{n-2}^1, a_{n-2}^2, a_{n-2}^3, a_{n-2}^4]$, etc.

Writing S_{-i}^k for $[a_{n-i}^1, \dots, a_{n-i}^k]$, then we may symbolise the avalgenidentities between successive sections through an avalsequence by:

$$S_0^1 a \equiv S_{-1}^2 a \equiv S_{-2}^4, \text{ etc.}$$

Further, we have

$$\begin{array}{l}
 S_i^a \equiv S_j \rightarrow \forall a_i \in S_i (a_i^a = S_j) \\
 a_i^a = b_0 \rightarrow \exists x_i y_i \dots [a_i, x_i, y_i, \dots] a \equiv b_0
 \end{array}$$

A single biological formation may occur more than once as element of a single section through an avalsequence. And avalsequences may intersect: a section may belong to distinct complete avalsequences, although

it may occur as the 0-sequence in only one avalsequence (op. cit., pp. 90–94). Elements of the same generation in an avalsequence may be wholly non-contemporaneous; contemporaneous biological formations may belong to distinct generations. In contrast to the physical case, the elements of an avalsequence are not temporally punctual entities, but entities exhibiting their own individual temporal development.

Lewin also considers the relation of genidentity between successive temporal sections through biological individuals (including not only complete organisms, but also individual organs, cells, etc.). Writing ' $a' = b'$ ' for ' a is simply individual-genidentical to b' ' (as, for example, an ameoba is simply individual-genidentical to either half of itself after splitting), and ' $a' \equiv b'$ ' for ' a is completely individual-genidentical to b' ', we have:

$$a' = b' \rightarrow \exists xy \dots x'y' \dots ([a, x, y, \dots]' \equiv [b, x', y', \dots]).$$

Individual-genidentity is a transitive relation. It is characterised, like physical and avalgenidentity, by continuity, but also by the existence of a youngest section (having no individual-genidentical predecessors) and an oldest section (*der Todesschnitt*, having no individual-genidentical successors).¹⁷⁵

6.3 In our discussion of the Gestalt-psychological concept of summative whole in § 6.1 above, we pointed out that the definition given by Köhler in his *Die physischen Gestalten in Ruhe und im stationären Zustand* (p. 42, cf. p. 67 above) did not succeed in demarcating a single formal ontological concept. The task of reconstructing Köhler's definition was undertaken by his student, Edwin Rausch, in his work "Über Summativität und Nichtsummativität" (On Summativity and Non-Summativity) of 1937. This work is of interest since, building upon a close familiarity with the range of physical and psychological examples investigated by Köhler and his associates, Rausch formulates a taxonomy of the corresponding types of wholes and parts whose generality rests essentially on exploitation of the flexibility of modern symbolic logic (and specifically in its capacity to represent *multiple generality*).

Rausch takes as his starting point the concept of a finite, spatially extended, static *manifold* or *collective* (*Zusammen*), *Z*, a concept interpreted broadly enough to include not only collective wholes occurring in objective physical space, but also spatial manifolds occurring in the

phenomenal fields of conscious subjects. The constituents of a collective Z may therefore be either physical things or perceptual contents arising, e. g., in the observation of spatial signs.¹⁷⁶

A *partition* (*Einteilung*), E , of Z is a purely conceptual division of Z , and of the space (or phenomenal field) surrounding Z , into a finite number n of discrete, more or less arbitrarily demarcated parts, t_i (Z^E), $i = 1, 2, 3, \dots, n$, which exhaust the totality of Z .¹⁷⁷ It is as if E imposes upon Z (and upon the surrounding space), according to its dimensionality, a 1-, 2- or 3-dimensional *grid*. A line of n motorcycles, for example, has a *natural partition* into its constituent vehicles, but it may also be partitioned into e. g. $2n$ wheels plus a single residual constituent.¹⁷⁸

Given a manifold Z and a partition E we may consider properties of the resultant partitioned manifold Z^E and of its parts t_i (Z^E), $i = 1, \dots, n$, either from the point of view of conceptually possible *removal* of parts, or from the point of view of conceptually possible processes of *assembly*. Köhler, as we saw, held these two modes of consideration to be such as to lead to equivalent definitions of summativity (see p. 67 above). It may indeed be the case that the respective definitions are extensionally equivalent, – or that they are extensionally equivalent over all example-domains familiar to workers within existing scientific disciplines. Rausch, however, argues that in the absence of more detailed analyses it would be illegitimate to presuppose that they are logically equivalent.

Consider, first of all, the concept of *invariance* of an arbitrary part t_i (Z^E) under removal or subtraction from the collective Z . We shall employ ' $\Phi(t_i)$ ' to designate the physical removal, by some specific process (e. g. pruning of a leaf, unscrewing of a bolt, etc.), of t_i from Z .

$$t_i(Z^E) \text{inv} \Phi(t_i)$$

will express the proposition that t_i is invariant under removal from Z^E by process Φ . The first concept of summativity distinguished by Rausch is then defined as follows (Rausch, p. 216):

$$S_{\text{inv}}^{\Phi}(Z^E) := \forall i (t_i(Z^E) \text{inv} \Phi(t_i)).$$

Clearly Köhler's collection of spatially disparate stones fulfils this definition under the natural partition, where ' Φ ' signifies simple physical removal.

A second concept of summativity is obtained if we consider not the invariance of an arbitrary part upon removal, but rather the invariance of the *residue*:

$$S_{\text{inv}}^{\Phi}(Z^E): = \forall i ((Z^E - t_i) \text{inv} \Phi(t_i)).^{179}$$

We can also impose the condition of invariance, under removal of an arbitrary part, not of the residue-manifold taken as a whole, but of any arbitrary part of the residue-manifold:

$$S_{\text{inv}}^{\Phi'}(Z^E): = \forall i \forall j (t_j (Z^E - t_i) \text{inv} \Phi(t_j)),$$

or the weaker condition of invariance of at least one part of the residue-manifold:

$$\forall i \exists j (t_j (Z^E - t_i) \text{inv} \Phi(t_j)).$$

A process Φ of removal or separation may be associated with a converse operation $\bar{\Phi}$ of *addition* of parts. An initial approximation to the first of Köhler's two definitions of summativity might then be:

$$S_{\text{K}}^{\bar{\Phi}}(Z^E): = \forall i (t_i (Z^E) \text{inv} \bar{\Phi}(t_i))$$

where $t_i (Z^E) \text{inv} \bar{\Phi}(t_i)$ is to signify that t_i remains invariant under addition to $Z^E - t_i$.

We can define the following variant concepts:

$$S_{\text{K}}^{\bar{\Phi}'}(Z^E): = \forall i ((Z^E - t_i) \text{inv} \bar{\Phi}(t_i))$$

i. e. each sub-manifold $Z^E - t_i$ remains invariant under augmentation by the corresponding t_i ; an n -fold whole is summative in this sense iff every $(n - 1)$ -fold sub-whole is invariant under augmentation;

$$S_{\text{K}}^{\bar{\Phi}''}(Z^E): = \forall i \forall j (t_j (Z^E - t_i) \text{inv} \bar{\Phi}(t_j));$$

$$S_{\text{K}}^{\bar{\Phi}'''}(Z^E): = \forall i \exists j (t_j (Z^E - t_i) \text{inv} \bar{\Phi}(t_j)).$$

A more adequate approximation to Köhler's two definitions is achieved however if we consider invariance not merely under separation or addition of single members, but rather invariance under complete or total de-

composition (or construction) of the whole Z . An n -manifold Z^E exhibits $n! \sum_{i=1}^{n-1} \frac{1}{i!}$ in principle possible piece-wise decompositions (Rausch, pp. 233–30). Clearly there may be types of collectives in which the order of decomposition plays no role, others in which certain orders of decomposition have distinct effects, or are physically impossible. Summativity will be associated with collectives belonging to the first of these groups.

If we imagine the tree of possible decompositions of the n -manifold Z^E , any given t_i will be contained in $2^{n-1} - 1$ of the (proper and improper) sub-manifolds which form the nodes of the tree. Introducing the expression ' $M_{ik}(Z^E)$ ' to designate the k th respective t_i -containing sub-manifold of Z^E under some lexicographical ordering, $k = 1, 2, 3, \dots, 2^{n-1} - 1$, (Rausch, p. 230f), we can define the following strengthened form of Köhler-summativity:

$$S_{KK}^\Phi(Z^E) := \forall i \forall k (t_i(M_{ik}(Z^E)) \text{inv} \Phi(t_i)),^{180}$$

with the variants:

$$S_{KK}^\Phi(Z^E) := \forall i \forall k ((M_{ik} - t_i) \text{inv} \Phi(t_i)),$$

etc. (see Rausch, pp. 235–37).¹⁸¹

Köhler's original definitions refer merely to 'assembly' and to 'separation'. Thus they pay no regard to the possibility that variance and invariance properties may be dependent upon the manner in which parts are joined or removed (suddenly or gradually, violently or cautiously; 'with a hammer, with a screwdriver', etc.). We can take into consideration the range of possible processes of addition/subtraction by treating ' Φ ' as a variable expression, thereby obtaining the following amended version of S_{inv}^Φ :

$$S_{inv}^\Phi(Z^E) := \forall i \forall \Phi (t_i(Z^E) \text{inv} \Phi(t_i)),$$

and the much weaker condition:

$$S_{inv}^\circ(Z^E) := \exists \Phi \forall i (t_i(Z^E) \text{inv} \Phi(t_i)),$$

and correlately for each of the definitions S_{inv}^Φ , etc.

Köhler's definitions refer further merely to 'invariance of parts'. It may however be fruitful to consider invariance not of parts, but of *properties* of parts: a banana is invariant under removal from a bunch in regard to its weight, not e. g. in regard to its spatial position. Introducing ' $e(t_i)$ ' as a variable functional expression ranging over properties of t_i , we may generate a further sequence of summativity concepts as follows:

$$S_{\text{inv}}^{\Phi^*}(Z^E) := \forall i \forall e (e(t_i) \text{inv}\Phi(t_i))$$

and

$$S_{\text{inv}}^{\Phi^{**}}(Z^E) := \exists e \forall i (e(t_i) \text{inv}\Phi(t_i)),$$

etc.

In relation to $S_{\text{inv}}^{\Phi^{**}}$ (and certain variants) it is possible to define a concept of summativity for properties: a property e is said to be $S_{\text{inv}}^{\Phi^{**}}$ -summative relative to Z and E iff it satisfies

$$\forall i (e(t_i(Z^E)) \text{inv}\Phi(t_i))$$

(Rausch, p. 267). e is absolutely $\bar{S}_{\text{inv}}^{\Phi^{**}}$ -summative iff it satisfies

$$\forall Z \forall E \forall i (e(t_i(Z^E)) \text{inv}\Phi(t_i)).$$

Similarly we can define concepts of summativity-with-regard-to-property- e (e. g. mass, colour, etc.) by, e. g.

$$S_{\text{inv}}^{\Phi e}(Z^E) := \forall i (e(t_i(Z^E)) \text{inv}\Phi(t_i)).$$

The summativity concepts defined so far all relate to manifolds subject to arbitrarily determined partitions E , conceived as imposed by the experimenter. If we wish to define a concept of *ontological summativity*, a concept which would involve no relativisation to any arbitrarily imposed partition, i. e. which would relate directly to the underlying manifold itself, then this can be achieved by treating the hitherto constant term ' E ' as a variable, and quantifying over partitions of Z as follows:

$$S_{\text{ont}}^X := \forall E (S^X(Z^E))$$

where X ranges over the various S-indices introduced above.

A further sequence of ontological summativity concepts is obtained by means of the schema:

$$S_{\text{ont}}^{X^o} := \exists E(S^X(Z^E)).$$

Besides subtraction $\bar{\Psi}$, and addition $\bar{\Psi}$, of parts, we can consider also other operations on a collective. In particular we can consider various types of *variation* ψ (of the parts of a physical whole or of a psychological content).¹⁸² Thus for example

$$S_{\text{inv}}^{\psi}(Z^E) := \forall i((Z^E - t_i)\text{inv}\psi(t_i))$$

signifies that Z^E is such that each $Z^E - t_i$ is invariant under the process of ψ -variation (e. g. increase in size, or intensity of electric charge) of t_i .¹⁸³

Rausch considers the following condition of non-summativity relative to ψ -variation:

$$N^{\psi}(Z^E) := \exists e \exists e' \forall i (e(t_i(Z^E))\text{var}\psi(e'(t_i)))$$

i. e. that, for a given manifold and partition, there should exist a pair of properties, e and e' which are such that, for each member t_i of Z , ψ -variation of the first property brings about a consequent variation of the second property. It would be fruitful to investigate the relationship between this condition and the condition of one-sided dependence of properties considered by Stumpf and Husserl.

e can be said to be dependent in Z upon e' if

$$\forall E \forall i (e(t_i(Z^E))\text{var}\psi(e'(t_i)));$$

and e is absolutely dependent upon e' if

$$\forall Z \forall E \forall i (e(t_i(Z^E))\text{var}\psi(e'(t_i))).$$

Thus consider a system L of n conductors \bar{L}_i , connected together by wire whose capacity is negligible in relation to that of the L_i and placed so far apart that they do not influence each other through field effects. Writing ' $c(L_i)$ ' for 'the capacity of L_i in L ' and ' $q(L_i)$ ' for 'the quantity of charge in L_i ', then the state of equilibrium of the system is given by

$$q(L_i) = q(L) \frac{c(L_i)}{\sum_{j=1}^n c(L_j)}$$

The condition N^ψ given above is satisfied by the system L under the natural partition defined by $t_i = L_i$, where $e = c$, and $e' = q$. That is:

$$\forall i (q(L_i(L)) \text{var}\psi(c(L_i))),$$

where ' ψ ' signifies a specific variation of capacity (cf. Rausch, p. 275f).

Similarly we have:

$$\forall i \forall j (q(L_j(L - L_i)) \text{var}\psi(c(L_i)))$$

i. e. a change in the capacity of an arbitrary conductor brings about an adjustment in the charge of all remaining conductors: the charge of a conductor depends not only on its own capacity (on the local conditions), but on the capacities of all the remaining conductors in the system.

The above is a treatment of only a selection of the summativity and non-summativity concepts distinguished by Rausch. They are of relevance not only in the areas of physics and perceptual psychology but also, in principle, to wholes of every kind. Thus we can make a distinction between two kinds of summativity according to the *order* in which the elements of a whole may be separated from it (see n. 180 above), and such a distinction between kinds of wholes can be seen at work in, for example linguistics, in the familiar distinction between restrictive or defining and non-restrictive or non-defining relative clauses. This distinction might be illustrated by respectively:

the man in the corner who is looking at his hands is a philosopher

and: *the man in the corner*, who is looking at his hands, *is a philosopher.*

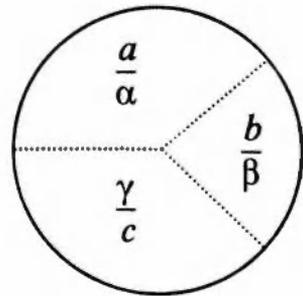
Or: *knowledge which comes from books* is power

and: *knowledge*, which comes from books, *is power.*

The non-italicised portion of the second example is in each case detachable without loss of meaning. This is not the case in regard to the

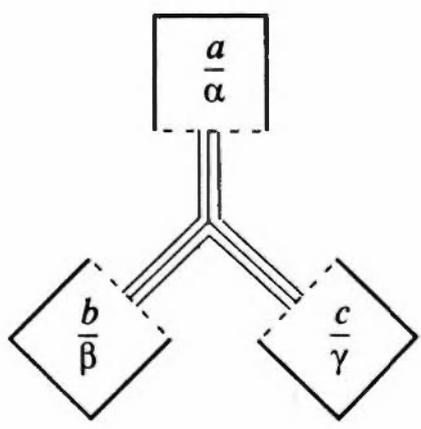
corresponding portion of the first example: here the italicised portion must be separated from the sentence as a whole.

6.4 We should like, by way of an appendix, to consider briefly a number of notational issues raised by the Stumpf-Husserl theory of wholes and parts.¹⁸⁴ The dependence diagram,

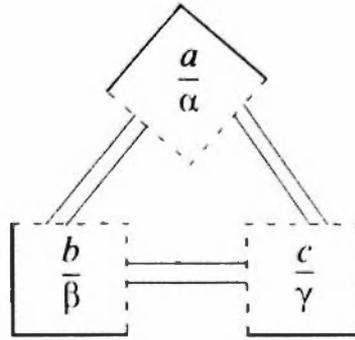


as already noted at n. 72 above, is ambiguous; it may signify either the obtaining of three distinct relations of pairwise mutual dependence, of *a* on *b*, *b* on *c*, and *a* on *c*, or the inherence in *a*, *b* and *c* of a single relation of three-fold mutual dependence. That this distinction is not an empty one is seen by considering the case of a polymorphously polygamous tribe which allows marriage both between pairs and between triples of individuals. If a single 3-marriage bond holds between *a*, *b* and *c*, then divorce between *a* and *b* can be effected only at some cost to *c*, which is not the case where *a*, *b* and *c* are respectively 2-married to each other.¹⁸⁵

Three-fold reciprocal dependence may be unambiguously represented by means of a diagram such as the following:



(and similarly for 2-, 4- and n-fold reciprocal dependence). The complex of relations of pair-wise dependence may then be symbolised by:

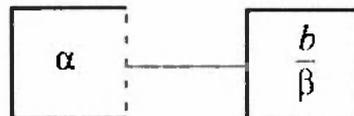


Mutual dependence relations (whether 2- or n-fold) are in each case represented by multiple lines. One-sided dependence may be represented by means of single lines:



where independence of an object or content is symbolised by solid walls.

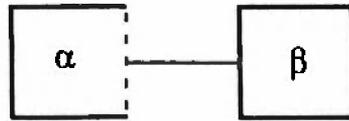
The above diagram may represent, say, the inference of a specific moment of redness a in a beetroot, b . It is as if a and b lay behind the page, capable of being viewed as articulated in some specific way by the windows which make up the figure. This articulation may be more or less crude.



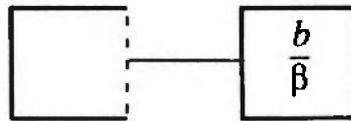
for example, might symbolise 'the tomato b is red';



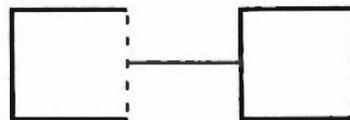
'the individual redness-accident a inheres in a tomato';



'a tomato is red';



'something inheres in the tomato b ';



'something inheres in something';

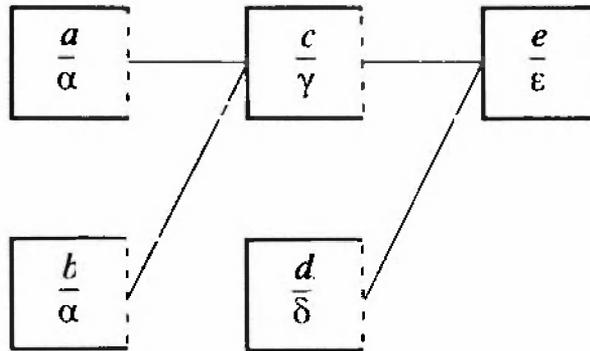
and finally perhaps also



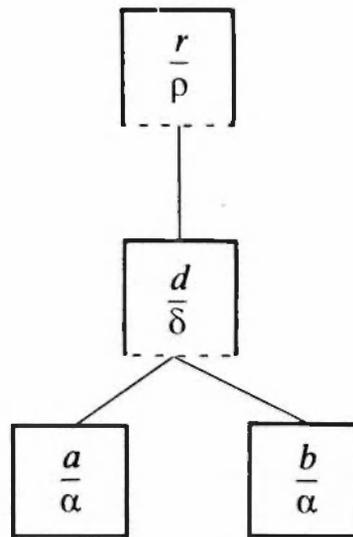
'something (some independent whole) exists'.

Such devices, which go some way to rendering superfluous the quantifier machinery of modern predicate logic and at the same time to restor-

ing the common noun to philosophical respectability,¹⁸⁶ have been thoroughly explored by Wolfgang Degen in unpublished writings.¹⁸⁷



might signify the dependence of two successive bouts of fever, *a* and *b*, on the disease *c*, inhering, along with the knighthood, *d*, in the individual *e*.



might signify the dependence of the reconciliation *r* upon the disagreement *d*, which is in turn a two-place temporally extended moment inhering in the pair of individuals *a*, *b*.¹⁸⁸

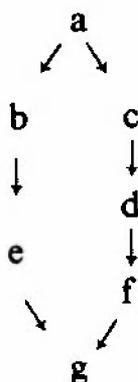
The analogy of the window introduced above is not an arbitrary one. Dependence diagrams are *pictures* of states of affairs, and may be taken as propositional signs in the sense of the *Tractatus*.¹⁸⁹ The directly depicting language thereby determined reveals a hitherto unnoticed point of contact between Wittgenstein's early logical work and the primitive

experiments of Euler and Venn on a two-dimensional logical notation.¹⁹⁰

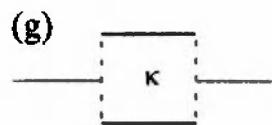
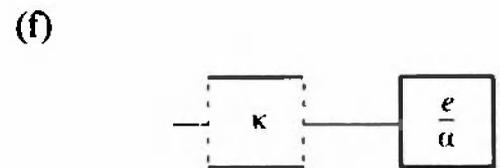
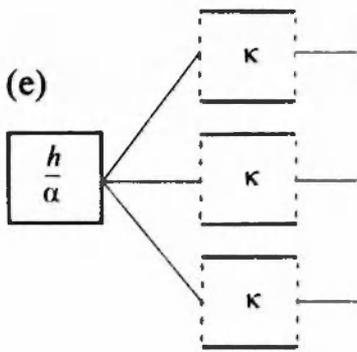
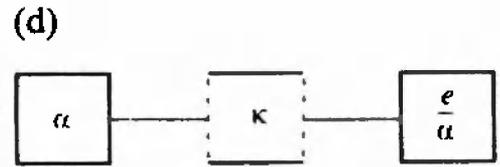
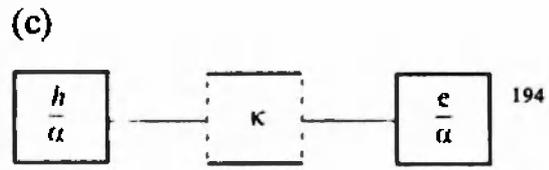
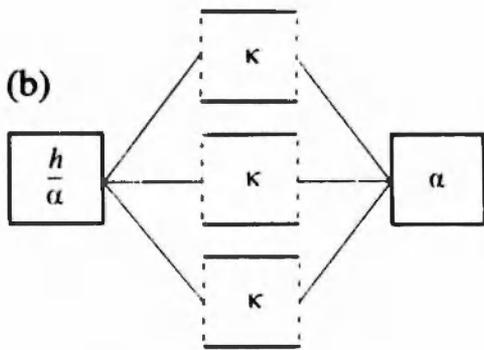
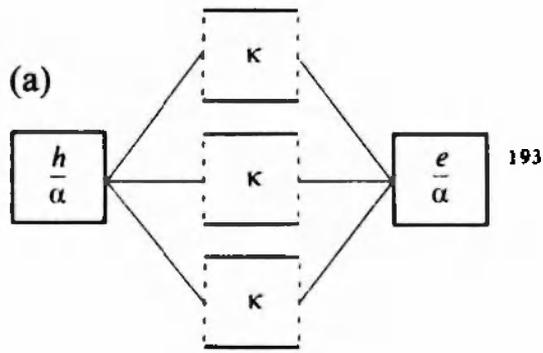
The parts of an existing state of affairs also exist.¹⁹¹ Hence a directly depicting language must satisfy the condition that *every well-formed constituent of a propositional sign Γ admits of being inferred as a consequence of Γ* . Degen has pointed out that a language which satisfies this requirement allows the perspicuous representation of certain forms of inference (which he calls *grammatical* as opposed to *logical* inference) in a fashion which avoids the highly unnatural detour through quantification theory imposed by predicate logic. Thus consider the sentences:

- (a) Hans kisses Erna three times.
- (b) Hans kisses someone three times.
- (c) Hans kisses Erna.
- (d) Someone kisses Erna.
- (e) Hans kisses three times.
- (f) Erna is kissed.
- (g) There is kissing.

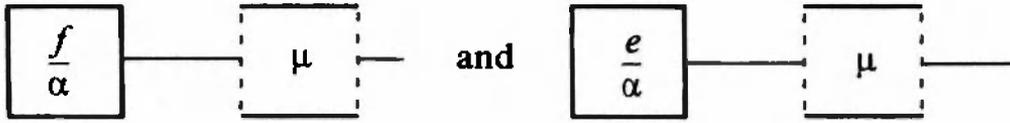
Part of what is involved in the mastery of the grammar of a language such as English is the ability to recognise immediately the inferential relations between sentences of this kind, relations which may, in this case, be represented as follows:



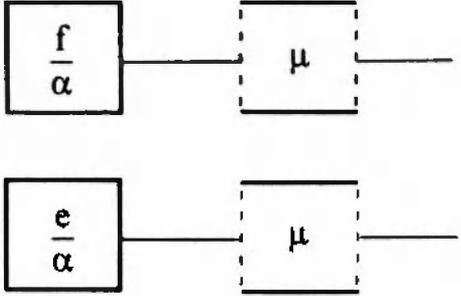
We are criticising not only the artificiality of the quantification-theoretical translations of such sentences¹⁹², but also the inability of predicate logic to provide any direct representation of the grammatical inferences involved. Within our framework these inferences are simple and immediate applications of the sub-formula rule stated above:



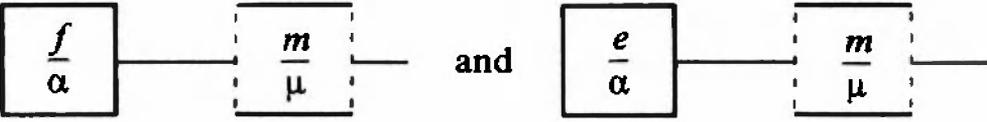
The converse of the sub-formula rule: *from Γ and Δ infer Ξ* , whose parts are precisely the parts of Γ and Δ cannot, however, be accepted. For consider the two formulae



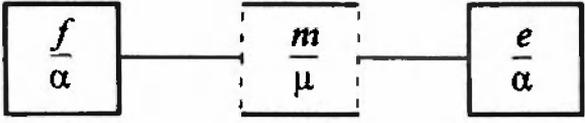
i.e. 'Franz is married' and 'Erna is married'. The composite of the two states of affairs pictured by these formulae would be represented by:



which presupposes (in a directly depicting language¹⁹⁵) that two (distinct) accidents of marriage are involved. This presupposition rules out the possibility that Franz and Erna are married to each other. From



however, i.e. from the propositions which involve the presupposition that the same accident of marriage inheres in both Franz and Erna, we may infer:



This suggests that we adopt the rule: *from Γ and Δ , two propositional formulae having only properly (as opposed to generically) designating*

*sub-formulae in common, infer Ξ , whose parts are precisely the parts of Γ and Δ .*¹⁹⁶

The rule-schemata so far considered are purely formal in nature: they apply to all material contents (to all formulae, irrespective of the common noun expressions they may contain). Grammatical deduction however typically rests also upon certain material rules of inference, i. e. on rules of inference specific to the *matters* (or associated common noun expressions) involved. This is in virtue of the fact that such matters are not implicatively independent of each other. From 'Hans kisses Erna', for example, we may infer 'Hans does something (is physically active in relation) to Erna', and in general, wherever ' κ ' occurs in a valid propositional context, the substitution of ' φ ' (for 'physical action in relation to') is validity-preserving:

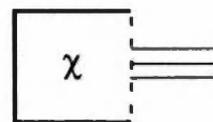
$$\frac{---\kappa \dots}{---\varphi \dots}$$

Similarly from either 'Hans has pyopericarditis (π)' or 'Hans has paroxysmal tachycardia (τ)' we may infer 'Hans has (a) cardiovascular disease (γ)', and from this we may infer 'Hans has a disease (δ)'. I. e., in general:

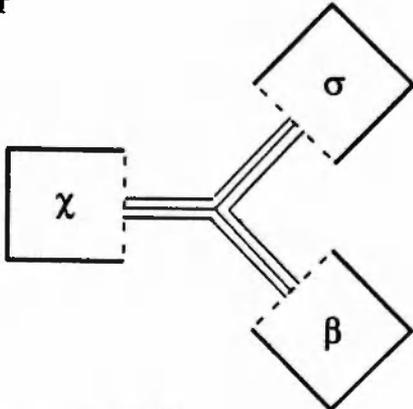
$$\frac{\frac{---\pi \dots \quad \text{---}\tau \dots}{---\gamma \dots}}{---\delta \dots}$$

A system of material substitution rules typically exhibits the structure of a tree, (isomorphic to the structure of the Porphyrian tree constituted by the material essences associated with the common noun expressions which are involved).¹⁹⁷

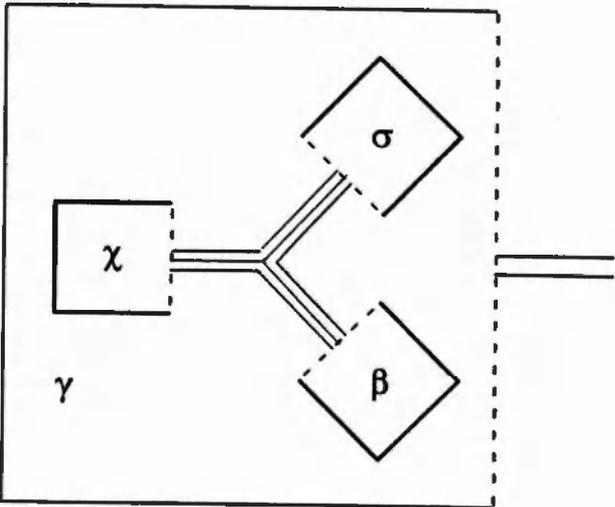
There are not only vertical relationships amongst essences but also horizontal relationships, i. e. relationships of dependence between essences, for example between the hue (χ), saturation (σ), and brightness (β) of a given colour-moment (γ), and between the colour-moment and a moment of extension (ϵ). These give rise to further categories of material inference rules. Thus from



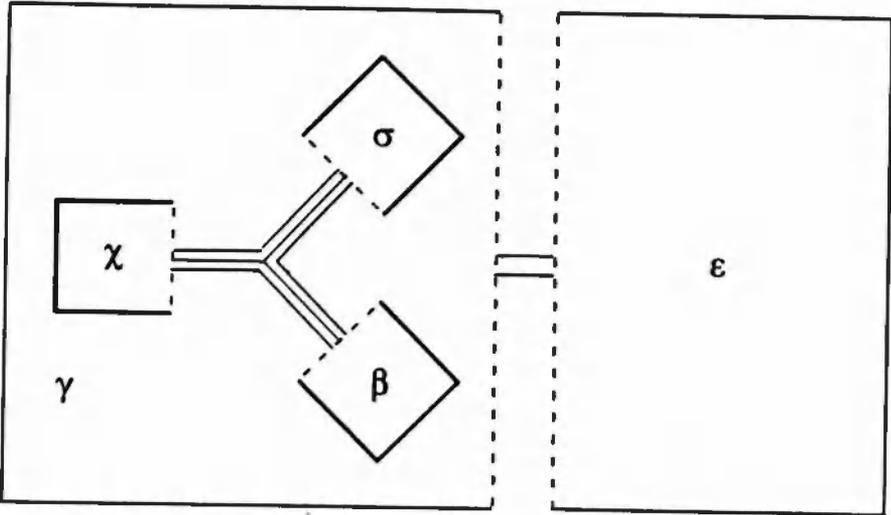
we can first of all infer



from which we can in turn infer¹⁹⁸

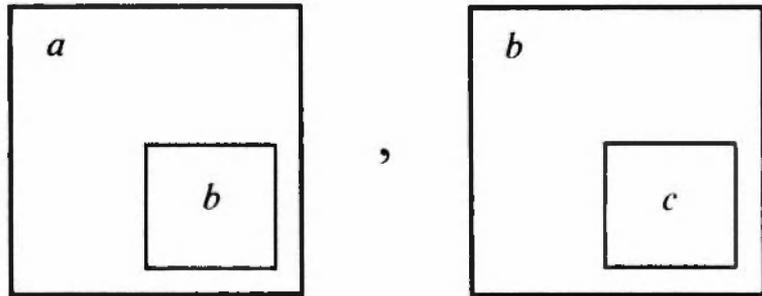


and finally

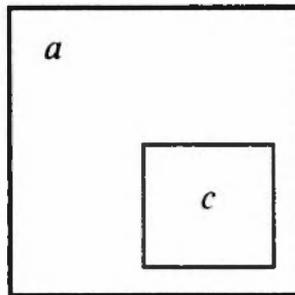


It remains to consider briefly the relationship between the two-dimensional system sketched above and the logic of Euler diagrams. Clearly there is no problem in principle in incorporating overlapping figures into the present framework, along with other elements of the Euler system which preserve the properties of a directly depicting language. A number of formidable combinatorial problems are raised, however, as soon as we begin to investigate the inferential structure of the language which results.

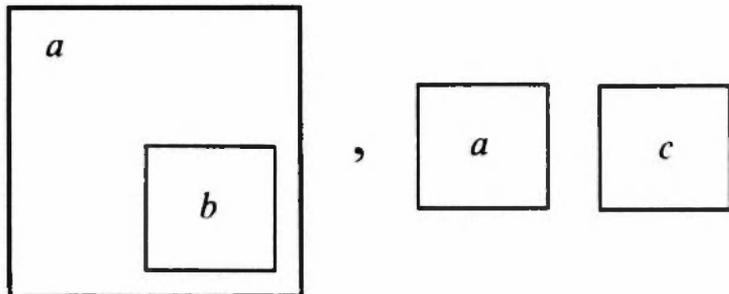
Thus whilst from, say,



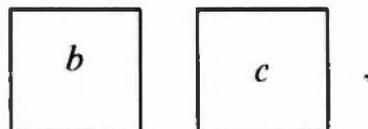
we may unproblematically infer



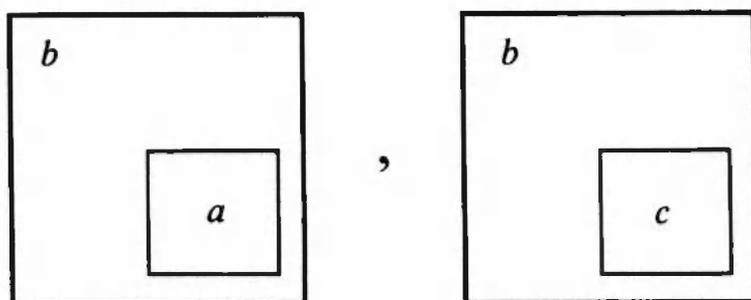
and from



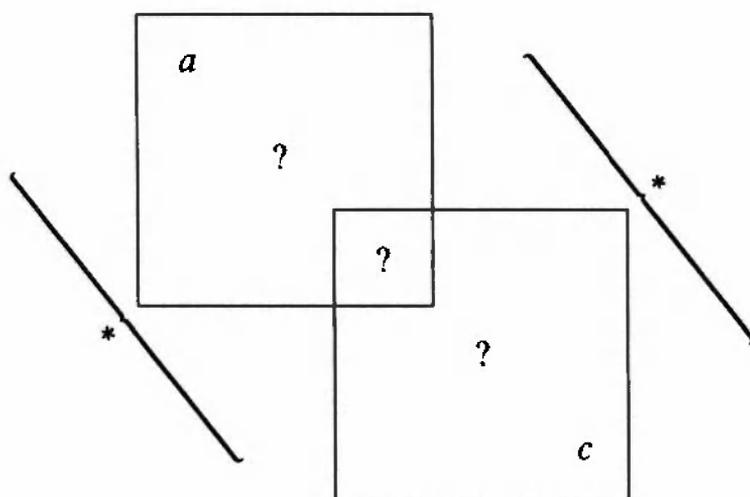
infer



no similar unproblematic conclusion concerning the relation between a and c can be drawn from



At best we can infer something of the form:



where '*' and '?' signify, respectively, 'is known to be occupied' and 'is not known to be occupied'. Not only does the introduction of '*' and '?' threaten to involve a radical departure from the principles of a directly depicting language; it also places almost insuperable difficulties in the way of any statement of the inference rules of the resultant system.¹⁹⁹

This completes our sketch of one possible symbolic framework for formal ontology. Issues not considered include the linearisation of systems of this sort,²⁰⁰ and they include the relation of the formal ontology presupposed here to other formal ontological theories, for example the formal ontology of measurement (the theory of extensive and intensive magnitudes), the formal ontology of probability and possibility, of fusion or *Verschmelzung*, and of time and causality. We hope to take up these problems in future investigations.

Notes

- ¹ In Wittgenstein's words (*Tractatus*, 4.221): "wie kommt der Satzverband zustande?"
- ² References given in this form are to works listed in the bibliography of writings on whole-part theory at the end of the volume. References to works not in this bibliography are given in full in the notes below.
- ³ A detailed analysis of the various form-matter concepts in the tradition is given by Ingarden in ch. 7 of his 1964/65.
- ⁴ Compare Leśniewski, 1929, p. 14, and Gardies, 1975.
- ⁵ This classification of accidents, due to Wolfgang Degen, closely resembles the classification of Gestalt-qualities given by Ehrenfels in his 1890. Compare also the appendix to Smith, 1981.
- ⁶ Cf. I. Angelelli, "On Individual Relations", *Studia Leibnitiana*, forthcoming. Angelelli contrasts what he calls the Siamese twin theory of relational accidents with the belief in the reality of (individual) relational states of affairs, propounded for example by Aquinas. Without such relational states of affairs "there is no real 'order' among the *entia* in the world" (§ 2). Cf. also Habel, 1960, part 2.
- ⁷ *Wissenschaftslehre*, § 127: "Closer consideration shows that *all* propositions have three parts, a subject-idea, the concept of having, and a predicate idea, as indicated in the expression '*A* has *b*'."
- ⁸ See also § 58 on mediate and immediate parts, and compare §§ 18 ff of Husserl's 3rd Logical Investigation.
- ⁹ See especially §§ 9 f of Twardowski, 1894. On the influence of Twardowski, Marty and Husserl on Leśniewski and other Polish logicians see Dąbwska, 1978, Luschei, 1962, Surma, 1977 and § 5.1 below.
- ¹⁰ On the influence of Bolzano on Husserl see § 9 of the latter's 1913. It is worth mentioning that, along with Brentano and Twardowski, Husserl was principally responsible for the rediscovery of Bolzano's logic after his work had lain fallow for several decades.
- ¹¹ See the introduction to the bibliography at the end of this volume, and especially the diagram on p. 87.
- ¹² Compare Twardowski, 1894, § 12 "Das Verhältnis des Vorstellungsgegenstandes zum Vorstellungsinhalt".
- ¹³ Cf. also Fechner, *Über die physikalische und philosophische Atomenlehre*, 2nd ed., Leipzig: Mendelssohn, 1864, p. 260, and Husserl, LU V, § 18:
"Not every unitary experience compounded out of acts is for that reason a *compound* act, just as every concatenation of machines is not a compound machine. . . . A compound machine is a machine compounded out of machines, but so compounded, that it has a total performance into which the performances of the partial machines flow, and the like is the case in regard to compounded acts."
- ¹⁴ Such complexes are illustrated by the kind of intermeshing depicted in, say, a model of a traffic accident or in a drawing of a pair of swordsmen *en face*: for a discussion of the place of these examples in Wittgenstein's philosophy see Smith, "Law and Eschatology in Wittgenstein's Early Thought", *Inquiry*, 21, 1979, 425–41, esp. p. 428. On the influence of legal science in the universities of Central Europe see Smith, "On the Production of Ideas. Notes on Austrian Intellectual History from Bolzano to Wittgenstein", in Smith, ed., 1981, 211–35; "Kafka and Brentano: A Study in Descriptive Psychology", *op. cit.*, 113–60; and also "Law and Eschatology", pp. 425 ff.
- ¹⁵ We shall return to this discussion of dependence relations amongst the elements of legal and other complexes in the introduction to the essay by Reinach below. Beling,

1906 and Bierling, *Juristische Prinzipienlehre*, 5 vols., Freiburg and Leipzig: Mohr, 1894/1917, esp. vol. 1, provide a mass of examples of such *a priori* dependence relations in the fields of criminal law and of civil and constitutional law respectively. The philosophical foundations of this conception of the objects of law are set forth by Reinach in his "Die apriorischen Grundlagen des bürgerlichen Rechts", 1913, discussed in § 3 of the introduction to the essay "On the Theory of the Negative Judgment". On *a priori* tendencies see Duncker, 1941.

¹⁶ A *Rechtsverhältnis* is simply that kind of *gegenseitig sich zu einander Verhalten* amongst individual subjects within the world which is, against the background of a given legal system, a legally relevant whole.

In the classical account, presented by Savigny in his *System des heutigen römischen Rechts*, 9 vols. (of which seven are devoted to the study of various types of *Rechtsverhältnisse*), Berlin: Veit, 1840/49, the matter or *Stoff* of a *Rechtsverhältnis* is distinguished from its form, i. e. from the 'juridical determination of this stuff', through which 'underlying factual relations between individuals are elevated to the form of law'. Bierling, *op. cit.*, distinguishes amongst the mutual relations of human beings in general *Rechtsverhältnisse* (obligations, claims, matrimonial and property relations, etc.) and *spontaneous* relations amongst individuals which admit of no juridical articulation; see esp. vol. 1, p. 194 *et passim*. His full classification of *Rechtsverhältnisse* is given on pp. 275–331. Most important, for our purposes, is his distinction between one-sided and n-sided *Rechtsverhältnisse*, his account of *Teilverhältnisse* (p. 316) and of *Rechtsverhältnisse höherer Ordnung* (pp. 327 ff). Compare also his analysis of causality in the realm of legally relevant states of affairs in vol. 3.

This ontological classification of *Rechtsverhältnisse* by philosophers of law in the 19th century ran parallel with the classification by logicians of the *Grundverhältnisse* – above all the substance-accident relation and the relations of causality and of space and time – amongst objects in general. See above all Beneke's *System der Logik*, part 1, Berlin: Dümmler, 1842. On the relations between these two traditions, – which merged in the *Sachverhalt*-ontologies developed by Stumpf and by the early phenomenologists, especially Husserl, Pfänder and Reinach – see the article "Sachverhalt. I" in K. Gründer, ed., *Historisches Wörterbuch der Philosophie*, Basel: Schwabe, forthcoming.

¹⁷ 2nd ed., Leipzig: Breitkopf and Härtel, 1869.

¹⁸ *Op. cit.*, vol. II.2, pp. 328 ff. Cf. also 4th ed., vol. III. 1, pp. 182 ff. The passage quoted is discussed at length in Engisch, 1953, ch. 5, pp. 238 ff.

¹⁹ Cf. n. 13 to Smith, "Law and Eschatology".

²⁰ The influence of Boolean/Schröderian ideas on mathematicians and philosophers, and particularly of Schröder's *Algebra der Logik* on Husserl (see his 1890, 1891a, 1908/09) and on Skolem and Löwenheim, has perhaps been insufficiently emphasised. One remnant of the Boolean algebra of classes appears in Bernays' and Gödel's theories of sets and classes (see P. Bernays, "A system of axiomatic set theory. I", *Journal of Symbolic Logic*, 2, 1937, 65–77 and K. Gödel, *The Consistency of the Continuum Hypothesis*, Princeton: Princeton U.P., 1940). Where within (first order) Zermelo-Fraenkel set theory one has to adopt for each first order property a separate replacement axiom, in the Bernays-Gödel theory one has a single replacement axiom for classes, from which individual replacement axioms for all first order properties of sets can then be derived. This is achieved by systematically replacing such properties by classes derived from other classes by means of operations which mirror the operations of the algebra of logic.

²¹ In modern properly mathematical work in set theory the distinction between \in and \subseteq has, ironically enough, ceased to play any crucial role, since the levels of the cumulative hierarchy which form the objects of mathematical investigation are all of them

transitive sets, so that the systems of structural relations generated respectively by \in and \subseteq prove to be mathematically interchangeable.

- ²² And cf. the third of Simons' three papers below. Cf. also Simons, 1980.
- ²³ This pragmatic standpoint has come to imply that set theory is no longer conceived as a theory which describes its own appropriately structured world; it is conceived rather as an arsenal of mathematical tools with the help of which other, quite heterogeneous mathematical theories (arithmetic, analysis, topology, etc.) can be built up. But this implies that, with respect to any single mathematical theory, set theory is superfluously strong, its object-universe monstrously overgrown.
- ²⁴ It may be helpful in what follows provisionally to identify 'moment' with 'abstract part', 'piece' with 'concrete part'. Compare § 17 of Husserl's 3rd Logical Investigation where Husserl explains why this identification is of only limited validity.
- ²⁵ See e.g. Drobisch, *Neue Darstellung der Logik nach ihren einfachsten Verhältnissen*, 4th ed., Leipzig: Voss, 1875, part 1, *passim*; Ueberweg, *System der Logik und Geschichte der logischen Lehren*, 4th ed., Bonn: Marcus, 1874, § 4, "Das Merkmal und die Theilvorstellung".
- ²⁶ In the light of the discussion of German legal ontology in § 1.3 it is interesting that Stumpf began his university career as a student of law (see Lewin, 1937). Stumpf's pupils in psychology included Wertheimer, Köhler and Koffka, who went on to found the Berlin School of Gestalt psychology. As a philosopher however (cf. his 1906 f and 1939/40, see also the work of E. Becher), he has had an almost negligible influence. This is to a large extent a result of the fact that his philosophical method – which in some ways resembles the piecemeal conceptual clarification of contemporary analytic philosophy – was alien to the German intellectual climate of the inter-war period.
- ²⁷ "Were they merely members of a sum, then it would perhaps be conceivable, to put it simply, that when the extension disappears, so too does the quality (that they do not exist independently); but that the quality disappears gradually in this way, and disappears – without its changing as a quality – with the mere decrease and disappearance of the quantity, would be incomprehensible" (op. cit., p. 113).
- ²⁸ Stumpf has 'Starke' instead of 'saturation' [*Sättigung*]. Cf. Stumpf, 1917.
- ²⁹ Such that they "... merely keep each other company as a matter of habit..." (Stumpf, 1939/40, vol. 1, p. 183).
- ³⁰ I. e. precisely the Wertheimer-Köhler-Koffka Gestalt psychology; see § 6 below.
- ³¹ This corresponds to an underlying Spinozist metaphysic: cf. Stumpf's "Spinozastudien", *Abhandlungen der preussischen Akademie*. Berlin, 1919. Compare also the entry for 1. 8. 1916 in Wittgenstein's diary: "Gott ist, wie sich alles verhält" (*Notebooks 1914–16*, p. 79). On one-sided correlations see Stumpf, 1899, 1907, 1916.
- ³² For a full account of the evolution of Stumpf's early thought it would be relevant to note that Lotze, who supervised Stumpf's work on presentation of space, had anticipated the bare outlines of its general approach – albeit within a semi-idealistic framework – particularly in his work on local signs in spatial perception.
- ³³ Brentano, *Die Psychologie des Aristoteles, insbesondere seine Lehre vom Nous Poietikos*, Mainz: Kirchheim, repr. Darmstadt: Wissenschaftliche Buchgesellschaft, Eng. trans. by R. George, Berkeley: University of California Press, 1977.
- ³⁴ See also Smith, 1981, § 2 and G. Küng, 1978.
- ³⁵ Brentano, *Wahrheit und Evidenz*, ed. O. Kraus, Leipzig: Meiner, 1930, Eng. trans., *The True and the Evident*, London: Routledge and Kegan Paul, 1966.
- ³⁶ *Psychologie*, book 2, ch. 6.
- ³⁷ The Brentanian treatment of complete and incomplete expressions (cf. Marty, 1908), can usefully be contrasted with Frege's historically more influential treatment of sentential parts.
- ³⁸ Brentano, *Psychologie*, chs. 5–7, *Vom Ursprung sittlicher Erkenntnis*, 3rd ed., with in-

roduction and notes by O. Kraus, Leipzig: Meiner, 1934, p. 57; and cf. Heinrich, 1910, p. 10f. for a discussion of the relation between Brentano's views and those of Husserl, especially as put forward in the 5th Logical Investigation.

³⁹ *Psychologie*, book 2, ch. 7.

⁴⁰ Thus Brentano's philosophy influenced the development of Kotarbiński's 'reism' or 'concretism' (see the latter's 1929).

⁴¹ It is striking how much recent philosophy has returned to ideas first spelled out by Brentano around the turn of the century. This is particularly true of his commitment to an ontology of material things and persons, of the great stress he laid on linguistic analysis (see R. Haller, "Brentanos Sprachkritik", in *Die Philosophie Franz Brentanos*, R. M. Chisholm and R. Haller, eds., Amsterdam: Rodopi, 1978, 211–24), and also of his notion of the reflexivity of representation, which has surfaced in recent work on the pragmatic component in a theory of meaning (see F. Récanati, *La transparence et l'énonciation: pour introduire à la pragmatique*, Paris: Seuil, 1979, e.g. §§1 and 9). For Brentano, when I hear a sound (the primary object of my act of hearing), I am also obliquely aware of another object, namely this act itself (the secondary object in Brentano's terms). Similarly a linguistic act involves not only a content or reference but also an indication by the speaker of the sort of act involved. Kastil summarises Brentano's view as follows: ". . . when I say 'A exists' I want the person I address to judge that I believe in A ('s existence) . . . the attempt to communicate is always double and what is communicated secondarily is a judgment concerning ourselves. Exclamations, requests and orders are also communications of judgments about what is going on in me. They are abbreviations of 'I am conscious of the wish that something should exist', etc.'" (*Die Philosophie Franz Brentanos*, Bern: Francke, 1951, p. 100).

⁴² See Brentano, 1978, and the introduction to this work by Chisholm and Körner. Cf. also Kastil, op.cit., ch. 5. It is in the contrast between their respective treatments of the continuum that the comparison between Brentano and Husserl on part-whole relations is most fruitful. Briefly, where Brentano's treatment centres around the concept of a boundary, Husserl distinguishes what he calls moments of (spatial or temporal) extension. A moment of extension has the following property, possessed by no other moments, that it, and moments founded on it, can be pieced (see § 3 below).

⁴³ This emphasis on structural relations signified a radical break with the subject-object centred tradition of German idealism. The neglect of Husserlian whole-part theory and of the work of e.g. Selz or Burkamp is almost certainly to be ascribed to the continuing dominance of this tradition far into the 20th century.

⁴⁴ Cf. E. Heinrich, 1910, §§13 f, and § 16 of Husserl's 3rd Logical Investigation.

⁴⁵ Even here however Brentano did not conceive his work as contributing to a theory of wholes and parts as such. Following Aristotle, he regarded his work much more as a contribution to the theory of relations (or, better, of *relativa*: cf. *Kategorienlehre*, pp. 166–99, *Raum, Zeit und Kontinuum*, part 2, essay 8, "Das Zeitliche als Relatives", and Kastil, op. cit., p. 132 f). Examples of *relativa* include a cause and its effect, the parts of a collective and the collective itself, a continuum and its boundary, a thinker and a thought, and all comparatives. The concept thus has the same extension as 'foundation relation' in Husserl (see § 3 below). Husserl however saw clearly that the concept of foundation relation is a formal concept (applying to all matters whatsoever), and he was explicit that his theory represented "a segment of the *a priori* theory of wholes and parts, i.e. of forms of connection (*Verbindung*) and unity in general" (Husserl, 1913, p. 131).

⁴⁶ Accidents, that is to say, fall outside the universe of discourse of any scientific theory, (a view which echoes, of course, the opinion of Aristotle). Thus despite the subtlety of Brentano's concept of *thing* he suffers, in the end, from the same kind of naturalism criticised by Husserl in Part I of the *Krisis*. The analytic philosopher's emphasis on ma-

terial things (or, more generally, on the bearers of proper names) is a related form of this naturalism.

- ⁴⁷ See Marty, 1884/95 and 1908, and also Kraus, 1930, in which Brentano's and Marty's work on syncategoremata and on *Sachverhalte* is compared with Russell's early philosophy and with the *Tractatus*.
- ⁴⁸ On the influence of Schroder on Husserl see the latter's 1890 and pp. 180–93 of his 1908/09. More general information on Husserl's mathematical background is provided in Scrimieri, 1965, Voltaggio, 1965, Husserl, 1907, pp. xxvff and Smith, 1976.
- ⁴⁹ Compare Cantor's notion of a set as a 'collection into a whole (*Zusammenfassung zu einem Ganzen*) of definite and separate objects of our intuition or our thought' ("Beitrage zur Begründung der transfiniten Mengenlehre", *Mathematische Annalen*, 46, 1895/97, 481–512, 49, 207–46, as repr. in Cantor's *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts*, Hildesheim: Olms, 1966, p. 282).
- ⁵⁰ Cf. PdA, p. 19. Here Brentano was drawing on a great deal of contemporary work on ontology in the Aristotelian tradition by, above all, his teacher Trendelenburg.
- ⁵¹ Cf. Stout, 1900 and Hicks, 1930/31. Contrast this approach with e.g. the following passage from Hume's *Treatise*, I, part IV, sect. III, "Of the Ancient Philosophy": "... our ideas of bodies are nothing but collections formed by the mind of the ideas of the several distinct sensible qualities, of which objects are composed, and which we find to have a constant union with each other."
- ⁵² Cf. *Tractatus*, 2.021: "Objects make up the substance of the world. That is why they cannot be composite," and Trendelenburg, *Logische Untersuchungen*, 3rd ed., Leipzig: Hirzel, 1870, vol. 1, p. 185.
- ⁵³ Cf. § 1 of the first edition of LU III (Passage deleted in second edition) and compare also *Ideen*, I, § 12 and the discussion of *Morphenverschmelzung* in Hering, 1921, ch. 2.
- ⁵⁴ Husserl told Kneale, on a visit to Freiburg in 1928, that the 3rd Logical Investigation was the best starting point for a study of his works. See *Journal of the British Society for Phenomenology*, 2, 1971, p. 78. In the forward to the second edition of the *Logical Investigations* Husserl wrote of the 3rd Investigation: "I have the impression that this Investigation is all too little read . . . it is an essential presupposition of the full understanding of the Investigations which follow". Cf. also Holenstein, 1973, *passim*.
- ⁵⁵ The *Logical Investigations* are dedicated to Stumpf, Husserl's teacher and colleague in Halle from 1886. During his time in Halle, Husserl was on friendly terms also with Hermann Grassmann, and with Cantor (see Schuhmann, *Husserl-Chronik*, p. 22 and A. Fraenkel, "Das Leben Georg Cantors", in Cantor, *Gesammelte Abhandlungen*, p. 477). Stumpf and Cantor were indeed, with the physicist Knoblauch, Husserl's Habilitation examiners (Stumpf examining Husserl on topics which included Lotze's theory of spatial perception, the history of theories of space, and the relations between mathematics and logic – Schuhmann, *op. cit.*, p. 19). Nothing is known of any contact between Cantor and Stumpf. The mathematical works published by Stumpf in this period under the influence of Felix Klein ("Über die Anwendung des mathematischen Wahrscheinlichkeitsbegriffes auf Teile eines Kontinuums" and "Über den Begriff der mathematischen Wahrscheinlichkeit", both published in the *Sitzungsberichte der bayrischen Akademie der Wissenschaften* for 1892) had an incidental philosophical significance in that they influenced mathematicians such as Czuber to conceive probability statements as relating not to events but to statal entities (*Tatbestände* or, to use the term favoured by Stumpf, *Sachverhalte*). See e.g. pp. 5, 85 of Czuber's *Wahrscheinlichkeitsrechnung und ihre Anwendung auf Fehlerausgleichung, Statistik und Lebensversicherung*, Leipzig: Teubner, 1903.
- ⁵⁶ The concept of variation as a mode of gaining knowledge of essences played a crucial role in Husserl's later philosophy and in that of his students (see e.g. the account in E.

Swiderski, "Some Salient Features of Ingarden's Ontology", *Journal of the British Society for Phenomenology*, 6, 1975, 81–90, esp. p. 87 f). It was anticipated by Bolzano (*Wissenschaftslehre*, § 147) and appears already in Husserl's 1894, an important connecting link between the *Philosophy of Arithmetic* and the *Logical Investigations*.

⁵⁷ Cf. § 2 of LU III. In his translation of the *Logical Investigations* Findlay coined the term 'non-independent', as a translation of Husserl's *unselbständig* which would bring out the negativity of the latter (see p. 39 of his introduction). Our reasons for preferring the straightforward 'dependent' are given by Simons in his introductory note to the essay by Ginsberg translated below.

⁵⁸ These essences exist autonomously, i.e. independently of all cognitive acts (and thus, *a fortiori*, of the linguistic conventions or grammatical rules adopted by subjects who may gain access to them). See Ingarden, 1925 and 1964/65, vol. II/1, and Swiderski, *op. cit.*

⁵⁹ At least some of these truths are, we shall argue, not only *a priori*, but also synthetic. We do not however wish to rule out the possibility that others may belong to the realm of the analytic *a priori* as this is delineated by Husserl. See § 4 below and Smith, 1981.

⁶⁰ Unpublished work by Wolfgang Degen on the formal ontology of quantum mechanics suggests that it may be fruitful to regard certain types of purported sub-atomic particles as dependent moments of more familiar entities.

⁶¹ See Husserl, *Ideen I*, § 11.

⁶² Compare Ehrenfels, 1890, the paper which instigated Meinong's introduction of the terminology of foundation.

⁶³ See the discussion of representation on pp. 782–800 in Reinach, 1913, and § 4 of the introduction to the essay by Reinach below.

⁶⁴ This aspect of part-whole theory is stressed by Burkamp in his 1929.

⁶⁵ A preliminary classification of more than one thousand different types of whole is presented by Rausch in his 1937. See § 6 below.

⁶⁶ This example is taken from Ingarden, 1964/65, vol. I, § 15.

⁶⁷ See n. 72 below.

⁶⁸ Findlay translates Husserl's 'Satz' (in '1. Satz', '2. Satz', etc.) by 'Proposition' which draws attention away from the fact that Husserl provides a proof of all but Theorem I (which he finds axiomatically self-evident). This fact is further obscured in the English edition by the typographical running together of the proof of Theorem VI with the statement of the theorem. This edition also has 'subordinate' in place of 'superordinate' in Theorem II.

⁶⁹ Compare § 84 of the *Wissenschaftslehre*.

⁷⁰ In his *A History of Experimental Psychology*, 2nd ed., New York: Appleton-Century-Croft, 1950, E. G. Boring speaks of a nativist tradition in psychology standing in opposition to the empiricist tradition of, say, Hume, Lotze, Helmholtz, Wundt und Külpe. Nativism had its origins in Kant's theory of space and also in work on colour-theory by Goethe and Purkinje. Its principal representatives were Johannes Müller, E. Hering (successor of Purkinje in Prague), Mach and Stumpf (also both for a period holders of chairs in Prague), and the Gestalt-psychologists Wertheimer, Koffka and Köhler.

⁷¹ The following distinctive features of the English (stop) consonantal system: ± Labial, ± Dental, ± Velar, are mutually incompatible moments; and they are genuine moments: they never appear in isolation. See § 5 below.

A relation of foundation is exhibited between the ingredients of *sense*, *tone* and *force* in a linguistic meaning as this is conceived by Frege. They correspond to three distinct dimensions in the geometry of meaning:

"a difference between two expressions, or two sentences, in respect of any these three features . . . would ordinarily be accounted a difference in meaning; a mistake about the sense, tone or force intended to be understood as attached to a sentence or expres-

sion would ordinarily be accounted a misunderstanding of its meaning” (Dummett. *Frege. Philosophy of Language*, London: Duckworth, 1973, p. 84).

Appeal is made to the concept of foundation also – we suggest – by Strawson, in his account of the *statement* as an action or event standing (in effect) in internal relations to the circumstances in which it is made; see D. Willard “Husserl’s Critique of Extensionalist Logic: ‘A Logic that does not understand itself’”, *Idealistic Studies*, 9, 1979, 143–64, pp. 154f, Tugendhat, 1976 and Mulligan, 1980. It is remarkable how seldom analytic philosophers have paused to reflect on the ontological presuppositions of their talk of ingredient, dependence, aspect, feature, etc.

- ⁷² Here, as elsewhere, lower case italic Roman letters are proper names of individual existents. Lower case Greek letters are common nouns or common noun phrases, signifying species or types (*middle C*, *prussian blue*, etc.). $\frac{a}{\alpha}$ may be read, according to context: ‘*a* which is (an) α ’, ‘*a*, *qua* or in its capacity as α ’. (A similar notation is suggested by Husserl in his 1908/09.) The broken lines signify mutual dependence relations amongst the contents designated. The form of representation given in the text is inadequate in a number of ways. In any complete account it would be necessary, for example, to distinguish between *n*-sided mutual dependence amongst moments and an *n*-fold pairwise dependence amongst the moments taken separately. It would be necessary also to distinguish relations of dependence obtaining amongst species *in abstracto* (where part-whole imagery may be out of place) from the corresponding relations obtaining amongst factual instantiations (*a qua* α , *b qua* β), etc.). Two pictures would then be necessary: a picture of the relevant *Wesensverhalt* on the level of essence, and a correspondingly articulated picture of the underlying individual *Sachverhalt* on the level of fact. A more detailed account of the directly depicting language which then results – taking dependence-diagrams as propositional signs in the sense of the *Tractatus* – is given in § 6.4 below.

- ⁷³ See especially his *Zur Lehre vom Lichtsinne; sechs Mittheilungen an die Kaiserliche Akademie der Wissenschaften in Wien*. Vienna: Gerold, 1878, and for further references pp. 115 and 379 of Boring, op. cit., and also Stumpf, 1918.

- ⁷⁴ It was originally Herbart who recognised that expressions such as ‘between’, ‘among’, ‘high’, ‘low’, ‘side’, etc., having their primary meaning in the geometrical sphere, can be used non-metaphorically for all quality continua. Cf. also the work on quality-spaces by Helmholtz, e.g. his “Kürzeste Linien im Farbensystem”, *Zeitschrift für Psychologie*, 3, 1892, 108–22. The idea of a *Farbengeometrie* was developed systematically by Meinong in his “Bemerkungen über den Farbenkörper und das Mischungsgesetz”, *Zeitschrift für Psychologie*, 33, 1903, 1–80, repr. in GA I. Cf. also Stumpf’s *Tonpsychologie*, 1883/90, esp. vol. I, 189, his 1906a, p. 28f, Gilman, 1892, Selz, 1930ff, and Mulligan, 1980. Against this background the apparent gulf between Husserl’s formal ontological work on the theory of manifolds and his more strictly phenomenological writings on perception disappears completely.

It is perhaps worth pointing out that the word ‘*Farbenraum*’ employed by Meinong (loc. cit., § 5, “Die Farbenraum und seine Dimensionen”) is used also by Wittgenstein, e.g. at 2.0131 in the *Tractatus* and on p. 51 of the *Philosophische Bemerkungen*.

- ⁷⁵ Stumpf employed the term ‘phenomenology’ to designate one such *Vorwissenschaft* – that which would explore the domain of possible contents of immediate sensory and memory experience. Given the considerable influence amongst experimental psychologists of the lecture “Zur Einteilung der Wissenschaften” in which this theory of descriptive *a priori* sciences is set out, it is interesting to note the similarities between Stumpf’s use of ‘phenomenology’ and the use made of it by Wittgenstein (cf. Spiegelberg, “The Puzzle of Ludwig Wittgenstein’s *Phänomenologie* (1929–?)”, *American Philosophical Quarterly*, 5, 1968, 244–56). “Physics”, writes Wittgenstein, “differs from

phenomenology by its aim of establishing laws. Phenomenology establishes merely possibilities. Thus phenomenology would be the grammar for the description of those facts upon which physics erects its theories" (*Philosophische Bemerkungen*, p. 51). "Physics wants to establish regularities; it does not look for what is possible. This is the reason why physics, even when it is completely developed, does not offer a description of the phenomenological state of affairs. Phenomenology always deals only with possibility, i.e., with meaning, not with truth and falsehood. Physics focuses, as it were, on certain points in a continuum and uses these for constructing a lawlike series. It does not care about anything else" (*Wittgenstein und der Wiener Kreis*, p. 63). "But what kind of a proposition is that, that blending in white removes the colouredness from the colour? As I mean it, it can't be a proposition of physics. Here the temptation to believe in a phenomenology, something midway between science and logic, is very great" (*On Colour*, p. 15, written ca. 1950–51). The possibility of an influence of Stumpf on Wittgenstein has been raised in another context by Smith in his 1978 and also in "Wittgenstein and the Background of Austrian Philosophy", *Wittgenstein and his Impact on Contemporary Thought*, Vienna: Hölder-Pichler-Tempsky and Dordrecht: Reidel, 1978, 31–35.

⁷⁶ See e.g. Husserl, 1913, § 6.

⁷⁷ The term 'material *a priori*' is used instead of the more usual 'synthetic *a priori*' in order to avoid unnecessary confusion with Kantian uses of this term. An excellent account of the confusion, in Kant's critical philosophy, between logical and material necessity is provided by Reinach in his 1911 (see also the introduction to the essay by Reinach translated below, and also G. Davie, "Edmund Husserl and 'the as yet, in its most important respect, unrecognised greatness of Hume'", in G. Morice, ed., *David Hume Bicentenary Papers*, Edinburgh University Press, 1977, 69–76).

Analytic *a priori* truths in Husserl's sense (see LU III, § 10 ff), are either purely formal laws or the results of specification (substitution) in these. Synthetic *a priori* truths derive their truth from necessary connections not amongst forms, but amongst *matters*; they reflect necessary connections amongst the things referred to, characteristic of some material domain or region. In the terminology of the present essay such connections are referred to as material *a priori* connections.

This line of demarcation may be less than sharp if under 'the results of specification' we allow ourselves to understand not merely those sentences obtained by uniform syntactic substitution (taking 'an α which is a β is a β ' to: 'an aardwolf which is a predator is a predator'), but also sentences obtained by substitutions resting on problematic semantic equivalences. Consider, for example, the following specification of 'an α which is a β is a β ': 'a declaration which occasions a tendency to bring about Φ occasions a tendency to bring about Φ '. If we can endorse some semantic equivalence along the lines of:

act of promising to do Φ \equiv declaration which occasions a tendency to bring about Φ
then this would seem to yield as an ultimate result of specification the apparently synthetic 'an act of promising to do Φ occasions a tendency to bring about Φ '. What had appeared synthetic would then have to be counted as implicitly analytic. Compare *Wissenschaftslehre*, § 148, Note 1, where Bolzano grapples with a similar problem.

⁷⁸ Cf. e.g. *Ideen I*, § 49: "Consciousness . . . must be reckoned as a *self-contained system of being, a system of absolute being*, into which nothing can penetrate, and from which nothing can escape; which has no spatio-temporal exterior, and can be inside no spatio-temporal system; which cannot experience causality from anything nor exert causality upon anything . . .". Investigation of the range of possible combinations of dependence and independence between consciousness and world forms the basis of Ingarden's ontology as set forth in his *Der Streit um die Existenz der Welt*. As will have been clear from the passage quoted from Berkeley's *Treatise* at the head of § 2

above, it is not only the metaphysics of phenomenologists which admits of expression in whole-part terms. The absolute idealism of Bradley, to take just one example, is founded upon a theory of internal relations which has many points of contact with the theory of foundation relations here developed, though marred by the lack of any respect for the distinction of matter and form. Bradley held, simply put, that there is only one independent whole (the world), that all other entities are moments (in his eyes inadmissibly abstracted moments) of this whole. At the opposite end of a spectrum of metaphysical positions we would find some form of absolutist atomism, a view according to which the furniture of the universe is constituted exclusively by mutually independent simples (apparent ontological commitment to higher order objects founded on these simples being treated as a dispensable *façon de parler*). The truth, which is to be found in Ingarden, 1964/65, falls between these two extremes. See also § 6.1 below.

- ⁷⁹ See *Ideen II* (Hua IV), pp. 133 f, 253 ff; *Erste Philosophie 1923/24 I* (Hua VII), 403; Kern, *Husserl and Kant*, Haag: Nijhoff, 1964, § 9.
- ⁸⁰ A third early criticism is to be found in the *Philosophie der Arithmetik* (Hua XII), p. 42: "Kant overlooked the fact that many contentual connections (*inhaltliche Verbindungen*) are given to us which do not involve any noticeable trace of synthetic activity bringing them about".
- ⁸¹ Cf. Ch. 1 of *Ideen III* (Hua V), "Die verschiedenen Regionen der Realität".
- ⁸² In the double sense indicated in the second half of note 72 above.
- ⁸³ Cf. the accounts in ch. 1 of Habbel, 1960 and in ch. 7 of Gardies, 1975.
- ⁸⁴ Cf. Reinach, 1921 a, p. 53 of reprint; Eng. trans., p. 211. Consider also the laws holding in the region of musical tones (as discussed by Stumpf, e.g. in §§ 8 and 10 of vol. 1 of the *Tonpsychologie*); for example the laws relating to the one-dimensionality of the tone-sequence, that given three tones of different pitch, one is always in between the others, etc.
- ⁸⁵ This view is of course already present in Aristotle. Reinach's 1909 Habilitationsschrift on *Wesen und Systematik des Urteils*, the manuscript of which has unfortunately been lost, can be presumed to have contained further steps towards the realisation of this project. The idea of an ontological foundation for logic, which Reinach shared with Pfänder, can be discerned also as underlying Meinong's work on logic and *Gegenstandstheorie*. The project formed the subject-matter of Martin Honecker's *Gegenstandslogik und Denklogik. Versuch einer Neugestaltung der Logik*, Berlin und Bonn: Dümmlers, 1921. For an account of the opposition between the (Reinachian) ontological approach to the *a priori* and the logico-grammatical approach of modern analytic philosophy see Delius, 1963. The audacity of Reinach's theory is made manifest – at a time when so much ink is spilt on the synthetic *a priori* by philosophers prepared to acknowledge at most a small fraction of Kant's original range of examples, – in its implication that there are vastly *more* cases of synthetic *a priori* truths than Kant had recognised.
- ⁸⁶ It is clear that in the classification of *a priori* dependence relations sketched briefly in § 1.3 above (text to n.15) the dimension of temporality has to be incorporated: relations of necessitation, exclusion, etc., may hold between moments standing in determinate relations of succession. A complete classification would thereby exhibit at least three dimensions: according to the type of relation involved (necessitation, exclusion, etc.); according to the temporal structure of the relevant moment-whole; and according to the formal structure of this whole (which may be one-sided, two-sided, n-sided, etc., or a combination of these). The first dimension has been discussed in detail already by linguists, especially Jakobson and his associates; see Holenstein, 1974, ch. 2, and § 5 below. The second dimension, considered by Husserl in LU III, §§ 13 and 25, is discussed briefly in the introduction to the essay by Reinach translated below; the third dimension in § 6.4 of the present essay.

- ³⁷ Where, in regard to the jigsaw puzzle which is a language, analytic philosophers have been ready to accept that there are complicated restrictions of fit between the constituent elements, they have been less willing to accept the existence of such restrictions in regard to the constituent elements of human life. They have dismissed as accidents of convention such material *a priori* truths as: *it is impossible to flick one's wrist with jealousy* or *all human languages contain words for black and white* or even *all human societies are patriarchal*. This is because what is materially excluded in a given region is often perfectly well *imaginable*, by those who have taken no steps to familiarise themselves with the laws holding in that region.
- ³⁸ Cf. e.g. Strawson, *Individuals*, London: Methuen, 1959, ch. I and Tugendhat, *Vorlesungen zur Einführung in die sprachanalytischen Philosophie*, Frankfurt: Suhrkamp, 1976, esp. chs. 23ff.
- ³⁹ This notion of an object's being embedded in a system or net of *Sachverhalte* is discussed, from different formal-ontological points of view, by Ingarden (1931, e.g. § 24 and 1964/65, vol. II/1, ch. XI), Burkamp, 1927, and Hazay, 1915.
- ⁴⁰ See e.g. Pariente, 1973, Burkamp, 1933, § 188 and B. Erdmann, *Logik*, 3rd ed., Berlin: de Gruyter, 1923, § 169.
- ⁴¹ LU III, § 12, Note 1.
- ⁴² On regional ontologies see Husserl's *Ideen III* (Hua V), ch. I, and also L. Landgrebe, "Seinsregione und regionale Ontologien in Husserls Phänomenologie", *Studium Generale*, 9, 1956, 313–24. Around 1918 Husserl seems to have worked on a never completed 'philosophical book for mathematicians' and 'mathematical book for philosophers' dealing with the problems of a formal theory of science in relation to the theory of individuation. Cf. Hua, III, 1: "Einleitung des Herausgebers", p. XLVf, and also Beilagen 27 and 30–32 to Hua III, 2.
- ⁴³ See Ingarden, 1935 and 1964/65, III/1.
- ⁴⁴ See vol. III of Ingarden's *Der Streit um die Existenz der Welt: Über die kausale Struktur der realen Welt*, 1974; compare also § 25 of LU III (1st ed.).
- ⁴⁵ LU III, § 22. This sentence is used by Jakobson as the motto to his 1940/42.
- ⁴⁶ See, in particular, Stout's two papers, "Bradley's Theory of Relations" (1900), "Russell's Theory of Judgment" (*Proceedings of the Aristotelian Society*, 15, 1915, 332–52, repr. in Stout, 1930, 239–57) and also Moore's 1919.
- ⁴⁷ See e.g. his 1960 and 1967.
- ⁴⁸ See LU III, § 22.
- ⁴⁹ See Husserl's *Zur Phänomenologie des inneren Zeitbewußtseins (1893–1917)*, Haag: Nijhoff, 1966 (Hua X), e.g. p. 24 f. Cf. also Seebohm, 1972 and 1973, and Sokolowski, 1974, ch. 6. Every experienced *now*, Husserl argues, contains as dependent moments a backward-looking *retention* and a forward-looking *protention*.
- ¹⁰⁰ Cf. *Ideen II* (Hua IV), summarised in part by Claesges, 1964, §§7 f. On Husserl's work on the theory of colours see Piana, 1966. Detailed unpublished material by Husserl on secondary qualities drawing on the whole-part framework is discussed by G. Witschel in his 1964. Husserl distinguishes, for example, between those sense-qualities which, though localisable, do not have an extension which admits of immediate piecing (*Zerstückung*), warmth, for example; and those which do, e.g. colour.
- ¹⁰¹ See part II of Husserl's *Krisis* (Hua VI).
- ¹⁰² Cf. also *Ideen I*, §§ 95, 116.
- ¹⁰³ For an account of the relation between this theory and the formal ontology of part and whole see Smith, 1980a.
- ¹⁰⁴ In both LU III and V Husserl carefully distinguished between whole-part relations amongst acts and sensory contents on the one hand, and phenomena of fusion (of qualitative continuity or *Verschmelzung*) on the other. Cf. Stumpf, *Tonpsychologie*, passim, and also e.g. Hicks, 1930/31, and Mulligan, 1980.

- ¹⁰⁵ Principles of an ontology of fitting are employed also by Ingarden in his theory of 'pure epistemology' which considers the question: "what conditions must be met by an object and by an act of knowing, if this cognition of the object which is carried out in the given act is to have this or that epistemic value" (*Über die Stellung der Erkenntnistheorie im System der Philosophie*, Habilitationsvortrag, Halle: Karras, Krober and Nitschmann, 1925, p. 30f. Cf. also his "Über die Gefahr einer Petitio Principii in der Erkenntnistheorie", in *Jahrbuch für Philosophie und phänomenologische Forschung*, 4, 1921, 545–68, and G. Küng, "Zum Lebenswerk von Roman Ingarden. Ontologie, Erkenntnistheorie und Metaphysik", in H. Kuhn, et al., eds., *Die Münchener Phänomenologie*, Haag: Nijhoff, 1975, 158–73.)
- ¹⁰⁶ Cf. the passages from § 18 of LU V quoted above, and Ingarden's "The Hypothetical Judgment", *Philosophy and Phenomenological Research*, 18, 1958, 435–50.
- ¹⁰⁷ The formal ontology of fitting is related, in particular, to topology. This is seen by the fact that an extensional fragment of this logic is simply the geometry of solid sheets. As primitive terms of this fragment we might select: *fits partially but not perfectly (fits with free play) into*, *fits perfectly into*, *is incapable of being fitted into (is skew to)*. Symbolising these relations by, respectively, \subset , \sqsubset and $\not\sqsubset$ we can assert, for example, that $a \subset b \rightarrow \exists x. a \cup x \sqsubset b$; $a \sqsubset b \rightarrow \forall x. a \cup x \not\sqsubset b$; $a \not\sqsubset b \rightarrow \forall x. a \cup x \not\sqsubset b$. Here 'U' symbolises geometrical colligation. These implications hold, like all truths of geometry, as a matter of *a priori* necessity.
- The topological analogy was exploited – somewhat overenthusiastically – by Kurt Lewin in his *Principles of Topological Psychology*, 1936, which presents a metricised whole-part-theoretical framework within which the relations of fit obtaining amongst psychological formations, particularly those associated with goal-seeking, can be formally expressed.
- ¹⁰⁸ The analyses presented in E & U, unlike those in the *Ideen*, represent a true continuation of and advance over those of the 3rd Logical Investigation.
- ¹⁰⁹ Cf. E&U §§ 28–32. On the relation between Husserl's concept of absolute substrate and the concept of an absolutely simple object see especially §§ 28f and Piana, 1977. Ingarden, 1925, contains an early, complementary discussion of explanation.
- ¹¹⁰ Pfänder's *Logik*, 1921, 182–206, contains a detailed account of the different ontological relations expressed by the copula. Cf. also Husserl, 1952, Beilage 30. Husserl's argument for the introduction of species is presented in LU II.
- ¹¹¹ Cf. n. 105. It is Ingarden, more than any other philosopher, who has developed Husserl's insight into the possibility of a descriptive epistemology.
- ¹¹² Such material connections will of course be obscured if the object-domains with which one deals consist not of objects (or meanings) themselves, but of set-theoretical models of such objects or meanings. Examples of material connections among meanings are provided e.g. by J. Trier, "Das sprachliche Feld. Eine Auseinandersetzung", *Neue Jahrbücher für Wissenschaft und Jugendbildung*, 10, 1934, 428–49.
- ¹¹³ We are grateful to Peter Simons and to Professor Czeslaw Lejewski for their help in the composition of this section.
- ¹¹⁴ See Leśniewski, 1929; Ajdukiewicz, 1935; Rickey, 1972 ff. For a more general account of Leśniewski's work see Luschei, 1962, which also provides ample bibliographical material. For a more historical perspective see Surma, 1977.
- ¹¹⁵ It was Leśniewski who produced the first ever formalisation of the relation of part to whole in the strict sense of 'formalisation' as understood within post-Fregean philosophy. *Formal laws* of this relation had however been expressed by many authors before Leśniewski: the law of transitivity of parts, for example, was formulated by Bolzano, Twardowski and Husserl, and was of course recognised within the Boole-Peirce-Schroder tradition.

- ¹¹⁶ It is only in connection with his *ontology* (see below), that Leśniewski, in his published writings, mentions Schroder.
- ¹¹⁷ *O Zasadzie Sprzeczności u Aristotelesa: Studium krytyczna*, Krakow, 1910.
- ¹¹⁸ This terminology first appears in Kotarbiński, 1929: it appears in Leśniewski's own published works only later.
- ¹¹⁹ He was quite simply unable to make any sense of the axiomatic set theory of Zermelo: see C. Lejewski's article on Leśniewski in P. Edwards, ed., *The Encyclopedia of Philosophy*, 4, 441–43, p. 442.
- ¹²⁰ A single axiom for mereology on the basis of the primitive 'discrete' was formulated by Lejewski already in 1977; see (c) on p. 62f and § 7 of the Appendix to B. Sobociński, "On Well Constructed Axiom Systems", *Polskie Towarzystwo Naukowe na Obczyźnie. Rocznik*, 6, 1855/56, 54–70. Welsh, 1978, offers a variety of further possible starting places for the same theory of whole and part.
- ¹²¹ Note that Leśniewski was not concerned with the explication of the term 'is' as this appears in any actually existing natural language; his aim, rather, was to produce a logically perfected copula, defined exclusively by its behaviour within the system.
- ¹²² Husserl's reciprocal dependence is thus effectively absent from Leśniewskian grammar. Indeed, a categorial grammar taking 2- or n-sided foundation as basic remains a desideratum even today.
- ¹²³ This development may have received its initial impetus from purely terminological considerations. When the new, strictly formal-logical meaning of the term 'semantics' that had been propounded by Tarski came to be accepted by logicians in the 30's and 40's, the characterisation of Husserlian meaning-categories as 'semantic categories' became no longer viable. Certain elements of the theories inspired by Husserl, Leśniewski and Ajdukiewicz could however properly be designated as 'syntax' under the new dispensation, and it has been these elements, at the expense of investigations of purely formal relations amongst *meanings*, that have continued to be developed by workers in the field.
- ¹²⁴ See Lejewski, 1973.
- ¹²⁵ Leśniewski himself conceived protothetic and ontology as properly logical theories, mereology as belonging to mathematics. Nowhere in his published works, however, does he provide a criterion for this distinction.
- ¹²⁶ To characterise mereology as a true theory implies, of course, that one understands the theory under its intended interpretation. The Leśniewskian approach to logic was, like that of Frege, exclusively a first order approach: the statements of formal logic are conceived as immediately and unproblematically true of the world itself, which is the only model held worthy of consideration. Contemporary logicians, in contrast, ignore the world, directing their attention instead to families of set-theoretic models of logical systems.
- ¹²⁷ On the and-functor see § 15 of Reinach's "On the Theory of the Negative Judgment" below, Husserl, PdA, p. 75f, Bolzano, *Paradoxien des Unendlichen*, § 3, and §§ 70–74 of Russell, 1903 (cf. the discussion in § 1.2 of Essay II by Simons, below). On the structure of the hypothetical judgment see Ingarden, "The Hypothetical Judgment", *Philosophy and Phenomenological Research*, 18, 1957/58, 435–50.
- ¹²⁸ This would in many respects parallel the formal taxonomy of all possible relations between chemical elements as determined by atomic theory: for further discussion of this parallel see § 6.4 below.
- ¹²⁹ See C. Serrus, *Le Parallelisme logico-grammatical*, Paris: Alcan, 1934, part 2, ch. 5, 317–81, and perhaps also Bar-Hillel, 1956.
- ¹³⁰ Now collected in Husserl, 1979.
- ¹³¹ "Syntaktische Formen und syntaktische Stoffe. Kernformen und Kernstoffe", Hua, XVII, 299–313.

- ¹³² See Hua, XII, pp. 475, 485 ff, "Das Gebiet eines Axiomensystems/Axiomensystem. Operationsystem", and Mulligan, 1980.
- ¹³³ See LU IV, §§ 12, 14, and the work referred to in n. 131 above.
- ¹³⁴ The term 'modification' signifies an *a priori* possible meaning transformation (e. g. the nominalisation of a sentential expression such as 'this book is red' to form the nominal 'this red book'). See LU IV, §§ 11, 13, V, §§ 39–40, E&U, § 55.
- ¹³⁵ Thus Jakobson, for example, distinguishes between phonetics as a *Stofflehre* and phonology as a *Formlehre*: cf. Jakobson, 1939, and compare the remarks on Savigny on p. 93 above. The form/core distinction emerges in the syntax of natural languages as a distinction between syntactic categories and syntactic features.
- ¹³⁶ See Holenstein, "Prague Structuralism – A Branch of the Phenomenological Movement", in J. Odmark, ed., *Language, Literature and Meaning I: Problems of Literary Theory*, Amsterdam: John Benjamins, 1979, 71–98.
- ¹³⁷ *Logitscheskija Izsledowanija. Tschastj perwaja. Prolegomeny k tschishoi Logike*, trans. E. A. Berstein and S. L. Frank, St. Petersburg: Obrazovanije.
- ¹³⁸ See Holenstein, 1975, which contains a full account of Husserl's influence and a discussion of the associated substantive issues. Jakobson also draws on Stumpf's work on the psychology of sound, arguing, for example, that not only colours but also sounds exhibit dimensions of brightness and saturation.
- ¹³⁹ See e.g. ch. 3 of Holenstein, 1974, and the papers referred to in n. 150 below.
- ¹⁴⁰ Jakobson, "Un manuel de phonologie générale", 1939, SW I, 311–16, p. 315.
- ¹⁴¹ Jakobson, "Phoneme and Phonology", 1932, SW I, 231–33, p. 231.
- ¹⁴² "Un manuel de phonologie générale", loc. cit. Jakobson was able to show that natural languages make use of an inventory of not more than a dozen such distinctive features (Jakobson and Halle, 1956, p. 471 f). These 'inherent features' are distinguished from prosodic features which 'bind the phoneme as such to the time axis', i.e. in the terminology of the 3rd Investigation, necessarily involve a moment of temporal extension.
- ¹⁴³ On gradual (non-binary) phonological features see T. Vennemann, "On the theory of syllabic phonology", *Linguistische Berichte*, 18, 1972, 1–18.
- ¹⁴⁴ Cf. Jakobson, "Retrospect (1961)", SW I, 629–58, p. 645.
- ¹⁴⁵ See Jakobson, "The identification of phonemic entities", 1949, SW I, 418–25, p. 421. This point is made also by the phenomenologist and linguist H. Pos in his 1938.
- ¹⁴⁶ Jakobson and Halle, 1956, p. 471 f.
- ¹⁴⁷ Jakobson, "Aphasia as a Linguistic Topic", 1955, SW II, 229–38, p. 233.
- ¹⁴⁸ In his 1974 Holenstein discusses a type of opposition studied by Jakobson of extreme importance in natural languages and other sign-systems: the opposition between marked and unmarked terms. Husserl and Stumpf had isolated the phenomenon of *Verschmelzung* (blending or fusion) as this pertains to the phenomenal continuity of perceptual or act moments. It seems that unmarkedness is a linguistic correlate of continuity for objects and acts. Cf. n. 104 above.
- ¹⁴⁹ See especially G. Ungeheuer, 1959, but also A. Juilland and H.-H. Lieb, *Klasse und Klassifikation in der Sprachwissenschaft*, Haag: Mouton, 1968, p. 58 f, and J. W. F. Mulder, *Sets and Relations in Phonology. An Axiomatic Approach to the Description of Speech*, Oxford: Clarendon Press, 1968.
- ¹⁵⁰ On implicational laws see also Jakobson, "Results of a Joint Conference of Anthropologists and Linguists", 1953, SW II, 554–67, p. 563 and also his 1963. Compare Holenstein 1974, p. 36 of Eng. trans., and § 4 of "Zur Begrifflichkeit der Universalienforschung in Linguistik und Anthropologie", *Arbeiten des Kölner Universalien-Projekts*, 35, 1979. On linguistic universals in general see J. H. Greenberg, ed., *Universals of Language*, Cambridge, Mass.: M.I.T. Press, 1963. See also text to nn. 15 and 86 above.
- ¹⁵¹ SW I, p. 320. Further examples are provided throughout Jakobson's works. See also Holenstein, 1976a and 1976b.

- ¹⁵² Jakobson's work on aphasia is complemented by the investigations of the consequences of brain damage by Gelb and Goldstein listed in the bibliography at the end of this volume.
- ¹⁵³ SW I, p. 321.
- ¹⁵⁴ See also the work of Hjelmslev on dependence, as presented e.g. in B. Siertsma, *A Study of Glossematics*, Haag: Nijhoff, 1955 (here 'interdependence' = our 'reciprocal dependence', 'determination' = 'one-sided dependence', 'constellation' = 'mutual compatibility'). Literature on semantic features tends to be more recent, but see, on word-fields, J. Trier, "Das sprachliche Feld . . ." (n. 112 above).
- ¹⁵⁵ See Holenstein, 1974.
- ¹⁵⁶ In a more complete account it would be necessary to discuss also the work of R. A. Hudson, particularly his recent *Arguments for a Non-Transformational Grammar*, 1976. Whilst by no means falling under the influence of Husserl, Hudson has nevertheless contributed substantially to our understanding of the project of a Husserlian grammar. The work in question sets out a typology of the dependence relations to which different grammatical theories are committed. He points out that whereas European dependency grammars such as those of L. Tesnière, *Éléments de Syntaxe Structurale*, Paris: Klincksieck, 1959 and H.-J. Heringer, *Theorie der deutschen Syntax*, Munich: Hüber, 1970, have made use of bi- and multilateral dependence (horizontal or 'sisterhood') relations between parts, not only did the 'immediate constituent' analysis of American structuralism concentrate exclusively on whole-part (vertical or 'mother-daughter') relations in which larger syntactic units are seen as being quite literally composed of smaller ones, this one-sided emphasis on vertical relations was taken over by transformational grammar. Hudson's grammar, like the pure grammar outlined by Husserl, makes use of both sorts of relations and shows the importance of dependency relations both between parts i.e. categories (such as noun-phrase and verb) and between features (such as [+ transitive] and [+ nominal]).
- ¹⁵⁷ The given properties and effects are, in the terminology of the philosophy of science, *emergent*: "A property \bar{P} of a whole w is an *emergent wholistic property* (relative to the theory T , a decomposition D of w into parts, and a set G of properties) if ' $\bar{P}(w)$ ' cannot be deduced from the body of knowledge consisting of (1) the theory T , and (2) a characterization with regard to all the properties of G of the parts of w which are members of the decomposition D ". This definition, given by Rescher in his reply to Madden, 1952 (*Philosophy of Science*, 20, 1953, p. 327) and adapted from the definition given by C. G. Hempel and P. Oppenheim in "Studies in the Logic of Explanation", *Philosophy of Science*, 15, 1948, 135-75, provides only an approximation to the concept of absolute emergence (as contrasted with emergence relative to a theory T) defended by Köhler (see e.g., *op. cit.*, pp. 34, 169).
- ¹⁵⁸ The first criterion, of 'Übersummativität', is formulated by Ehrenfels on pp. 13-16 of his 1890; the second criterion of 'Transponierbarkeit' on pp. 18-21. For literature on Ehrenfels' criteria see Gelb, 1911.
- ¹⁵⁹ The notion of functional proximity or functional interdependence is very close to Husserl's notion of mutual foundation discussed above.
- ¹⁶⁰ Other examples considered by Köhler include magnetic fields, membranes, osmotic systems, energy fields (e.g. high temperature systems giving rise to work, in the physical sense, and kinetic energy structures in hydrodynamics), and electric circuits. Köhler notes that the non-additive character of all of these examples had been recognised long before 'Gestalten' were discovered by Ehrenfels in 1890 (*op. cit.*, p. 92, 124, *et passim*). The concept of *relative difference* has played an important role in more recent work in structuralist linguistics and elsewhere.
- ¹⁶¹ The idea that physical structures could throw light on psychological structures was rejected by the Ganzheits-psychologists (Krueger, Volkelt, Sander, *et al.*), a parallel

movement to the Gestalt school, taking as their starting point the absolute heterogeneity of the two types of holistic formations, (Cf. Köhler, p.x.) Whilst many of Köhler's, and the other Gestalt-psychologists', specific claims concerning the physical properties of stimulus fields were later refuted by independent physiological evidence – a fact which led to an almost total abandonment of the Gestaltist research-programme in mainstream psychology – Köhler's arguments for the importance of non-summativity, in both physics and psychology, retain their validity. Maddens's argument in his 1952 to the effect that all that is of scientific importance which can be expressed in Gestaltist language can be re-expressed in (atomistic) analytic terms, an argument which echoes Gustav Bergmann's early work on Gestalt theory, misses the point. It could equally be argued that all that is mathematically valuable in contemporary mathematics can be re-expressed in the language of *Principia Mathematica*. There is however no one who would defend the view that this re-expression would succeed in making more perspicuous the (referential) content of mathematical propositions.

¹⁶² These definitions apply equally to continuous as to discrete sums.

¹⁶³ We use the term 'Gestalt structure', with Köhler, as synonymous with 'Übersummativität', i.e. as implying merely the satisfaction of Ehrenfels' first criterion. On the various possible definitions of 'Übersummativität' see § 6.3 below.

¹⁶⁴ By 'thing' we are to understand 'non-living bodies in a stable state... i.e. the greater part of the inorganic world insofar as this normally falls under our attention' (loc. cit.).

¹⁶⁵ The moments of an *Eigenstruktur* are, further, non-extensive moments: they cannot be subjected to piecing in the way in which e.g. the colour-distribution across a surface can be pieced. Köhler was aware of Husserl's 3rd Logical Investigation and frequently employs the piece/moment opposition in his work, without however accepting Husserl's theory of foundation (see below).

The non-extensive character of *Eigenstrukturmomente* is illustrated by the example of current flowing through a liquid or through a non-homogeneous conductor (p. 137 f): "There are no partial currents to be encountered in particular regions of the conductor and which might therefore also be encountered in the absence of the remainder of the current; in our theoretical conceptions of the current in specific regions we have much rather to deal, as in the electrostatic case, with *moments* which *carry* the remaining current as well as being themselves supported by it; for as independent formations they are, in relation to the given physical form, not capable of existence."

Köhler discusses not only the concept of an *Eigenstruktur* but also the complementary concept of a *Feldstruktur* (field structure). The former is illustrated by the magnetic properties intrinsic to a magnetised body, the latter by the magnetic properties of the environment of the body. The field is, for Köhler, not a mathematical fiction (as in Faraday's electrostatic theory) but a *realer Zustand des Mediums* (p. 70). The properties of the field are univocally determined by the *Eigenstruktur* and thus also by the physical form of the underlying material. The two structures cannot however exist in isolation from each other: they are inseparable, equally real sides of a single formation (p. 71).

¹⁶⁶ The works acknowledged by Köhler himself as having contributed to the logic of non-summative wholes are Wertheimer, 1912 (particularly the remarks on the theory of categories); Stumpf, 1906a; and Krueger, *Über Entwicklungspsychologie, ihre sachliche und geschichtliche Notwendigkeit*, Leipzig: Engelmann, 1915, (see op. cit., p. 58n).

¹⁶⁷ See the second half of n. 78 above.

¹⁶⁸ The classic statement of the atomist position in 20th century philosophy is Wittgenstein's *Tractatus*. The concept of independence can be applied not merely to objects but also to states of affairs and to events. Again, Wittgenstein's *Tractatus* offers an extreme view as to the lack of interdependence of states of affairs, implying a position ac-

ording to which "There is no special object peculiar to probability propositions" (5.1511). This position was criticised, along lines very similar to those defended by Köhler, in J. v. Kries, *Die Principien der Wahrscheinlichkeitsrechnung. Eine logische Untersuchung*. Freiburg: Mohr (Siebeck), 1886 and in Meinong's, 1915.

¹⁶⁹ Similarly the fact that ordinary everyday experience exhibits mainly additive wholes has led many philosophers to overestimate the power of set theory as a basis for philosophical ontology.

¹⁷⁰ See also Lewin, 1923, 'Wissenschaftslehre' in Lewin's work is to be understood in the sense of Stumpf, 1906a.

¹⁷¹ It is clear that no thing can be genidentical with an event (see vol. I of Ingarden, 1964/65). For the distinction between genidentity sequences of things and of events Lewin refers to Lotze, *Metaphysik*. Leipzig: Hirzel, 1880, p. 8.

¹⁷² ' \wedge ' may be defined in terms of ' \leq ' as follows: $a \wedge b := \exists x (x \leq a \ \& \ x \leq b)$.

¹⁷³ Lewin designates '[a, b, \dots]' as the 'mathematische Zusammenfassung' (i.e. the mereological sum) of discrete a, b, \dots .

¹⁷⁴ Here numerical subscripts indicate successive generations.

¹⁷⁵ One further species of genidentity is considered by Lewin (pp. 121–200), the relation of *Stammgenidentität* obtaining between temporally distinct sections through phylogenetic *Stämme* (a concept broad enough to include biological species, races, tribes, individual families, and in principle also nations). This concept is used by Lewin to throw light on the relation of consanguinity, on the problem of providing identity criteria for biological species, and in particular on the possibility of splitting and merging of *Stämme*.

¹⁷⁶ Thus Rausch will consider only extensive wholes and their piece-wise partitions (op. cit., p. 213). He will however find it possible to find a place for certain non-extensive relations between properties of a whole: see the discussion of N^\forall below.

¹⁷⁷ Whilst Rausch considers only finite partitions, the taxonomy he develops is in principle generalisable to the infinite case.

¹⁷⁸ "The partition" writes Rausch, "relates primarily not to the substantial object, but to a specific spatial region. The object (the objectual manifold) is only secondarily subject to partition in that it occupies the given spatial region or is brought into it. It is however precisely this secondary partition which will be here of interest, since we are not concerned with purely geometrical problems, but rather with real existent things. Thus we impose the condition that every cell of the grid should contain at least some 'objectual substance'..." (p. 214).

¹⁷⁹ ' $(Z^E - t_i)$ ' designates simply the result of removing t_i from Z^E ; ' t_i ' in what follows, shall serve as an abbreviation for ' $t_i(Z^E)$ '.

¹⁸⁰ Still stronger versions of Köhler-summativity can be obtained by considering decompositions which involve the removal not only of single elements, but also of groups of elements. Variant concepts can be defined which take account of the *order* of removal. See Rausch, op. cit., pp. 231–34.

¹⁸¹ To all of the above concepts we can assign correlated concepts of non-summativity in a number of non-equivalent ways. We may, first of all, simply negate the formula on the right of the definition. Or we may substitute *variance* for *invariance* in this formula (yielding definitions which are once more negatable, giving rise to further concepts of summativity). See Rausch, pp. 239–41.

¹⁸² The relation of Rausch's discussion here to Husserl's concept of variation and to analytic philosophical work on variables and functions would merit an investigation of its own.

¹⁸³ Two concepts of variation are at work here and in the formula which follows: the *formal* concept var, signifying (formal) variance under some operation or other, and the *material* concept, ψ , signifying some specific type of variation (e. g. a specific 2°C-increase in temperature). Cf. Rausch, op. cit., p. 274.

- ¹⁸⁴ Many of the ideas in this section have their origins in work by our colleague Wolfgang Degen, of the University of Erlangen, particularly his "Skizze zur rationalen Grammatik" (MS, 96pp.) of 1979. Further details are provided in Smith, 1981 and in Smith and Mulligan, 1982.
- ¹⁸⁵ For those who prefer realistic examples, we might remark that precisely parallel distinctions can be found e.g. in partnership law, and in the law of several property.
- ¹⁸⁶ We should also hope that it might contribute to the rehabilitation of the term 'something' as a *bona fide* referring expression.
- ¹⁸⁷ See also the first and third of the three papers by Simons below, as well as his paper "Logic and Common Nouns", *Analysis*, 38, 1978, 161–67.
- ¹⁸⁸ Whilst, *qua* individual human beings, *a* and *b* are independent wholes, *qua* parties to a disagreement they are mutually dependent, a state of affairs which may be represented by:



- ¹⁸⁹ Thus compare the diagram at the head of this section with that given on p. 119 of Black's *Companion to Wittgenstein's Tractatus*, Cambridge: Cambridge University Press, 1971.
- ¹⁹⁰ There is an obvious connection also to chemical notation. An early attempt to develop a generalised theory of bonds, on the basis of chemical valency theory, was made by Clifford and Sylvester. See Clifford's paper "On an application of the new atomic theory to the graphical representation of the invariants and covariants of binary quantics", *American Journal of Mathematics*, 1, 1878, 64–125 and Sylvesters's note, "Chemistry and Algebra" on pp. 103–4 of his *Collected Mathematical Papers*, Cambridge: Cambridge University Press, 1909. The graph-theoretic properties of the diagrammatic forms considered in the present paper have been investigated by Wolfgang Degen.
- ¹⁹¹ This *principle of downward closure* does not, of course, imply that all constituents of existing states of affairs exist as independent wholes. Stronger principles of downward closure, which do have this implication, are characteristic of ontological atomism. A principle of intermediate strength would assert that only those constituents of states of affairs which are independent wholes (and which correspond to saturated sub-formulae of the corresponding propositional signs) can properly be said to exist. A formal system which accepts the principle of downward closure in any of these forms thereby renders itself incapable of formalising either negation or disjunction – a consequence of the fact that negative and disjunctive states of affairs do not exist (or do not exist in the same sense as positive states of affairs). Since, however, a directly depicting language is capable, as we have already seen, of simulating certain types of existential quantification, it follows that the orthodox interpretation of existential quantification as a form of multiple disjunction has to be rejected (a surely intuitively acceptable consequence of the fact that a statement such as 'there is an apple on the table' does not involve the type of running through of all the objects in the universe which is involved in a reading of ' $\exists x(x$ is on the table)' as ' x_1 is on the table $\vee x_2$ is on the table $\vee x_3$ is on the table $\vee \dots$ ').
- ¹⁹² For example of the type advanced by D. Davidson in his "The Logical Form of Action Sentences", in Rescher, ed., *The Logic of Decision and Action*, Pittsburgh: University of Pittsburgh Press, 1967, 81–95.
- ¹⁹³ Strictly speaking a complete representation of 'Hans kisses Erna . . .' would have to take account of the fact that Hans here serves as agent, Erna as patient. Degen proposes to represent the *actio* and *passio* involved by distinguishing two kinds of inherence, for example by:



with additional symbols of this sort corresponding to further types of categorial relations among substances and accidents, for example, to the relations of causality or of temporal succession. An alternative solution would consist in distinguishing, within Hans and Erna themselves, constituent moments of agency and patiency, thereby obviating the need to move away from a system embodying the single, purely formal relation of inherence or one-sided foundation.

¹⁹⁴ Again, a complete formal representation of 'Hans kisses Erna' would have to take account of the differences between 'Hans kisses Erna *a number of times*', 'Hans kisses Erna *as a matter of habit*', etc.

¹⁹⁵ In some developments of the system such disjoint formulae would be disallowed, on the basis of an appeal to the fact that disconnected states of affairs do not exist (or do not exist in the same sense as connected states of affairs); *im Sachverhalt hängen die Gegenstände ineinander wie die Glieder einer Kette*.

¹⁹⁶ This rule corresponds to the acceptance of a *principle of upward closure*, and again, a number of distinct principles of upward closure, of varying strengths, might prove to be acceptable. Bradley, for example, would seem to be committed to the view that from any (abstractly demarcated) state of affairs we may infer the single all-embracing state of affairs which is the world as a whole.

¹⁹⁷ More precisely, we should have to say that, in contrast to the views of the scholastic philosophers (and of Husserl in the *Logical Investigations*), material essences do not constitute trees, but more complex structures, loosely identifiable as graph-theoretical products of trees). Work on these structures (or on the logic of common noun expressions) is however in its infancy (again, as a result of the dominance of set theory and of predicate logic amongst contemporary philosophers).

¹⁹⁸ There is no limit to the degree to which the constituent figures of a propositional sign may be nested inside each other (though clearly, if we are to preserve the directly depicting property of the language, we must insist that no figure be either mediately or immediately properly nested inside itself). The question arises as to whether we can accept the purely formal inference rule:

$$\frac{\Gamma}{\boxed{\Delta}},$$

where Δ is a propositional formula including Γ as sub-formula. This rule would correspond to a principle ruling out the possibility of free accidents (or of free non-independent wholes in general), i. e. to the view that all accidents are constituents of larger, independent wholes.

¹⁹⁹ Thus for example the range of possible consequences of the pair \boxed{a} , \boxed{b} of propositional signs includes as many as 9 distinct cases, a number which is increased by several orders of magnitude when 3, 4 or 5 propositional signs are taken as starting point.

²⁰⁰ This presents no significant problems other than the combinatorial difficulties mentioned above. Cf. Smith and Mulligan, 1982.

Three Essays in Formal Ontology

Prefatory Note

When first reading Husserl's *Logical Investigations* it is very easy to pass by the third as a minor detour from the high road of Husserl's major concerns. In common with many other readers, I initially held this view: the many distinctions Husserl makes seemed to me to be, to use his own words about Twardowski, 'as subtle as they are queer'. To anyone accustomed only to the extensional whole-part theories of Lesniewski or Goodman this is a natural reaction. My change of view was influenced partly by Kevin Mulligan's insistence on the pivotal role of the third investigation in Husserl's work, and also by the increasing recognition of the themes of unity, dependence and self-sufficiency treated by Husserl, as concepts echoing loudly throughout the history of ontology. It was also Kevin Mulligan who unearthed Ginsberg's 1929 article on Husserl's six theorems, and discontent with her criticisms spurred me to attempt a formalised reconstruction of Husserl's ideas, which met with various difficulties on the way to the first of these essays.

At the same time I was attempting to use mereological considerations to offer an alternative to what I consider the unacceptable account of number put forward by Frege, using Schröderian and Husserlian ideas suggested to me by Barry Smith. My original view was that numbers are properties of what I then called manifolds, i.e. aggregates considered as composed in some determinate way. This is what I should now call a group or aggregate theory of number. In the second essay I present the considerations which forced me to abandon such a view and to recognise the distinctive nature of pluralities as against aggregated individuals. This in turn led me to reappraise the notions of reference and set, with the result seen in the third essay, where a formal theory of mani-

folds, now reconstrued as comprising both individuals and pluralities, is developed. Some manifolds are aggregable: to such aggregates mereological considerations still apply. These issues are dealt with in the second essay, where the opposition to Frege is also explicitly set out.

At each turn I found voices of encouragement from the past, some from unexpected quarters. Harkening to these has convinced me that the logical and philosophical harvest of the fecund years between Husserl's *Philosophie der Arithmetik* in 1891 and Russell's *Principles of Mathematics* in 1903 is yet far from being reaped in full.

I. The Formalisation of Husserl's Theory of Wholes and Parts¹

§ 1 Introduction

Husserl's third Logical Investigation is called "*On the theory of wholes and parts*". It has probably received less attention from commentators than any of the other investigations, including the shorter fourth, which Husserl himself saw as an application of the ideas of the third to questions of grammar. The ideas put forward in the third investigation play a crucial role in Husserl's subsequent philosophy, and he was able to recommend them, even much later in his life, as offering the best way into his philosophy.² Although they did not perhaps present such an attractive clarion-call to research, they might, had Husserl's advice been followed, have made a much greater contribution to philosophical work than in fact they did. I should like to suggest that it is not too late to learn from the third investigation, and that, in a tidier form than they there receive, the ideas could become indispensable weapons in the conceptual armoury of the philosopher interested in ontology. This paper has the more modest purpose of attempting to clarify and interpret what Husserl was trying to say, with a view to eventually offering a rigorous treatment of the most important notions, and I wish also briefly to suggest where such notions might prove important in ontology.

It is important to distinguish formalisation from mere symbolisation. Any expression may be symbolised: one simply introduces symbols for various words or other expressions: the difference is merely one of the graphic shape of the expression. However, symbols, unlike the natural language expressions they can conventionally replace, derive their sense from the specific convention setting up their use, whereas this freedom of interpretation is not available for the original natural language expressions. For this reason symbols are more easily detachable from their specific interpretation, and may be manipulated purely syntactically, without interpretation. It is this feature which makes symbolisation such a useful way of presenting a formal theory. A formal theory, in Husserl's sense,³ is one in which no mention is made of any particular things or kinds of things, but which deals with objects in complete ab-

straction from their specific natures. A formal theory need not even be expressed symbolically: a statement such as, 'If a thing bears a relation to another thing, then the second thing bears the converse relation to the first',⁴ contains no restriction to particular domains of application, but consists purely of logical constants and formal concepts, such as *thing*, *relation*, and concepts such as *converse* definable in terms of these. It is advantageous to present formal theories symbolically because we may use symbols which are not given any fixed interpretation, but belong to a grammatical class which corresponds to a formal concept; they are then free to vary in interpretation over any entities whatever falling under that formal concept. So, if we allow the usual sorts of formal grammar, the above formal statement, could be symbolised (If aRb then $b\check{R}a$). Symbolisation usually proceeds further, with symbolisation of the logical constants, which may indeed be necessary if they need some degree of regimentation for the specific purpose in hand. In this sense, a symbolised presentation of a purely formal theory in Husserl's sense fulfils the conditions suggested by Wittgenstein as marking an adequate *Begriffsschrift*.⁵ Each formal concept corresponds to a different type of variable, i. e. symbol with variable interpretation. Only the logical constants are fixed.⁶

Husserl thought that a purely formal theory of part and whole was possible, and regarded the second part of the third investigation as offering the beginnings of such a theory. But, for all its detail, the investigation remains only a sketch of what a fully developed formal theory would look like, and like all philosophical sketches, presents problems of interpretation, lacunae, and vagueness, as well as being highly suggestive of possible fruitful developments. Although Husserl makes a brief and somewhat half-hearted venture into a partial symbolisation of a few theorems, the investigation is largely couched in Husserl's semi-technical German, and he nowhere attempts to set up a formal language in the modern logistic sense, which means that his formal treatment falls well short of modern standards in terms of the rigour of its symbolisation. While Husserl was by no means unfamiliar with symbolic logic as such, he was less interested in symbolisation for its own sake than in the philosophical treatment of concepts, even those concepts where, as in logic and mathematics, symbolisation had become indispensable to progress. He never believed that problems could be resolved purely by recourse to symbolisation, and rejected strongly formalist tendencies in mathematics, which would have us believe that mathematics is simply a

game with symbols which do not themselves have any meaning.⁷ It might be suggested that a theory of whole and part cannot be formal in Husserl's sense, since where – as in the work of Leśniewski and Goodman – it has been formalised hitherto, it has proved to be a proper extension of logic in the normally accepted sense.⁸ Against this it must be pointed out that Husserl clearly states that *whole* and *part* are purely formal concepts.⁹ Whether Husserl is correct on this, depends on what is taken as the criterion for being a formal concept. I do not believe that enough has as yet been done in clarifying the idea of a formal concept to give a definitive answer on this point. To that extent, the title of this essay promises something which it is not clear can be given. However, to the extent that we can eliminate from the theory all *other* concepts which are clearly not formal, to that extent we have succeeded in outlining what Husserl would call a theory of the pure forms of whole and part.¹⁰

Although advertised as a theory of whole and part, Husserl's investigation spends as much time on the concepts of dependence and independence, which, while they bear crucially on Husserl's particular brand of whole-part theory, cannot be counted as purely mereological notions. However, Husserl lays great stress on the distinction between dependent and independent parts as being the chief distinction among parts, and since it is in this distinction that Husserl's theory is distinguished from later and symbolically more adequate whole-part theories, I shall also consider the question of dependence and independence in some detail.

Husserl draws a distinction in the investigation between two different kinds of part or constituent of a whole. Some parts, those normally so-called, could exist alone, detached from the whole of which they happen to be part. These Husserl calls 'pieces' or 'independent parts' of the whole. On the other hand there are parts or constituents of a whole which could not exist apart from the whole or sort of whole of which they are part. These Husserl calls 'moments' or 'dependent parts' of the whole. For example: the board which makes up the top of a table is a piece of the table, while the surface of the table, or its particular individual colour-aspect, are moments of it. This distinction amongst kinds of parts is certainly not new: indeed it may be claimed to go back to the *Categories* of Aristotle.¹¹ Husserl himself certainly derived the distinction from his teacher Stumpf, who used the terms 'partial content' and 'independent content', in his discussion of the distinction within the realm of phenomenological psychology.¹² Husserl first used the distinc-

tion himself in his 1894 article “Psychological studies in elementary logic”, where many of the distinctions later made in the *Logical Investigations* are already to be found.¹³ The later exposition contains two major advances on the earlier version: firstly a recognition that the dependent/independent distinction has application outside the sphere of psychological contents to ontology generally, and secondly, connected with this, the idea of a formal theory of whole and part, which, as we have said, Husserl sketches but does not completely execute in the second half of the investigation.

In the hands of later whole-part theorists such as Leśniewski and Goodman, whole-part theory has become associated with nominalism and extensionalism, where its general applicability and algebraic similarities with set theory make it a substitute for set theory more acceptable to those who have ontological objections to sets as abstract objects.¹⁴ Part of the interest in examining Husserl’s whole-part theory is that it is free from such nominalist scruples, being conceived within the richly Platonist ontology of pure species adopted by Husserl at the time. It is, furthermore, non-extensional, making indispensable use of the concepts of essence and necessity. The basic distinction Husserl makes between dependent and independent parts is not even expressible in an extensional language. However it seems to me that one need not buy Husserl’s package of Platonism and non-extensional language as a whole in order to make use of his whole-part theory. It is usually taken for granted that a non-extensional language brings ontological commitment to Platonic entities of some kind, whether species, meanings, or something like possible worlds. But it is far from clear that we can even manage to make reasonable sense of the actual world in a purely extensional language. It may, further, be possible to use a whole-part theory of Husserl’s type to buttress a more sophisticated nominalistic approach to universals *via* Husserlian moments, so the usual yoking together of Platonism and non-extensionalism is far from clearly established.

One of the problems with the interpretation of the third investigation is that not all traces of Husserl’s earlier psychological approach and interests have been expunged. This affects both the language within which Husserl makes his points, and the range of examples to which he generally makes recourse. Thus the word ‘content’ is frequently used where the word ‘object’ is also appropriate, and where the latter ought to be used in preference. This is despite Husserl’s acceptance that his remarks hold for all objects generally, and not just psychological contents. The

examples are drawn almost exclusively from the phenomenological psychology of perception; for instance, that in the visual perception of a coloured thing, the moment of colour and the moment of spatial extension are both dependent parts of the thing as a whole, and require each other's co-occurrence in the thing. When this observation is transposed from the phenomenological to the ontological mode, this yields the proposition that the moment of colour and moment of extension of the thing itself (rather than the thing as perceived) are dependent parts of it. In this case the transposition seems to go quite smoothly, and I believe that it was Husserl's opinion that this would be so quite generally: for 'content' substitute 'object' and the theory has been in principle extended. It seems to me questionable whether the extension of the theory to objects in general is in fact so easy. Particular attention must be paid to the fact that some objects at least may belong to more than one kind at once, and that its dependence relations vis-à-vis other objects may vary according to the kind. This consideration is lacking from the psychological case, and so may have been at work in moulding Husserl's thoughts about the general properties of the more important part-whole relations. It is often difficult to tell, at crucial junctures in the text, whether the unthematized background of examples was playing a part, and if so, what part.¹⁵

Arising from this is the fact that it is in general possible to give the concepts of dependence and independence a much wider application outside the theory of whole and part. Husserl may not have been unaware of this, but he does not embark on any such general development. I have therefore allowed myself to go beyond the range of Husserl's examples in order to open up the question of such a generalised theory of dependence. The attendant risks of distortion and misrepresentation of Husserl's own position are I believe worth running if we are to put his ideas to work quite generally.

§ 2 Problems of Formalisation

There is a wide range of formal languages among which to choose when we attempt to formalise Husserl's ideas. Choice among these must be motivated by considerations partly external to whole-part theory as such. But whichever language is chosen, it cannot, if it is to do justice to Husserl's ideas, be extensional. The whole-part theories of Leśniewski

on the one hand and Leonard and Goodman on the other are both extensional. So a minimalist solution to the choice problem would be to add to one of these a necessity operator and axioms for it. One could for instance take the axioms and rules of S4 and graft these on to the Leonard-Goodman calculus of individuals. This approach has all the merits of timidity: it causes least disturbance. But there are drawbacks as well. Since Husserl was writing before it was appreciated how modal logic would proliferate different systems, there is no chance of receiving a direct answer from his writings as to which of the many available would be the best to choose. In view however of formula (3) below, which tells us that whenever species stand in a relation of foundation they do so of necessity, it appears that any modal system used would have to contain the characteristic S4 axiom $\Box p \supset \Box \Box p$ as a theorem. One obvious candidate modal system is accordingly S4. However, since the applications of modal considerations in the present context do not seem to require that we decide among alternatives whose differences do not show up in the sorts of formula we shall be considering, I shall in fact shirk the choice, and suggest merely that the modal axioms be not weaker than S4.

There is a problem about using a propositional necessity operator at all, in that traditionally the term 'essence' has related not to propositions but to properties, to *de re* rather than *de dicto* necessity. Husserl's writings show a willingness to accept both that individuals of certain kinds possess essential properties, and that there are general essences or *eide*, which are the abstract objects of imaginative variation among possibilities. For this reason I suggest that in addition to a necessity operator on propositions it is advantageous to consider a necessity operator on predicates, or property-abstracts. I shall use the expression 'nec' for this purpose. The operator was introduced by David Wiggins,¹⁶ who has given strong reasons, independent of Husserlian considerations, for believing that such an operator is indispensable to our ordinary conceptual scheme. It remains to be seen how 'nec' and ' \Box ' should be taken to interact, indeed whether a unified theory of them is possible at all. Because of these uncertainties, the account given in this paper must be regarded as only a tentative investigation into essentialistic whole-part theory.

There is yet a further reason for disquiet over simply grafting modal operators onto extensional mereology. For in extensional mereology (which I take to comprise both Leśniewski's mereology and the Leonard-Goodman calculus of individuals) a thing is identified with the

sum of its parts; indeed Goodman *defines* the identity of things as consisting in their having the same parts. But this rules out in advance the possibility of different things merely *coinciding* spatio-temporally. The case where such coincidence does not extend throughout the total life-span of both things is usually handled within extensional mereology by reconstruing things as four-dimensional space-time worms, and pointing out that temporary coincidence merely involves two such entities overlapping in a certain spatio-temporal region. However, there may also be cases in which we should wish to say that two things coincided over their total life-span, yet were not identical. This is connected with the fact that according to the everyday notion of a material thing, a thing can both gain and lose parts without prejudice to its identity, as can, most obviously, an organism.¹⁷ But a whole which conforms to the sum-principle of extensional mereology cannot lose any part. One way of avoiding recourse to four-dimensional objects, but which preserves the sum-principle, is Roderick Chisholm's theory of *entia successiva*.¹⁸ However, it seems somewhat drastic to abandon the paradigmatic role of organisms among material individuals for the sake of an abstract principle, when the normal three-dimensional thing-concept has not conclusively been shown to be beyond redemption. It would further be premature to abandon the normal conception in expounding Husserl's whole-part theory, if there is, as I believe, a chance that this very theory could provide assistance in explicating the normal conception of a thing.¹⁹

So I shall not be following a minimalist line: our mereology will not have the principle that coincident things are identical, and we shall use a *de re* necessity operator. It follows that the suggestions contained in this paper are largely exploratory: like Husserl's this is not a formal presentation with axioms and theorems, but an attempt to set out some of the possibilities and clarify some of the issues which need to be resolved before a formalisation of Husserl's ideas which is both intuitively and formally adequate can be presented.

One respect in which Husserl's whole-part theory is distinctive is its essential use of what Husserl calls *pure species*. I shall use lower-case Greek letters α , β etc. for such species, and lower-case Italic letters *a*, *b*, *c*, etc. for arbitrary members of α , β , γ respectively. Where we are treating an individual as such, in abstraction, as far as possible, from considerations of which species it belongs to, I shall use the letters *s*, *t*. Expressions of the form ' $s \in \alpha$ ' will mean '*s* belongs to the species α '. But there

is here a problem of interpretation. What are such species? Do they indeed exist? If we follow Husserl in assuming that they do, we run the risk of building too many ontological presuppositions into the formalisation in advance. I shall accordingly give expressions of the form ' $s \in \alpha$ ' as far as possible a merely syntactic reading, allowing ' α ' to replace a common noun, and reading it as ' s is an α '. This leaves it open until later whether we should treat α as a proper name of a pure species, or of a set, or merely as a common name for s and maybe various other individuals. One thing to note, however, if we are to remain faithful to Husserl's way of construing species, is that we cannot allow contradictory species. Every species is, for Husserl, such that it *could* have members, even if it in fact does not. We shall accordingly make the informal stipulation that substituends for α , β etc. should be such that ' $\diamond (\exists x) (x \in \alpha)$ ' should be true.²⁰

Husserl explicitly warns the reader that he is using the term 'part' in a wider sense than it is usually given. He wishes it to comprise not only detachable pieces but also anything else discernible in an object, anything that is an actual constituent of it, apart from relational characteristics.²¹ In Aristotelian terminology, Husserl's parts would comprise parts normally so-called, accidents, and also boundaries.²² Doubts about the propriety of such a treatment are expressed by Findlay in the introduction to his English translation of the *Logical Investigations*. Findlay suggests that while there may be analogies between parts in the usual sense and individual accidents or moments, the two do not belong to the same category and it is therefore a mistake to treat them together ontologically. This does not recognise the expressly formal nature of Husserl's theory, for it is precisely the independence of restrictions to any particular category or region which mark what Husserl calls a formal theory. Husserl's account proceeds independently of doctrines concerning categories and category-mistakes.²³ The only way in which Husserl could be, in his own terms, mistaken, would be if he had confused either two formal concepts, or one formal concept and one material. Given only that Husserl does believe in individual accidents or property-instances, he cannot but treat them as falling within the formal concept of part. It is true that many philosophers have disputed whether there are such accidents. In answer it can be pointed out that not all the examples Husserl adduces as moments are property-instances; there are also boundaries, although he did not expressly include the latter until the later work *Experience and Judgment*.²⁴ It would be uncharitable to expect Husserl to pro-

duce a justification for treating of moments along with other parts in advance of judging how well the theory so produced managed to solve problems of unity and predication by comparison with other competing theories.

§ 3 Husserl's Basic Concepts: Whole and Foundation

The two most important concepts employed by Husserl in the third investigation are those of *whole* and *foundation*. Unfortunately, both these terms are ambiguous, and we must recognise their various senses before we can make clear sense of Husserl's theory. By contrast, Husserl does not make thematic the marks of the general concept *part* as such, but proceeds rather to make distinctions among the various kinds of part. It must be assumed that he considers the concept too primitive, being a formal concept, to allow of substantive elucidation.

Husserl distinguishes three different concepts of whole, a *narrow* concept, a *wide* concept, and a *pregnant* concept. The first two terms are mine; the last is Husserl's own. It is characteristic of Husserl's approach in the *Investigations* that he is reluctant to coin special terminology, even where he recognises ambiguities and is attempting to avoid them. This is in contrast to his later willingness to develop a specifically phenomenological vocabulary.

A narrow whole is one in which a number of entities are bound together into a unit by a further entity which Husserl calls a 'unifying moment' (*Einheitsmoment*). Narrow wholes are a rather special kind of whole, and cannot comprise all the wholes that there are. The supposition that all wholes are narrow in this sense leads, as Husserl points out in a passage reminiscent of Bradley,²⁵ to every complex being, appearances notwithstanding, infinitely complex. For if A and B are bound together by U, then A and U must be bound together by U₁, and so on *ad infinitum*. Husserl's own theory offers a way out of this regress of parts, by suggesting that some kinds of entity come together to form wholes just because they are the kinds of entity that they are, and thereby require partners, without requiring anything else which joins them together.²⁶

The wide concept of whole seems to me to be very like Goodman's concept of an individual; no restrictions are placed on how tightly or loosely connected the various parts of the whole are, whether they are

scattered or not, so long as we can still regard the whole as a single thing. It is indeed in the possibility of being regarded as a single thing that Husserl considers that bare unity consists.²⁷ This does not mean however that there are only individuals. Husserl expressly contrasts *unity* and *plurality* as formal concepts.²⁸ But any plurality may be taken together as something unitary, thereby founding a new higher unity, whose unity is, however, extrinsic to it, in the collective act.²⁹ So I shall allow as individuals anything which can possess a (singular) proper name. This will include even arbitrary *collectiva*. This liberality is reflected in extensional mereologies by allowing that arbitrary sums of individuals are themselves individuals. The reason for this is not that we wish to take most of these arbitrary *collectiva* seriously, but rather that it is not clear in advance where to draw the line between things which are wholes in this widest and weakest sense, and those which have some more intrinsic unity.³⁰

The third or pregnant concept of whole is defined by Husserl in terms of the concept of foundation. A pregnant whole is one each of whose parts is foundationally connected, directly or indirectly, with every other, and no part of the whole so formed is founded on anything else outside the whole. This of course presupposes Husserl's own concept of foundation, which means that Husserl attempts to define one sense of *whole* in terms of foundation, which in fact itself presupposes another concept of whole, the wide concept, which is, as Husserl points out, not a real or determining predicate.³¹ The unity of a pregnant whole is intrinsic to it, by contrast with the extrinsic unity of a mere sum or aggregate.

When we turn to foundation, matters are not so clear. It is most important to clarify Husserl's meaning here, since the concept of foundation turns out to be the most basic one of the whole investigation. I believe that we must distinguish two very different types of relation, both of which Husserl calls 'foundation'. There is a *generic* concept, which relates species, and there is an *individual* concept, which relates individuals which belong to species related according to the generic relation. It would in fact be more correct to speak of generic and individual concepts in the plural, since Husserl offers several formulations which do not exactly coincide, and it is possible to discern further definitional possibilities not considered by Husserl. It is chiefly in connection with the generic relations that one can speak, as Husserl does, of laws of essence. Husserl is mainly interested in the essential relations, and so does not offer an account of individual relations as such. But if one is to be

able to discuss the foundational relations of determinate individuals, such an account is needed, and there are crucial places in the investigation where Husserl is clearly talking about relations between determinate individuals, albeit individuals considered as belonging to a certain species. In his official introduction of the concept of foundation, Husserl, in addition to speaking of the case where the species α and β are foundationally related, also mentions the case where we should say that two members a and b of these respective species are themselves foundationally related.³² The definition of relative dependence and independence offered earlier speaks clearly of one thing's being dependent or independent relative to another.³³ In each case it is clear from either the context or the notation that the schematic letters used by Husserl are to be taken as singular terms.³⁴ Finally the notion of a pregnant whole requires that we talk about the foundational connectedness of the individual parts making up the whole. For these reasons an account of the foundational relatedness of individuals is necessary. However, Husserl was of the opinion that it is possible to move back and forth between talk about individuals and talk about species without difficulty, and so does not enlarge upon the difference.³⁵ It is however this difference which constitutes the major difficulty in developing a Husserlian whole-part theory.

Husserl defines foundation in the first instance as a relation holding between two pure species. The verbal rendering of the definition goes thus:³⁶

an α as such requires	foundation by a β	: =	there is an essential law to the effect that an α cannot exist as such except in a more comprehensive unity which associates it with a β .
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Later Husserl contends that the concept of whole or unity here employed is dispensable, and reformulates what he takes to be the same idea thus:³⁷ in virtue of the essential nature of an α , an α cannot exist as such unless a β also exists.

In this second version reference to a more comprehensive whole is missing. But this suffices to make the two concepts of foundation not equivalent. For according to the second definition, every species is self-founding. This means that, according to a statement Husserl elsewhere makes about absolute dependence,³⁸ everything is dependent absolutely. This is clearly not what Husserl wanted since it obliterates the distinc-

tion between dependent and independent objects. So the concept of foundation used in defining absolute and relative dependence cannot be the weaker second concept. One solution to this problem which readily suggests itself is that the species α and β have to be *different*. We could then say that something is dependent only if it is dependent on something belonging to another species.³⁹ This will not do, however, since it turns out that there are species which are non-trivially self-founding, which the suggestion does not allow.

So we shall revert to Husserl's first formulation, with its reference to a more comprehensive unity. This suggests that every α should be found together with a β in something of which the α in question is a *proper* part. In what follows, we shall use Goodman's symbols ' \leftarrow ' for 'is a proper part of' and '<' for 'is a part of', where the latter allows, while the former excludes, coincidence. Hence our suggestion for a rendering of the definition is:

$$(1) \quad \Box(\forall x) (x \in \alpha \supset (\exists yz) (y \in \beta \ \& \ x \leftarrow z \ \& \ y < z))$$

This condition appears still not to be strong enough. Whilst it captures the letter of Husserl's formulation it misses something of the spirit, in that in line with this definition the more comprehensive whole could be simply the β itself. This appears implausible as capturing the idea that we want: while we might say that the species *husband* is founded on that of *wife* (and vice versa of course) we should not want to say that because the existence of husbands required that of married couples that husbands are founded as such on married couples. This appears to have got marital carts before horses. Similar remarks would apply, *mutatis mutandis*, to foundation relations cited by Husserl, such as the mutual foundedness of colour-moments and moments of extension. Husserl takes foundation to be a relation of necessary *association*,⁴⁰ and the connotations of this word preclude either the α or the β in question from exhausting the more comprehensive whole of which each is a part. Indeed Husserl's formulation is itself ambiguous in that it could be read as implying that the whole is more comprehensive just than the α or as more comprehensive than both the α and the β , which is our second and preferred reading. Although Husserl does talk of wholes in the pregnant sense being founded upon the range of their parts,⁴¹ this is a regrettable equivocation, and probably stems from the etymology and previous use by others as well as Husserl of the word. In the sense we have formulat-

ed, everything which is founded on something is thereby dependent, whereas in this other sense we could describe even independent wholes as founded upon their parts. It would be better to describe such wholes as *constituted by* their parts, reserving the word 'foundation', despite its misleading etymology, for the associative relationship. However, we cannot merely strengthen the last conjunct ' $y < z$ ' of (I) to ' $y \ll z$ ', for it would follow from this that any species whose instances had to exist as part of some greater whole would thereby be a self-founding species. But while a lake cannot exist as such unless surrounded by land, and a child cannot exist as such unless it has parents, this cannot be regarded as making the species *lake* and *child* self-founding, whereas the species *sibling* clearly *is* self-founding, since a sibling cannot as such exist unless *another* sibling exists. So, using ' $\alpha \sqsupset \beta$ ' for ' α s are founded on β s' we arrive at the following definition of generic foundation:

$$(2) \quad \alpha \sqsupset \beta := \Box (\forall x) (x \in \alpha \supset (\exists y) (y \in \beta \ \& \ x \prec y \ \& \ y \prec x))$$

where ' $x \prec y$ ' abbreviates ' $\sim(x < y)$ '.

The essential nature of the foundation relation is expressed by the prefixed necessity operator. Since we have assumed the availability of the S4 principle $\Box p \supset \Box \Box p$, we have as a consequence that all generic foundation relationships hold of necessity:

$$(3) \quad (\alpha \sqsupset \beta) \supset \Box (\alpha \sqsupset \beta)$$

a result which would meet with Husserl's approval. It might be questioned, however, whether strict implication adequately fits the bill for expressing the relationship between α s and β s that we are aiming for. Should it perhaps involve some relationship of logical relevance, connecting the two species? For instance, would it be better to adopt the following as a definition of foundation:

$$(4) \quad (\forall x) (x \in \alpha \rightarrow (\exists y) (y \in \beta \ \& \ x \prec y \ \& \ y \prec x))$$

where the arrow represents the *entailment* connective?⁴² This would appear to be in harmony with Husserl's view of the relationship as arising out of the very nature of α s as such. It would preserve the theorem (3) above, since the logical system E of entailment has an S4 modal structure. And suppose that it is necessarily false that there be an α : would

not definition (2) make it trivially true that $\alpha \supset \beta$ for all species β ? This would also appear to favour an approach via entailment. But we have stipulated informally that no such case can arise, because it would violate the requirement that every species be such as to be capable of having instances, so this problem cannot arise as long as we remain within the limits imposed by this stipulation. While it would seem both possible and perhaps in the long run desirable to develop a foundation theory in terms of entailment or some other relevant connective, this course places additional difficulties in the path of interpreting Husserl, and so will not be followed here.

While the definition of foundation given above as (2) includes many important essential part-whole relationships, it does not include them all, so it is worth noting that the wider sense of foundation given by Husserl can be captured as follows:

$$(5) \quad \alpha \supset \beta := \square (\forall x) (x \in \alpha \supset (\exists y) (y \in \beta))$$

It is in this sense, rather than that of (2), that a whole which needs a part of a certain kind may be said to be founded on that part. For instance, it is essential to men that they possess brains, or tables that they possess tops. Such essential parts cannot however be described as being *associated* with the wholes which include them, since associated parts are co-ordinated, neither being the whole itself. It may be because Husserl was not quite clear which of the various possible essentialistic relations he wished to describe as foundation that we get from him more than one, non-equivalent definition. It is more to the point, however, simply to note the differences, remarking that both concepts of foundation have their uses. We shall in what follows concentrate predominantly on ' \supset ', since this appears to carry the greater weight for Husserl. However, as we shall see, some of the results which Husserl takes to hold for foundation in general hold for ' \supset ' but not for ' \supset '.

As an application of our definitions let us consider one of the propositions put forward by Husserl in § 14 of the Investigation, and for which he offers informal proofs. This is Husserl's Theorem I⁴³:

If an α as such requires to be founded on a β , every whole having an α , but not a β , as a part, requires a similar foundation.

To represent this we introduce by definition a complex general term ' $\alpha\beta$ ', to be read 'object which contains an α but not a β as part'. Defini-

tions of general terms take the form of showing what condition an individual must satisfy to fall under the term, and accordingly have the form ' $t \varepsilon \alpha := (\dots t \dots)$ ', where ' t ' is an arbitrary singular term and ' $(\dots t \dots)$ ' stands for a sentential context containing occurrences of ' t ' but not of ' α ' or any other term defined in terms of ' α '. Thus we give a definition of ' α) β ' as follows:

$$(6) \quad t \varepsilon \alpha) \beta := (\exists x) (x \varepsilon \alpha \ \& \ x < t) \ \& \ \sim (\exists x) (x \varepsilon \beta \ \& \ x < t)$$

It is understood that if an open sentence of the form ' $x \varepsilon \alpha$) β ' occurs in a proof, that when replaced by an open sentence corresponding to the right-hand side of the definition (6), we reletter bound variables if necessary so as to ensure that scope problems do not arise; otherwise the use of open sentences containing defined general terms is the same as that when there are no bound variables present.

Given (2) and (6) it is a simple matter of modal predicate logic, using only the transitivity property of '<' to prove

$$(7) \quad \alpha \supset \beta \supset \alpha) \beta \supset \beta$$

and its necessity, which is the obvious way of representing Husserl's Theorem I.⁴⁴ Thus what Husserl confidently calls its axiomatic self-evidence is seen to stand up in the present formalisation.

While the relation ' \supset ' is trivially reflexive, the relation ' \supset ' is not. Only certain species are self-founding in the stronger sense. The most obvious examples are those using derelativised nouns. These do not figure as such in Husserl's examples, although his exposition uses such nouns a good deal. We can offer the following as examples: sibling, spouse, partner, colleague, cousin, accomplice, companion, fellow, enemy, peer, associate. The last example uses the very idea Husserl employs to characterise the foundational tie as such. We might offer examples of non-self-founding species such as house, mountain, planet.

A crude grammatical test for whether a noun corresponds to a founded species or not is to see whether it is natural to describe an α as, say, an α *of* something or someone. So every colour is the colour of something, every spouse is the spouse of someone, every planet is the planet of some star, every monarch is the monarch of some realm, and so on. The test is only crude, however, in that some founded species are not so spoken of, e.g. we do not call a lake or an island a lake *of* the sur-

rounding land, or island *of* the surrounding sea, and the word 'of' can mean many other things. Nevertheless the test is a useful rough guide. For self-founding species, for instance, it often makes sense to say that every α is an (or the) α *of* another α .

Some foundation relations between species are symmetric; Husserl calls such relations *two-sided* or *mutual* foundation.⁴⁵ For example the species husband and wife, or colour and extension, are mutually founding.⁴⁶ On the other hand, some foundation relations are not symmetric; these are *one-sided*. Thus in Brentano's psychology judgments are one-sidedly founded on presentations or ideas, while feelings of love and hate are founded on judgments, again one-sidedly, and hence indirectly on ideas. To take our geographical example again, a lake is as such one-sidedly founded on dry land. Several terms from physical geography show such one-sided foundation, e.g. mountain, plateau, cwm, island, peninsula, and so on.

Whereas ' \supset ' is transitive, ' \supset ' is not. The definition (2) has to be examined to see why not. The conditions $\alpha \supset \beta$ and $\beta \supset \gamma$ do not suffice to show that $\alpha \supset \gamma$, because if we have $a \in \alpha$, $b \in \beta$ and $c \in \gamma$ satisfying the conditions for (2), the fact that neither a and b nor b and c are part of one another does not suffice to show that a cannot have c as part or vice versa. Examples of this are hard to come by, but the following suggests itself: there cannot be a person conducting the defence at a trial unless there is a trial, and there cannot be a trial unless there is a defendant or defendants. But there is nothing to stop the or a defendant conducting the defence at the trial. Another consideration which reinforces the position that ' \supset ' be not transitive is that if it were, all species two-sidedly founded on some other species, would by transitivity and symmetry be self-founding, in the strong sense, and this is surely not intended.

It is possible to define certain more general concepts relating to foundation if we allow ourselves to quantify over species, introducing bound general term variables. We may say that a species is *founded* or is *founding* according as there is a species it is founded upon or, respectively, founds:

$$(8) \quad \alpha \supset : = (\exists \xi)(\alpha \supset \xi)$$

$$(9) \quad \supset \alpha : = (\exists \xi)(\xi \supset \alpha)$$

Here we use a notational device which we find convenient elsewhere also: to represent existential generalisation by omission. We can also de-

fine an important concept of essential independence for species: α s as such are *essentially independent* when they are not founded:

$$(10) \quad I(\alpha) := \sim(\alpha \supset)$$

The other important concept of foundation concerns not the relations between species but those between individuals. Husserl, as we mentioned, brushes very lightly over the distinction, and in the 1929 commentary on the formal work in the third investigation by Eugenie Ginsberg, whose later article appears in this volume in translation, the distinction goes quite unnoticed.⁴⁷ If we are to be able to speak of foundational relationships between individuals at all, we cannot rest content with defining such individual foundation in terms of generic foundation, after a fashion such as this: a , as an α , is founded on b as a β := $a \in \alpha$ & $b \in \beta$ & $\alpha \supset \beta$. The first and most obvious reason is that just because $a \in \alpha$ and $b \in \beta$ and $\alpha \supset \beta$ it does not follow that a is founded on b . For b must be not just any β but *the right one*. If Alice, as a wife, is founded on Bob, as a husband, it is not sufficient that Alice be a wife and Bob a husband: they must be married *to one another*. Similarly, although, to use Husserl's example again, a moment of colouredness requires a moment of extension and vice versa, merely taking the colour-moment of one thing and the extension-moment of another does not yield the more independent colour-extension whole required.

A definition of individual in terms of generic foundation would be forthcoming were we able to specify a condition $F(a,b,\alpha,\beta)$ to be added as a conjunct to the right-hand side of the attempted definition above. I have been unable to find such a general condition, and indeed have come to believe, somewhat reluctantly, that there is none to be found. We can certainly find formulae for many particular cases: for example in the marital case we simply need the relation ' a is married to b '. But it is clear that in this case the species terms are derived from the relative by derelativisation. Furthermore, if Husserl is correct in saying that the foundational relations between species rest on the essential natures of the species in question, and not on formal considerations, we *ought not* to be able to find such a general formula.

One initially promising way of trying to define individual foundation from generic is to introduce different concepts of whole. For the whole formed by the colour of this + the extension of that is a mere whole, in the widest sense mentioned above, whereas the whole formed by the co-

lour of this + the extension of this is much more coherent. This coherence cannot consist in total independence however, since both the colour and extension may (and in this case do) require completion by something beyond them. Nor can it mean simply completability, since the colour of this + the extension of that can also be completed into a self-sufficient whole, namely this + that. Similarly, while Mr. Smith and Mrs. Jones do not form a maritally self-sufficient whole, together with Mrs. Smith and Mr. Jones they do, namely a pair of married couples. This sort of completion results in a whole which is in a certain sense too large, giving us the sum of two of the sort of whole we were looking for. So the whole resulting from completion must be specified more closely. It would seem that the best way to do this is to invoke the concept of a *pregnant* whole. The colour of this + the extension of this is part of a self-sufficient visual datum, say, which is not merely summed aggregatively with another. Similarly a single married couple is the smallest maritally independent whole; every member of the collection is maritally connected to every other, whereas in a pair of couples each member of a couple is maritally unconnected with the members of the other couple. The pregnant whole for the foundation relation in question offers the promise of being neither too large nor too small. But this concept is itself defined in terms of the relation of individual foundation, as we shall see below, so it cannot be invoked without circularity. I do not believe that Husserl saw the threat of circularity here, so it is not to be expected that we could find from his account any indication as to how it might be avoided.

Another suggestion would be that two foundationally related items can only be found together in one substance. This would mean restricting the examples of foundation relations unduly, since a planet would normally be regarded as a substance, yet planets as such cannot exist unless stars exist, for example. The suggestion is quite foreign to the spirit of Husserl's enterprise, for Husserl never speaks of substances. It would I think have been much more to his liking to work towards a definition or definitions of substance through his theory rather than the other way around. This is not to say that we cannot use the notion of substance to guide our investigation in various directions, merely that the general problem of individual foundation is not to be resolved by recourse to the notion. It accordingly seems best that we treat the concept of individual foundation as primitive.

That a particular α cannot exist without a β may be true: that it cannot

exist without the particular β which satisfies this requirement need not also be true: for Bob to be a husband he must be married, but he need not be married to Alice; he might have married Carol instead. So at the level of individual foundation a measure of unavoidable factuality enters in, though not to all relations of individual foundation.

One of the general problems facing a theory of individual foundation is the question as to how far we may be taking Husserl to be working within assumptions about logical form which are implicitly Aristotelian, and whether such assumptions must be rejected. This concerns in particular the question whether an individual may be an instance of different species such that its foundational relations to other individuals vary according to the species in question. For instance, Jupiter falls into the species *planet* and also *heavenly body*. *Qua* planet, Jupiter is foundationally related to the Sun, the substantive relation in this case being gravitational. But *qua* heavenly body, Jupiter is not founded on the Sun. The problem is that if we are to say, as Husserl appears to want to, that a given individual either is or is not founded on another individual, we have to either deny that an individual can belong to two co-ordinate species, or else insist that there is some one privileged species with respect to which all talk about foundational relatedness of an individual is to be carried on. This sort of supposition can be roughly characterised as Aristotelian, and there are indications of such a position in Husserl.⁴⁸ An obvious candidate for such a privileged species is an individual's *infima species*, the product species of all those to which it belongs, which would, on Husserl's view, have only that individual as extension. It would be what he calls an *eidetic singularity*.⁴⁹ The problem with this is that in order for such a species to guarantee individuation of the object in question it would, *pace* Leibniz, have to comprise relational characteristics. This is not in itself objectionable, since many of the clearest cases of foundation rest on relations. But again the contingency of many of the relationships into which a thing enters means that we should have to find a way to distinguish essential from accidental attributes of something in order to arrive at a stable and useful conception of individuals' relative dependence and independence. Also an *infima species* will almost certainly have an infinite intension, so it could not be a working tool for the investigation of individual foundation. To take this problem into account, we shall have to mark explicitly the species under which we are considering an individual's foundedness.

The individual foundational relations hold between individuals not

merely as such, then, but considered as belonging to given species. The only way in which this consideration can be excluded from explicit mention is either by generalising, or by assuming that for certain individuals there are species to which they could not but belong in order for them to exist at all, in other words to assume essentialism for individuals. We shall explore both possibilities.

The basic relation of individual foundation we can accordingly gloss, in full dress, as '*s, qua* α , is founded on *t, qua* β '. We shall symbolise this as ' $s_{\alpha} \sqsupset_{\beta} t$ '. The similarity of basic symbol is intentional, but note that it is flanked by singular rather than general terms, and is indexed by a pair of general terms. We must take care to distinguish this formulation from the similar sounding '*s, which is an* α , is founded on *t, which is a* β '. The latter, while mentioning the species to which *s* and *t* belong, does not, like the former, say that it is in virtue of belonging to these species that they are so related. If we take '*s* is founded on *t*' as merely meaning that *s* and *t* belong to *some* species whereby they are so related (cf. (20) below) then the latter form may be true while the former is false: e. g., it is true that Jupiter is a heavenly body, and is founded on the Sun, which is also a heavenly body, but it is not in virtue of Jupiter's being a heavenly body that it is founded on the Sun, but rather in virtue of its being a planet of the Sun.

Expressions like 'as such', '*qua*', 'in virtue of being', and others repeatedly used by Husserl and by ourselves in discussing foundation, are logically peculiar in that they do not form unrestrictive relative clauses as 'which', 'that' etc. do, but create an intensional context. To see this, let us take a pair of examples. Suppose the owner of the Casa Negra nightclub is also the husband of Dolores, its principal singer. Then while the following are true:

- (a) The owner of the Casa Negra cannot exist as such unless the Casa Negra exists.
- (b) The husband of Dolores cannot exist as such unless Dolores exists.

The sentences obtained by interchanging subjects of (a) and (b) are false. Similarly, supposing that all and only rational animals are featherless bipeds, it does not follow that

- (c) A rational animal as such (by nature) has two legs.
or that
- (d) Jones, *qua* rational animal, has two legs.

It follows that there is no such entity as Jones *qua* rational animal, which is to be distinguished from Jones *qua* loving father, for instance. Expressions like ‘Jones *qua* loving father’ are not genuine singular terms, but sentential fragments having the force e.g. of ‘Jones is a loving father, and as such, he . . .’ where the ‘as such’ creates the intensional context.⁵⁰ This property of ‘as such’ and related expressions throws into relief the difficulties about the connection between generic and individual foundation. To the extent that we have either to mention or otherwise assume, with expressions like ‘*qua*’ or ‘as such’, a general kind or species, Husserl is right in taking individuals to stand in foundational relations in virtue of their belonging to species which stand in generic foundational relations.⁵¹

We can give a specification of the connection between generic and individual foundation by the following axioms:

- (11) $\Box (s \sqsupset_{\alpha} t \supset (s \in \alpha \ \& \ t \in \beta \ \& \ \alpha \sqcap \beta \ \& \ s \prec t \ \& \ t \prec s))$
 (12) $\Box (\alpha \sqcap \beta \supset (\forall x) (x \in \alpha \supset (\exists y) (x \sqsupset_{\alpha} y)))$

The converse implication to that given in (12) follows from (11) together with the definition (2) of generic foundation. This gives us the desirable result that α s as such are founded on β s if and only if any α is as such founded on some β as such. The appearance of triviality of this result disappears when it is remarked that ‘founded on’ does not mean the same in both occurrences. The indefinability of the individual relation in terms of the generic amounts to the lack of a general formula $F(s, t, \alpha, \beta)$ which could be added as a conjunct on the right of the implication in (11) so as to turn it into an equivalence.

§ 4 Dependence

Having dealt at length with the problems of foundation, we should now turn to the more general concepts of dependence and independence, which will of course vary according to the conception of foundation by means of which they are defined. Given the definition of foundedness (8) and essential independence (10) for species, we can define related notions of dependence and independence for individuals: an individual is *partly dependent*, written ‘dep’, when some species it belongs to is founded:

$$(13) \quad \text{dep}(s) := (\exists \xi)(s \varepsilon \xi \ \& \ \xi \neg)$$

while an individual is *totally independent*, written 'ind', when it is not partly dependent, that is:

$$(14) \quad \text{ind}(s) := (\forall \xi)(s \varepsilon \xi \supset I(\xi))$$

One could similarly define partial independence and total dependence. It follows from (11), (12) and (14) that an individual is totally independent if and only if it is not founded in any way on any other individual. It should be noted that by definition no individual can be self-founding, since every individual is a part (albeit improper) of itself. By contrast, some species are, as we have seen, self-founding.

The condition of total independence is extraordinarily strong, because of the universal quantification. It might be wondered what, if anything, could satisfy it. Since, according to orthodox cosmology, God falls under the term 'creator', and there can be no creator without creatures, even God would not, according to this view, be totally independent, being reciprocally founded on his works.

Because of the strength and uncertainty of application of such conditions, it would appear advantageous to develop more readily applicable conditions. One way to do this is to attempt to distinguish in individuals those species to which they belong of necessity from those to which they belong adventitiously. It is here that we shall use Wiggins' *de re* operator 'nec'. This will be used to offer a faithful formal rendering of such expressions as 's must be an α ', 's is essentially/necessarily/by its very nature an α '. This would normally be written, using property-abstraction, as

$$[\text{nec } (\lambda x)(x \varepsilon \alpha)](s);$$

however, to avoid unnecessary symbolic complication, I shall adopt the abbreviation

$$s! \varepsilon \alpha$$

and in general, for any *simple* predicate, where there is no risk of confusion, *de re* necessity will be marked by an exclamation mark after the occurrences of terms of which the predicate holds of necessity; so 's! < t!' will be short for $[\text{nec } (\lambda x)(\lambda y)(x < y)](s, t)$ and so on.

We may now define an individual as being *essentially independent*, written 'essind', when every species to which it belongs of necessity is independent:

$$(15) \quad \text{essind}(s) := (\forall \xi)(s! \varepsilon \xi \supset I(\xi))$$

while an individual is *essentially dependent* when it is not essentially independent:

$$(16) \quad \text{essdep}(s) := (\exists \xi)(s! \varepsilon \xi \ \& \ \xi \neg)$$

Armed with these new concepts we may resolve the theological problem about God's dependence on the world, in a manner suggested by Aquinas,⁵² by noting that since God need not have created the world, his being a creator is not essential to him, so he can be secured essential independence. The world, on the other hand, is essentially dependent, at least according to the traditional cosmology. The only possible candidate for total independence on such a view would be the totality comprising both God and the world.

According to traditional theology, the world is dependent on God both because he created it and because he continuously sustains it. The Husserlian concept of dependence covers both kinds of dependence, because it makes no reference to time. So something which needs to be produced by something else, but which can thereafter survive without this, is dependent on it in a different way from that in which something is dependent on something which it requires to exist at every time at which it exists itself. It is worthwhile contrasting the views of Husserl on dependence with those of his student Ingarden. In his chief work, *Der Streit um die Existenz der Welt*, Ingarden distinguishes four basic senses of dependence/independence.⁵³ Since these are given in opposing pairs, we need only characterise one of each pair. They may be set out in a table as follows:

- | | |
|----------------------|-----------------------------|
| (1) Autonomy | – Heteronomy |
| (2) Originality | – Derivation |
| (3) Self-sufficiency | – Non-self-sufficiency |
| (4) Independence | – Dependence. ⁵⁴ |

An object is autonomous or self-existent if it has its existential foundation in itself, is immanently determined.⁵⁵ An object is original if, in its

essence, it cannot be produced by any other object.⁵⁶ An object is self-sufficient if it does not need, by virtue of its essence, to coexist with something else within a single whole.⁵⁷ Finally, an object which is self-sufficient is independent if it does not require, by virtue of its essence, the existence of any other object which is also self-sufficient.⁵⁸ Ingarden draws attention to Husserl's examination of dependence and independence in the third investigation, but regards the eight concepts he sets out as belonging to a kind of theory which Husserl did not recognise, which Ingarden calls existential ontology, and which he contrasts with both formal and material ontology. There are considerable differences of background between Husserl's and Ingarden's respective treatments of dependence and independence, which we cannot enter into here. It is clear however that Ingarden's distinctions (2)–(4) could be variously interpreted within Husserl's theory of foundation. Ingarden in particular models his (3) on Husserl's definition of foundation. The difference between (3) and (4) is not highlighted by Husserl, and in making it Ingarden must have in mind some concept of whole stronger than the wide concept employed by Husserl. The only one of Ingarden's pairs which does not obviously fall within the general Husserlian account of foundation is (1).⁵⁹

If an object t is essentially independent, it follows that it is possible that t has no supplement, i. e. that t could constitute all there is, there being no whole (in the wide sense) of which t were a part. This possibility, which shows the self-sufficiency of the object in a perspicuous light, coincides with the conception of an object which is something for itself (*Etwas-für-sich*) in the late ontology of Brentano, as formulated by Chisholm.⁶⁰

(17) t is *Etwas-für-sich* := $\diamond \sim (\exists x)(t \ll x)$

This coincidence of notions is an interesting sidelight on the otherwise very different worlds of Husserl's ontology of the *Logical Investigations* and the ontology of Brentano in the *Kategorienlehre*. It suggests that Husserl's concepts of dependence and independence could contribute valuable insights to the problem of substance, which looms much larger for Brentano.

If $s \sqsupset_{\alpha} t$ then, as we have pointed out, it may be quite accidental to t as such, even t as a β , that it should *satisfy* the requirement for s for a β . One fact which shows this clearly is the possibility in certain cases of *dis-*

junctive satisfaction. Suppose, for instance, that Brown is a cat-owner. Then, as such, he must possess some cat. But he may possess more than one, each of which would, on its own, be sufficient to render him a cat-owner. At any time at which he owned more than one cat, the loss of one would not affect his status as a cat-owner. Indeed, provided he replaced cats as they died or he lost them etc., he could, barring catastrophe, remain a cat-owner for a time-span far longer than the life of any of his cats. In a similar way, a man is biologically dependent for continued life upon a regular supply of oxygen, water and nutrients, but the particular consignment of such material which actually sustains him will vary widely over time.⁶¹ Similar considerations apply to those parts of a thing which are essential to its being the sort of thing it is, but which can suffer replacement without the thing's ceasing to exist, either because it has more than one, and can acquire more as need be, or if it can temporarily survive without one. The replacement of cells in organisms gives an example of the first kind, while the repairing of machines gives one of the second.⁶²

Having defined the dependence and independence, whether essential or not, of individuals, we should now define *relative* dependence and independence, concepts of which Husserl makes much use in the investigation. We first define some more general concepts of individual foundation, following the practice established earlier of marking generalisation by omission of symbols.

$$(18) \quad s_{\alpha} \sqsupset t := (\exists \xi)(s_{\alpha} \sqsupset_{\xi} t)$$

$$(19) \quad s \sqsupset_{\beta} t := (\exists \xi)(s_{\xi} \sqsupset_{\beta} t)$$

$$(20) \quad s \sqsupset t := (\exists \xi \eta)(s_{\xi} \sqsupset_{\eta} t)$$

The general concept of foundation given by (20) does not make it explicit why s is founded on t . This more general concept frequently occurs in Husserl's exposition.

Husserl defines relative dependence as follows:⁶³

A content α is relatively dependent with regard to a content β (or in regard to the total range of contents determined by β and all its parts), if a pure law, rooted in the peculiar character of the kinds of content in question, ensures that a content of the pure genus α has an *a priori* incapacity to exist except in, or as associated with, other contents from the total ranges of the pure genera of contents determined by β .

I have quoted this in full because it illustrates vividly the sorts of problem of interpretation we face in the investigation. In the middle of what purports to be a definition, which should therefore be totally unambiguous, one finds inserted hedges and adjustments, which make a significant difference to the sense. It also displays Husserl's indifference to the possible problems of an individual's belonging to various species, since the same schematic letters are used for species and for members of these.

Three possible concepts of relative dependence suggest themselves to me on the basis of this passage. The first is that relative dependence is nothing other than individual foundation. This is arrived at by simply ignoring the bracketed adjustments in the passage and the clause 'or as associated with'. This identification may not be exact, because of the ambiguity of the phrase 'determined by β ', which might refer to parts of β , or essential parts of β , or simply some species to which β belongs (it must be remembered that here we are following Husserl's ambiguous lettering). It may be that the concept of individual dependence here suggested is not quite the same as that given by our (11)–(12).

By taking account of the adjustments beginning 'or . . .' we may arrive at the reading that s , say, is not directly founded on t but on something 'in its range', i. e. something which is, in the widest sense, a part of t . So we have the following alternative concept of individual relative dependence:

$$(21) \quad \text{dep}_1(s,t) := (\exists x)(x < t \ \& \ s \sqsupset x)$$

According to (21) anything which is founded on something else is thereby dependent₁ with respect to it, a result which is quite in the spirit of Husserl's exposition. The converse to this is not true: an object may be dependent₁ on another without being founded on it. To take an example from Eugenie Ginsberg's discussion of the Investigation, the shape of a particular brick is founded upon other aspects of the brick, and so this individual shape is dependent₁ upon the wall of which the brick happens to be a part, yet the shape could hardly be said to be founded upon the wall.⁶⁴ Ginsberg does not however distinguish between foundation and relative dependence, and so some of her attempts to show that Husserl's theorems are not all valid are vitiated. The wide concept of dependence₁ here canvassed is perhaps somewhat unnatural, and we should perhaps take closer cognisance of the phrase 'the total range of contents determined by β and all its parts'. There is, I think, no telling exactly

what this phrase is intended to mean, but the Ginsberg example suggests that we choose not merely an adventitious part of the whole t as something upon which s is founded, but rather take a part which t could not but have, i. e. something u such that $u < t!$, using our abbreviated device for showing essential predicates. This then suggests a third possible concept of relative dependence:

$$(22) \quad \text{dep}_2(s,t): = (\exists x)(x < t! \ \& \ s \sqsubset x)$$

According to this sense, whenever an individual is founded on another, it is also dependent₂ upon it, since for any individual t it is true that $t < t!$.

Dependence₂ does not reduce to foundation however. For one thing, s may be dependent₂ on t and at the same time a part of it, which means that, according to (11), s cannot be founded on t . In a case such as this we may say that, in one sense at least, s is a *dependent part* of t :

$$(23) \quad \text{deppt}_2(s,t): = \text{dep}_2(s,t) \ \& \ s < t$$

while of course it is similarly possible to define another sense of 'dependent part' through dep_1 :

$$(24) \quad \text{deppt}_1(s,t): = \text{dep}_1(s,t) \ \& \ s < t$$

It is clear of course that if $\text{deppt}_2(s,t)$ then $\text{deppt}_1(s,t)$: the second sense is stronger than the first. The close connection between foundation and dependent parts may be seen by the following theorem:

$$(25) \quad s \sqsubset t \supset \text{deppt}_2(s,s+t)$$

where $s+t$ is the aggregate or sum of s and t : the theorem follows from definition (23) together with the result that $t < (s+t)!$; clearly the very sum $s+t$ could not but have had t as part.⁶⁵ This shows that anything which is founded on something else is thereby a dependent part of a whole which is more comprehensive than either the founded or the founding part. It is because of this that the three notions of foundation, dependence, and being a dependent part, are so readily confused. It may be that Husserl himself did not make the distinctions so clearly as we have drawn them, but there is, as has been shown, sufficient evidence

from the investigation to show that such fineness of distinction can, and was perhaps intended to be, read from the text.

Recalling the supposition that entities may essentially belong to certain species, being the individuals they are, we can introduce various notions of essential foundation and dependence which are stronger than those we have used hitherto. We can for instance describe an individual s as *essentially founded* on an individual t when s is founded on t in virtue of some species to which s belongs essentially:

$$(26) \quad \text{essfd}(s,t) : = (\exists \xi)(s! \varepsilon \xi \ \& \ s_{\xi} \supset t)$$

while t essentially founds s when s is founded on t through a species to which t must belong:

$$(27) \quad \text{essfg}(s,t) : = (\exists \xi)(t! \varepsilon \xi \ \& \ s \supset_{\xi} t)$$

and a yet stronger relation can be obtained either by conjoining these, or, stronger yet, by insisting that the species α, β such that $s_{\alpha} \supset_{\beta} t$ are such that $s! \varepsilon \alpha$ and $t! \varepsilon \beta$.

We have already mentioned the possibility of disjunctive or generic satisfaction of an individual's need by other individuals. For though s may be in some sense essentially founded on t , this may not mean that s could not have been essentially founded on something other than t satisfying the same requirement. To take a biological example, an organism as such is, let us suppose, essentially founded at any time on some consignment of water, but any other consignment would have done equally well. Similarly an internal combustion engine is essentially founded on a supply of lubricant (here it is obvious that we mean a functioning engine, not a museum-piece), but again which particular mass of lubricant does the job is not important. A ship-launching ceremony might be thought to be essentially founded upon a bottle of champagne, but it need not have been just the one which was used. In other cases, however, an individual s is not only essentially founded on some other thing t , but it could only have been t upon which it was so founded. In such cases we may introduce definitions based on formulas such as $s \supset t!$, $s! \varepsilon \alpha \ \& \ s_{\alpha} \supset t!$, and $t! \varepsilon \beta \ \& \ s \supset_{\beta} t!$; which can themselves be used to further define notions of dependent parts. So we might be then equipped to say in what sense it is essential to a man that he has not just any brain, but this very brain, whereas it is not essential to him that he have this very

heart, or in what sense it is essential to a person that he or she should have the very parents he or she did have.⁶⁶ There is perhaps at present little point in doing more than indicating that there is here a wide range of questions and issues, some of them bearing on regularly-debated issues such as personal identity, together with a rich fund of possible concepts of dependence, all developed out of Husserl's ideas, and requiring further refinement.

We can however indicate a possible formulation of Husserl's attempt to define the pregnant concept of whole in terms of foundation. We need here individual foundation, as was argued earlier. Firstly we define *direct foundational relatedness*: two things are directly foundationally related when one is founded on the other:

$$(28) \quad \text{dfr}(s,t) := s \sqsupset t \vee t \sqsupset s$$

Then we define *foundational relatedness* as the (proper) ancestral of the relation of direct foundational relatedness:⁶⁷

$$(29) \quad \text{fr}(s,t) := \text{dfr}^{>0}(s,t)$$

Thus two entities are foundationally related if one founds the other, or both found or are founded on some third thing, or one founds and the other is founded on some third thing, etc. Then an entity is a *pregnant whole* when all its parts, in this case its proper parts, are foundationally related to one another, and no part is foundationally related to anything else outside this entity:

$$(30) \quad \text{Prwh}(s) := (\forall xy)(x \ll s \supset ((y \ll s \ \& \ x \neq y) \equiv \text{fr}(x,y)))$$

While it is thus not too difficult to express Husserl's idea symbolically it is much harder to see what it amounts to in practice. In theory the world should partition itself neatly into discrete entities, each of which is a pregnant whole. (Entities are discrete when they have no common part.) It might however be the case that every entity is foundationally related to every other, in which case there would be no partition, and only one pregnant whole, the world itself. This result would certainly be counted as in some sense monistic. It is possible that the sort of whole which Husserl had in mind when discussing pregnant wholes would be lesser in extent; to capture such wholes we might need to take a tighter foundation

relation as the basis for the definition of foundational connectedness. In general, the stronger such a relation, the tighter the organisation of the resulting wholes, the smaller in extent they are, and the more there are of them. So it seems that rather than there being a single concept of pregnant whole, there are several, having in common a recipe for generation from a concept of individual foundation. This is a characteristic outcome of studying the third investigation: ideas which at first sight seem sharp show themselves to hide various possible interpretations.

§ 5 Husserl's Six Theorems

An illustration of the difficulty is the attempt to interpret the six theorems of § 14: one has to use these as a guide to what Husserl meant at the same time as attempting to see whether they are valid or not. It is instructive to examine these and Husserl's proofs for them. We already saw above how Theorem I, interpreted as (7), is valid. Here is Theorem II:⁶⁸

A whole which includes a non-independent moment without including, as its part the supplement which that moment demands, is likewise non-independent, and is so relatively to every superordinate independent whole in which that non-independent moment is contained.

Husserl states that this follows from Theorem I as a corollary, given a definition of relative dependence. But he is wrong in this. Theorem I is stated in terms of species, whereas Theorem II relates to individuals. Here is a place where the transition between these two levels is not so simple as Husserl believes. We can give an example of things satisfying the intuitions represented by Theorem II which do not in any obvious way satisfy those of Theorem I. Let us call any expression which requires completion by only names or other singular terms to yield a sentence a *predicate*. Then the English verb 'loves' is a predicate, requiring completion by two names to obtain a sentence. In the sentence 'John loves Mary' the names 'John' and 'Mary' satisfy this double requirement. Now the predicate 'loves Mary' also has a requirement for supplementation by a name, and in the given sentence this requirement is met by the name 'John'. We might say that in the given sentence the predicate 'loves Mary' *inherits* from the predicate 'loves' that requirement which is met by the name

‘John’.⁶⁹ This is in conformity with the way in which the first part of Theorem II is phrased: the predicate ‘loves Mary’ does not contain *all* the supplements demanded by its part ‘loves’, and so inherits from the latter the demand satisfied by ‘John’. But the most obvious way of expressing this in the terms of Theorem I is to substitute the term ‘predicate’ for ‘ α ’ and ‘name’ for ‘ β ’. But in that case we should render ‘ $(\alpha)\beta$ ’ as ‘predicate which does not contain a name as part’: but precisely ‘loves John’ *is* a predicate which contains a name as part. It may be that this particular kind of multiple satisfaction was not considered by Husserl in his phrasing of Theorem I. To show that Theorem II does indeed follow from Theorem I we should have to be assured that *whenever* we have things satisfying the premisses in Theorem II we can always find a pair of species α and β such that Theorem I is satisfied with respect to the supplement which the larger whole inherits from the smaller moment. It seems to me dubious that we should be able to establish this in full generality, so it may be that Husserl’s theorems require another axiom to support them, such as the following:

$$(31) \quad (s \supset_{\beta} u \ \& \ s < t \ \& \ u \prec t) \supset t \supset_{\beta} u$$

Some such principle does indeed seem to be taken as self-evident by Husserl, but it cannot be directly proved from the proof of Theorem I, because it is compatible with the principles of this theorem that a is an α , b is a β such that $a \supset_{\beta} b$, and that c is an $(\alpha)\beta$, and so itself requires a β for completion, but rather than inheriting a ’s requirement satisfied by b , its requirement is satisfied by some further β , say b' . It is hard to find a convincing example of this state of affairs, which leads me to concur with Husserl. The nearest to a counterexample that I have managed is this: let ‘ α ’ be replaced by ‘represented district’ and ‘ β ’ by ‘representative’: the relevant whole being a district together with its representative. Now a council ward may be part of a parliamentary constituency, but the constituency, even if it does not contain the councillor who represents the ward, does not inherit the requirement for him, but has its own requirement met by its Member of Parliament. However, the force of this purported counterexample is somewhat blunted by the possible ambiguity in the notion of ‘district’, which might, one may say, have a bare geographical meaning and a more sophisticated administrative one. It might be argued that it is only in the administrative sense that a district’s representation requirements arise, whereas it is only in the geographical

sense that the ward is part of the constituency. In administrative terms the ward is not part of the constituency, but a completely different entity entering into quite different governmental arrangements. It is here that we face the problem of whether it is one and the same thing which is both a council ward and part of the parliamentary constituency, or rather whether these two *coincide*.

Given such uncertainties, it is far from apparent that Theorem II is, as Husserl takes it to be, a mere corollary of Theorem I. For this reason I shall confine myself to discussing the *consequences* of (31) taken as axiomatic, together with our other assumptions, rather than attempt to establish (31) or something like it. It can be seen that Theorem II follows very readily from (31), in its two parts, if interpreted as follows:

$$(32) \quad (s \cap u \& s < t \& u \prec t) \supset \text{dep}_1(t, u)$$

$$(33) \quad (s \cap u \& s < t \& u \prec t \& u < v) \supset \text{dep}_1(t, v)$$

In fact we can show not just (32), but the stronger formula obtained by replacing the consequent of (32) by ' $t \cap u$ '. Further, there does not appear to be any need for Husserl to restrict the superordinate wholes v merely to those which are independent. With these minor reservations, we can endorse Husserl's Theorem II provided we are prepared (a) to gloss 'dependent', as 'dependent₁' and provided (b) we accept (31).

Husserl's Theorem III is given in two versions: these both in effect amount to the transitivity of the relation 'is an independent part of'. We shall use therefore a simple version:

If s is an independent part of t and t is an independent part of u then s is an independent part of u .

To clarify this we must first give a definition of 'independent part'. The obvious one will do:

$$(34) \quad \text{indpt}_1(s, t) := s < t \& \sim \text{dep}_1(s, t)$$

One could also define similarly a relation indpt_2 based on the relation dep_2 but the one we have given here fits the bill more closely. For in the presence of (31–3) it becomes easy to prove that

$$(35) \quad (\text{indpt}_1(s, t) \& \text{indpt}_1(t, u)) \supset \text{indpt}_1(s, u)$$

by much the method Husserl uses in his informal proof of Theorem III, except that Husserl appeals both to Theorems I and II, whereas, because of the difficulties we have alluded to, we appeal only to (31) and its consequences.

Irrespective of the merits of (31), Husserl's fourth theorem is valid. His formulation is:⁷⁰

If s is a dependent part of a whole t , it is also a dependent part of every other whole of which t is a part.

We can represent this as

$$(36) \quad (\text{deppt}_1(s,t) \ \& \ t < u) \supset \text{deppt}_1(s,u)$$

and it follows immediately from the definition of deppt_1 and the transitivity of the part-whole relation ' $<$ '. In fact it is a more general thesis that

$$(37) \quad (\text{dep}_1(s,t) \ \& \ t < u) \supset \text{dep}_1(s,u)$$

It should be noticed that this was the assumption questioned by Ginsberg in her brick example, and the principle is harmless once the difference between individual foundation and the more general relation of relative dependence, in the sense of dep_1 , is made clear. One particular restriction of (36) yields the transitivity of deppt_1 . It must be noted that both deppt_1 and indpt_1 are transitive, but that the former is in many ways the more obvious notion. For as Husserl defines relative independence, it does not entail independence *tout court*, whereas this is true for relative dependence. The reason can be seen in the notion of independent part. That a is an independent part of b means only that a is not founded on anything within the range of b ; it does not mean that there is not something else outside b upon which a is founded. Husserl states this explicitly as his Theorem V: to represent this we must give some derelativised notions of dependence and independence derived from the relative notions we have been using. It is for instance possible to define 's is founded' as meaning simply 's is founded on something', and similarly for 's is dependent'. But because of the interrelation between dep_1 and \neg these amount to the same thing, so we shall simply say

$$(38) \quad \text{dep}_1(s) := (\exists x)(\text{dep}_1(s,x))$$

and define something as independent₁ when it is not dependent₁:

$$(39) \quad \text{ind}_1(s) := \sim \text{dep}_1(s)$$

The nice thing about this definition is that we can link now the notion of independence and dependence of an individual previously given as (13–14) in terms of its membership of a species, with the new derelativised notions stated in terms of individuals; by virtue of the principles (11–12) the following is a theorem:

$$(40) \quad \square (\forall x)(\text{ind}(x) \equiv \text{ind}_1(x))$$

so naturally the two contraries, *dep* and *dep*₁, are necessarily equivalent also. This shows that the detour through relative dependence and independence brings us back to the same position as we started from when considering the generic concept of foundation.

Husserl's Theorem V simply says

A relatively dependent object is also absolutely dependent, whereas a relatively independent object may be dependent in an absolute sense.

and we can see how, in our interpretation, this is unproblematically correct.

The final Theorem VI reads

If *a* and *b* are independent parts of some whole *c*, they are also independent relative to one another.

If we render this as

$$(41) \quad (\text{indpt}_1(a,c) \ \& \ \text{indpt}_1(b,c)) \supset \sim (\text{dep}_1(a,b) \vee \text{dep}_1(b,a))$$

then brief consideration shows that it is true, for were either *a* or *b* dependent on the other, since each is a part of *c*, the dependent one would by definition be dependent on *c*, contrary to the assumption; this is precisely the form of reasoning followed by Husserl in his proof.

We can thus see a way through the six theorems of § 14. Given the axioms and definitions hitherto suggested, the principle (31), which Husserl took to be self-evident, and the selection of *dep*₁ and not *dep*₂ as the relevant notion of dependence, all six follow. It is suggested then that

this constitutes an acceptable interpretation of what Husserl meant, which has the merit of making the theorems all valid if the axioms (11–12,31) are valid. This verdict on the semi-formal work of § 14 may be contrasted with that of Ginsberg,⁷¹ whom we suggested did not separate individual foundation from relative dependence, and whose criticisms of Husserl cannot therefore be accepted.

If, as suggested earlier, there are various possible concepts of dependence and independence which we could formulate without being unfaithful to Husserl's intentions, then it would be necessary to test these against the six theorems of § 14 in much the same way as we have done for the concepts connected with dep_1 . But the tests would be more complex, because of the essentialistic nature of many of the stronger definitions. After § 14 Husserl moves on to discuss various other whole-part notions which can be defined in his terms, such as mediate and immediate parts, abstractum and concretum, etc. These will obviously inherit any ambiguities possessed by the basic notions. Rather than follow up all the various possible interpretations, I shall instead turn to possible applications of Husserl's concepts within ontology. Applications in grammar, in particular the question of the dependence-status of different sentence-parts, and the structure of sentences, I hope to deal with elsewhere. For a summary of other applications which have been made, the reader should consult the essay by Smith and Mulligan earlier in this volume.

§ 6 Applications

One problem which was very much a live issue in Husserl's day, but which subsequently became buried, is the question of a distinction between ordinary or genuine objects and objects of higher order.⁷² Such a distinction was fundamental to Meinong's theory of objects, and suggests a kind of logical or ontological atomism whereby the basic objects are those of lowest order, there being aggregates, classes and complexes constituted on the basis of these. Husserl's account of categorial objects, or objects of the understanding, is very much in the same vein,⁷³ and Ingarden too, defends the difference between his concepts of self-sufficiency and independence by invoking this distinction.⁷⁴ Findlay has suggested, in commentary on Meinong, that the implied atomism is untenable.⁷⁵ We have, in Husserl's concepts of the third investigation, the

wherewithal for re-examining the issues. It may be simply misleading to regard objects with other objects as their pieces as somehow less self-sufficient than the pieces. The organs of an organism, while pieces of the organism in the sense that they are both separately presentable and physically separable,⁷⁶ considered as living tissue they are dependent for their continued existence on that of the organism of which they are part; in this sense they are moments rather than pieces of it. For the most vital organs, this dependence is reciprocal. The way is quite open to allow that some larger objects are in fact more self-sufficient than their smaller parts. One example which is mentioned by Husserl,⁷⁷ and which Findlay also cites as militating against the atomistic view of objects,⁷⁸ involves time. Temporal durations, considered not merely as abstractly extended parts of an abstract extended whole, but as concretely occupied by events and processes in the natural world, can no longer be seen as mere pieces, but must be regarded as dependent parts or moments of the whole. This suggests that the ontology which conceives of the world as made up of four-dimensional entities, of which the familiar three-dimensional objects of everyday experience constitute merely temporal cross-sections, is mistaken in supposing that temporally determined objects are sliceable in time in just the same way as a thing is sliceable in space. The theory of four-dimensional space-time objects can be accused of failing to distinguish between things and processes.

A similar consideration might help to dampen somewhat that perennially appealing aspect of all forms of atomism, micro-reductionism. If an entity can be shown to be complex, to consist of parts in a determinate relation to one another, it is the assumption of micro-reductionism that everything which could be meaningfully said about the complex could be expressed mentioning only its parts and their properties and relations. There is no doubt that in many areas of empirical investigation our understanding of entities is furthered by seeing how they are put together. The gains in understanding achieved fuel the drive to find ever more fundamental particles or constituents of matter in physics. It is sometimes suggested that there is no end to how far such reductions can be carried. But the assumption need not go unchallenged. At some stage of our knowledge of the physical world it might be reasonable for the philosopher to suggest that the bunch-of-grapes model of complexity is not the appropriate one. This might occur when the known fundamental particles fall into families by their characteristics, but there has been a prolonged inability to isolate the supposed constituents of these. Rather

than seeing the particles as consisting of more fundamental ones held together by a particularly strong natural glue, it might be hypothesised that the more fundamental parts of the isolable particles are not pieces but moments, which are mutually founding. As Husserl pointed out, such parts need not have any other part or constituent whose job was to hold them together, but require each other by their very nature. Such moments might be compared with the distinctive features of phonological theory,⁷⁹ which cannot be isolated but which explain the resemblances of phonemes, which can.

A rather similar but less universally appealing kind of unifying reduction of explanation is reduction *upwards*, macro-reduction, which seeks explanation of phenomena in terms of the objects in question belonging to some more inclusive totality with its own properties, a whole of which they can be seen to be mere moments. The supreme macro-reductionist was Hegel. Like the micro-reductionist, the macro-reductionist claims that nothing gets lost in his reductive explanation. An intermediate position might contend that micro- and macro-reductionism make opposite but cognate mistakes, the micro-reductionist taking all part-whole relations as relations of piece to whole, while the macro-reductionist takes all such relations as relations of moment to whole. The benefit of the observations drawn from Husserl is not just that it gives us a way to draw the parallels between the atomist and the holist, but that because there are various possible senses of dependence and cognate concepts, it can be made clear that there is not just one possible atomism or holism, but several, so that atomism of one kind might be quite compatible with holism of another. The atomist who sees a man as an aggregate of particles, and the holist who sees him as a mere mode or moment of some greater whole, may simply have different criteria for what it is to be an independent whole.

The question as to what constitutes a *natural* whole is probably not one which could receive a single answer. Which entities constitute natural wholes is something which cannot be settled *a priori*, but must be the concern of the empirical sciences. The sorts of object which we consider as having a tightness of organisation making it fitting to call them wholes in a natural sense seem to have a greater degree of causal coherence, and relative causal isolation from outside phenomena, than those which we should be less inclined to describe as natural wholes. The necessity to speak in terms of *degrees* of isolation and coherence suggests that there can be a spectrum of natural wholes of which some are more clearly

units than others. The paradigmatic examples of natural wholes would appear to be organisms, although these too can be from certain points of view taken as mere moments of some greater whole, involving say a species or an eco-system, while from other points of view they are aggregates of other wholes, such as cells, molecules etc., which have an integrity of their own. Other natural unities are not dissimilar from organisms e. g. in the manner in which they are able to utilise energy. Thunderstorms and river-systems have been suggested as examples.⁸⁰ Aristotle considered stars were not only natural but living unities, an opinion which is by no means so implausible as it appears at first sight.⁸¹ Such a readiness to see analogies between living or organic unities and other natural wholes need be neither anthropomorphic nor need it deny the ubiquity of causal explanation, since it is precisely the causal integrity of a natural whole or system which binds it together. This is not something imposed on reality from outside by our mode of cognition, but represents organisation which is intrinsic and which we discover.

According to this way of considering the multiplicity of ways in which things are connected in the physical world, the distinction between lower- and higher-order objects need not be an absolute one, with a single bedrock layer of natural units, but an object may be from one point of view a natural unit, from another it may coincide with an aggregate of differently organised units, or again be a moment of a greater whole. The fact that objects are naturally organised in many ways ensures that this relativity is not the mere imposition of a conceptual scheme on an otherwise unstructured world, but cuts along natural seams in reality.

When we move from considerations of units in nature to units in other spheres, such as social, legal and economic wholes, causal considerations are no longer so predominant, although they still apply. The unity of many man-machine wholes, such as a manned vehicle, is still predominantly one of relative causal self-containedness, while that of social wholes such as clubs, families, societies, or the various differently-sized units in an army or a business enterprise, require further considerations relating also, e. g., to functions and lines of control or authority. Such considerations may cut across those of causal or spatio-temporal proximity. It is, again, the merit of the vocabulary developed by Husserl that such matters can be discussed without an undue reliance on metaphor, and in full recognition that there will be very many different kinds of relation constituting the various kinds of whole brought into consideration.

One strand in the skein going to make up the traditional notion of substance is that a substance is what exists by itself, without needing the existence of anything beyond itself. In Husserl's terms, such an object is absolutely independent. Given the many different possible senses of 'independent' we could envisage various different senses of 'substance'. It might indeed be the case that some of the historic disputes over substance could be clarified by showing how different philosophers were operating with different concepts of independence. It is noteworthy that Husserl nowhere speaks in the third investigation of substance. His account is furthermore purely formal, and proceeds without assumptions as to which sorts of object are the most basic or paradigmatic independent wholes.

§ 7 Relations and Foundation

We have mentioned in several places the importance of relations between parts of a whole in constituting it as the whole it is. Many of our examples used nouns with a clearly derelativised sense, such as 'husband', 'sibling' and so on. We can very often generate one or more such nouns from a relative term, sometimes artificially. Sometimes the derelativised nouns are common enough to be etymologically unconnected with the relative term in question, as e. g. 'husband' and 'wife' have no etymological connection with the relative 'is married to', and may indeed be far more familiar than the relative notion which defines them. The term 'lake' for instance corresponds to no cognate verb expressing the relation of being land surrounding an expanse of water. Generally speaking, the more closely related things are affected in their properties by their particular relation, the more likely we are to have derelativised nouns to describe the relata as such. This is a partial explanation for the richness of the vocabulary of derelativised nouns dealing with human social and kinship relations, for the relations human beings have to one another mark and are marked by characteristic forms of behaviour of the people concerned.

It might be thought that we always can generate a foundation relation whenever we can obtain a pair of derelativised nouns from a relative term. Suppose for instance that given any binary relation R we define a pair of nouns by derelativisation as follows:

$$(42) \quad t \varepsilon R_1 := (\exists x)(tRx)$$

$$(43) \quad t \varepsilon R_2 := (\exists x)(xRt)$$

Does it follow automatically that $R_1 \supset R_2$ and $R_2 \supset R_1$? The answer is no: while we automatically get that $R_1 \supset R_2$ and vice versa, the stronger condition imposed by (2) means that R gives rise to foundation relations in the strong sense under these conditions:

$$(44) \quad R_1 \supset R_2 \text{ iff } \square (\forall x)(x \varepsilon R_1 \supset (\exists y)(xRy \ \& \ x \prec y \ \& \ y \prec x))$$

$$(45) \quad R_2 \supset R_1 \text{ iff } \square (\forall x)(x \varepsilon R_2 \supset (\exists y)(yRx \ \& \ x \prec y \ \& \ y \prec x))$$

Clearly any relation which is symmetric and for which (44–5) held, would give rise to a derelativised *self-founding* species term: for example from ‘possesses the same parents as and is different from’ we get ‘sibling’ while ‘is working together with’ gives ‘collaborator’.

Certain relative terms which possess etymologically related derelativised nouns fail this test, perhaps rather surprisingly. For example ‘employs’, ‘loves’, ‘shaves’, with their nouns ‘employer’/‘employee’, ‘lover’/‘loved’ etc. have neither of the cognate pair of nouns founding the other. The reason is that it is possible that all employers, lovers, shavers, etc. employ, love and shave only themselves. In general, so long as a relation *could* be reflexive, even by accident, i. e.

$$(46) \quad \diamond (\forall xy)(xRy \supset x = y)$$

then there is no reason why either of R_1, R_2 should be founded on the other. Of course, in the weaker sense of foundation given by ‘ \supset ’, there is always reciprocal foundation: there can be no employer without an employee, no lover without a loved one etc. But where general reflexivity is possible, this sort of requirement is not a requirement for an *associated* entity as such. It follows that any relative term possessing the logical property of reflexivity, including all equivalence relations, all partial orderings and especially identity, fails to give rise to foundation relations in the strong sense.

One obviously germane relation is the whole-part relation. In fact, if we consider the relation of being a proper part, symbolised ‘ \prec ’, we shall see that this gives rise to one self-founding derelativised term. For ‘ \prec_1 ’ is self-founding, whereas it is not true that $\prec_1 \supset \prec_2$, or that $\prec_2 \supset \prec_1$, or that $\prec_2 \supset \prec_2$. The reason that none of the last three is true is that we can

envisage the situation where the world consists of precisely two atoms, i. e. is a whole with only two proper parts. Again, it is certainly true that $\ll_1 \not\vdash \ll_2$ and vice versa, i. e. that there cannot be a proper part unless there is a proper whole or container, and vice versa, but the stronger relation of founding is ruled out by the restrictions of (2) as manifested in (44–5). The obvious noun-phrase corresponding to ‘ \ll_1 ’ is simply ‘proper part’. Because of the mereological law that to every proper part of a whole there must correspond a complementary proper part of that whole, i. e. an object disjoint from it (sharing no parts) which together with it makes up the whole, or, symbolically:

$$(47) \quad \Box (\forall xy)(x \ll y \equiv (\exists z)(x \sqcap z \ \& \ y = x + z))$$

it follows that \ll_1 or ‘proper part’ stands for a self-founding species. It turns out then that even the terms ‘whole’ and ‘part’ are derelativised from one or other of the relations ‘is part of’ or ‘is a proper part of’; this fact leads Husserl into local difficulties in expounding the idea of an independent part, since while it is natural to say that a (proper) part as such cannot exist apart from its whole, for independent parts we also want to say that the object which is here in fact a part *could* exist outside this particular whole.⁸² The difficulty is only one of expression, however, not of substance.

Having seen how foundation relations may arise of relative terms, we might turn the issue round and ask whether all foundation relations point back to some underlying and more basic relative term. The question must first be made more precise however, since for any pair of species α, β such that $\alpha \sqcap \beta$, we always have the relative term ‘ $\sqcap_{\alpha\beta}$ ’. We are trying to get beyond this however and ask whether an α which is founded on a β is so because of some relation which is not defined in terms of α and β , but which may indeed be used in definition of these terms, as in the case of derelativisation already mentioned. If we follow Husserl’s opinion on this, we should have to deny it. For Husserl claims that although *colour* and *extension* are mutually founding, there is nothing in the concepts colour and extension which points to any such underlying relation.⁸³ It is Husserl contends, precisely in this lack of a means to render the law of mutual dependence for colour and extension as an instance of a *logical* or formal principle that there consists the synthetic *a priori* status of the statement that colour is impossible without extension and vice versa. Were it possible to treat ‘colour’ and ‘extension’ as nouns

definable by derelativisation from some antecedently given relative term, the dependence in question would be analytic rather than synthetic. While it seems to me that Husserl's distinction between analytic and synthetic is not so sharp as he thought it was,⁸⁴ the mere possibility that there should be acceptable cases of foundation where the necessity is not obviously logical leaves in doubt the possibility of always finding an underlying relation.

It is worth considering a way of making a distinction among relations which can be found at its clearest perhaps in Meinong, who also brings this distinction into play when discussing the difference between genuine and higher-order objects.⁸⁵ Some relations, such as difference, similarity, being the same height, and the like, do not bring their terms into any real connection, but rather leave them quite unaffected by being thus related. Standing in such relations makes no difference to the properties of the terms; it is indeed often the case that they stand in such a relation in virtue of the separate properties that they possess. Such relations are themselves built or founded on their terms. We may call these *ideal* relations. Other relations, such as acting upon, magnetically attracting, playing tennis against, bring their terms into connection in that, had the relation not obtained, the properties of one or both of the terms would have been different. We may call these *real* relations. The most obvious examples of real relations involve some causal link. Now some foundation relations have underlying relative terms corresponding only to ideal relations, which means that the unity engendered by the foundation is in a sense extrinsic to the objects related. Many ideal relations are equivalence relations, and since these are reflexive they are in any case, by the result above, powerless to engender genuine foundation relations. But where there is some real connection between the terms of a relation, these terms, described in a way which implies the properties induced by the relation, will, if the relation in question satisfies one of (44–5), be foundationally related. We could then describe the relation as a moment of the whole uniting the parts. While these remarks are only schematic, it does seem to me that a theory of the unity of wholes can only be developed in conjunction with an adequate theory of relations: the two enterprises must proceed together. It is perhaps not accidental that the importance of the interconnection between relations and wholes only arises as a serious issue once the Leibnizian dogma that whatever exists is one is called into question.

One of the considerations we derive from examining the role of rela-

tions in engendering foundation is the impoverished role of reflexive relations, including especially equivalence relations. The role of the latter in modern theories of abstraction is well-known. But from the ontological point of view reflexive relations are as such highly dubious. While we may be perfectly prepared to allow a relative *term* to be flanked by a pair of names for the same thing, and yield a true sentence, it is a different matter again if we ask what *relation* corresponds to the term. The whole notion of a relation which holds between a thing and itself is suspect,⁸⁶ and the more especially when, in the case of identity, it can only hold between a thing and itself. This difficulty can be found for instance in Hume and Wittgenstein.⁸⁷ It is usual these days to dismiss their problem as a pseudo-problem resulting from the confusion of a sign with the thing signified.⁸⁸ But the objection is not that there is a certain kind of relative term which can generate true sentences. It is rather that nothing intrinsically relational is represented by this sign, if indeed anything at all is represented. Nor is this to deny the cognitive value of such relative terms. It is to object that they are ontologically sterile. Where a reflexive relation may also hold between different things, as e.g. 'is the same height as', it can always be traded in for the anti-reflexive variant, e.g. 'is the same height as and different from'. Such terms may now generate foundation relations between their derelativisations. Indeed those perplexing derelativisations like 'employer' / 'employee', etc. are most happily applied when reflexivity is not envisaged: it does sound wrong to describe a self-employed person as either an employer or an employee, or a narcissist as a lover, and it is because of such anti-reflexive uses that we have the derelativised nouns at all. It may be of more than etymological interest that many of the terms for equivalence relations are in fact derived from their associated adjectives or nouns, even, it should be noted, *identity*.

Notes

References in these notes are to works listed in the bibliography at the end of these three essays. Works are cited under the name and year in which they appear there.

References to Husserl's *Logische Untersuchungen* (Husserl, 1900–01) will be to the volume and page of the 5th edition of 1968, which will be abbreviated *LU*, and to the page of the English translation of Findlay 1970, abbreviated *LI*. Section numbers, unless otherwise specified, are to the third investigation.

- ¹ An earlier version of this essay was read at the Colloquium 'Whole-Part Theory and the History of Logic' held by the Seminar for Austro-German Philosophy at the University of Sheffield in May 1978. My thanks go especially to David Bell, Kevin Mulligan, Herman Philipse and Barry Smith for their help and constructive criticism.
- ² When William Kneale visited Husserl in Freiburg in January 1928, he relates that Husserl "told me that his essay *Zur Lehre von den Ganzen und Teilen* in his *Log. Unt.* was the best starting point for a study." (From part of a letter to Herbert Spiegelberg quoted in Spiegelberg, 1971, n. 25, p. 78.)
- ³ For Husserl on formal theories see *LUI* §§ 67–72. The ideas are expanded considerably in Husserl, 1929.
- ⁴ The example is Husserl's: *LUII/1* 254, *LI* 457. A purely formal proposition which is true is free of all existential assumptions, § 12 *ibid.*
- ⁵ Wittgenstein, 1961, 4.1272 tells us that words like 'object', 'concept', 'complex' and 'fact' signify formal concepts, and are represented in a *Begriffsschrift* by variables. Cf. also 3.325.
- ⁶ This concept of variability of all propositional constituents except the logical constants can be found already in Bolzano. Cf. his definition of logical analyticity and universal satisfaction in Bolzano, 1837, §§ 147–8, a work which influenced Husserl profoundly.
- ⁷ For explicit repudiations of formalism in mathematics cf. e.g. Husserl, 1929, § 39.
- ⁸ For an introduction to Lesniewski's work see Luschei, 1962 or Lejewski, 1958. For Lesniewski the division between logical and non-logical theories comes between his Ontology and his Mereology, so for him whole-part theory contains non-logical constants. Leonard and Goodman, 1940 or Goodman, 1977 blur such a distinction by defining identity mereologically. For an axiomatisation of the whole-part theory in Goodman, 1977 see Breitkopf, 1978.
- ⁹ *LUII/1* 252, *LI* 455.
- ¹⁰ Such a theory is explicitly canvassed at § 24.
- ¹¹ Aristotle, 1928, Ch. 2, where Aristotle contrasts being part of a subject with being in a subject in such a way as to be incapable of existence apart from it.
- ¹² Stumpf, 1873.
- ¹³ Husserl, 1894.
- ¹⁴ But compare my third essay below, where a nominalistically acceptable conception of set is described.
- ¹⁵ Cf. the discussion in § 18, where Husserl is not altogether clear whether it is possible to give examples of proper parts of a whole which are not proper parts of proper parts of this whole.
- ¹⁶ Wiggins, 1976. Cf. the mereological applications in Wiggins, 1980.
- ¹⁷ Cf. Locke, 1975, Book II, Ch. 27: "In the state of living Creatures, their Identity depends not on a Mass of the same Particles; but on something else. For in them the variation of great parcels of Matter alters not the Identity"; p. 330.
- ¹⁸ Chisholm, 1976, Ch. 3 and appendices A–B.
- ¹⁹ Wiggins, 1980 uses a whole-part theory strengthened with the operator *nec* to argue for this conception and against Chisholm's *entia successiva*.
- ²⁰ Cf. § 12 of the fourth investigation, where it is declared that 'round square' cannot correspond to any object: *LUII/1* 326, *LI* 517. Later, in Husserl, 1948, § 91 the extension of a pure species is said to comprise pure *possibilities*.
- ²¹ § 2, *LUII/1* 252, *LI* 455.
- ²² In commentary on Aristotle, Anscombe in fact replaces Aristotle's accident example by a boundary example: Anscombe and Geach, 1961, pp. 7–8.
- ²³ It is indicative of Husserl's low reliance on categories that he is very reluctant, by comparison with later philosophers, to brand sentences as nonsensical. Cf. his distinction in Investigation IV, § 12, between nonsense and absurdity.

- ²⁴ Husserl, 1948, § 32a. In § 32b Husserl adds *connections* as yet a further distinct kind of dependent part to accidents (which he calls 'qualities') and boundaries.
- ²⁵ § 22, *LUII/1* 280, *LI* 478–9.
- ²⁶ *Ibid.*, *LUII/1* 279, *LI* 477.
- ²⁷ As Husserl says, *LUII/1* 280, *LI* 478, "Unity is . . . a categorial predicate."
- ²⁸ § 11, *LUII/1* 252, *LI* 455.
- ²⁹ Perhaps the clearest statement of this is in § 148 of Husserl, 1913. Cf. Beilage 74 of the *Husserliana* edition (p. 625), where Husserl clarifies the statement in the text. It will become clear in my third essay below that I do not share Husserl's view that a nominalisation is necessary to constitute a set as a new object on the basis of plural reference.
- ³⁰ Cf. Husserl on *collectiva* at Investigation VI, § 51, *LUII/2* 159, *LI* 798. Though the section title also mentions *disjunctiva* the section has strangely nothing to say about such things.
- ³¹ § 22, *LUII/1* 279–80, *LI* 478.
- ³² § 14, *LUII/1* 261, *LI* 463.
- ³³ § 13, *LUII/1* 258, *LI* 460.
- ³⁴ In § 14 this is seen by the *ad hoc* use of a suffix, in § 13 by the lack of articles. Husserl's usage of symbols is sloppy by modern standards.
- ³⁵ Cf. the remark that we can use the same expressions for individuals and species as a 'harmless equivocation'. § 14, *LUII/1* 261, *LI* 463.
- ³⁶ *Ibid.*
- ³⁷ § 21, *LUII/1* 275, *LI* 475.
- ³⁸ Cf. Proposition 5 of § 14, *LUII/1* 263, *LI* 465.
- ³⁹ This suggestion was made to me by Barry Smith, as an improvement on an earlier formulation of mine which required that α s and β s be such that no α ever be part of a β or vice versa. Both these ideas are inadequate, as the case of self-founding species shows.
- ⁴⁰ § 14, *LUII/1* 261, *LI* 463.
- ⁴¹ § 21, *LUII/1* 276, *LI* 475.
- ⁴² On entailment see above all Anderson and Belnap 1975.
- ⁴³ § 14, *LUII/1* 262, *LI* 463.
- ⁴⁴ This is one place where the definition of foundation using entailment suggested at (4) is too strong for our purposes, because the definition of ' α) β ' is extensional while that of ' \neg ' in (4) is relevant and intensional, and the restrictions in E on e. g. importation make it impossible to prove (7) as it stands. However by strengthening the definition (6) a relevant version of (7) could be proved.
- ⁴⁵ § 16.
- ⁴⁶ Herman Philipse has objected that the husband/wife type examples are analytic, whereas Husserl is clearly interested in synthetic connections such as the colour/extension example. Two things may be said in reply. Firstly, the distinction Husserl draws between analytic and synthetic is not as sharp as he thought it was. This is an issue which I hope to take up elsewhere, though note the remarks in n. 77 of the opening essay by Smith and Mulligan. Secondly, as Husserl is really interested in an *a priori* theory (§ 24) no harm at all can be done by including analytic as well as synthetic examples.
- ⁴⁷ Ginsberg, 1929. Cf. my note to the translation of her later paper in this volume.
- ⁴⁸ Cf. Husserl, 1913, § 14.
- ⁴⁹ *Ibid.*, § 12.
- ⁵⁰ For an implicit recognition of this in Leibniz, cf. Leibniz 1903, 261. Cf. more explicitly Ishiguro, 1972, 16.
- ⁵¹ I had previously thought that it made some sense to talk of individuals simply as such, without mention or assumption of any kind to which they might belong. Many sources have dissuaded me of this view, but David Bell and Herman Philipse have done so most directly.

- ⁵² Aquinas, 1964–76, Ia, 44, 4; Vol. 8, p. 21, where Aquinas argues that in creating God does not act from need but ‘simply to give of his goodness’. At the same time the idea that God’s nature could have been other than it is is not particularly congenial for Aquinas, so his problem is not completely cleared.
- ⁵³ Ingarden, 1964–5, §§ 12–15.
- ⁵⁴ For a note on these translations cf. my introduction to Ginsberg’s paper.
- ⁵⁵ *Ibid.* § 12.
- ⁵⁶ *Ibid.* § 13.
- ⁵⁷ *Ibid.* § 14.
- ⁵⁸ *Ibid.* § 15.
- ⁵⁹ One of Ingarden’s examples of a heteronomous object would be any entity which is purely noematic, a correlate of consciousness, so Ingarden could fairly claim that the material for such a distinction exists already in Husserl.
- ⁶⁰ Chisholm, 1976, 208. This paper in general furnishes abundant evidence that late Brentano was working using whole-part theoretic considerations akin to those we find in Husserl, which is not surprising, given his influences on his pupils, in particular Husserl’s former teacher Stumpf. Unfortunately we have not here the space to compare Husserl and Brentano at length.
- ⁶¹ The example, though not the application of it, is drawn from Ingarden, 1964/5, § 15. The idea of disjunctive satisfaction of requirements clearly has applications in biology. The importance of such biological considerations is urged in the Preface to Wiggins, 1967.
- ⁶² Because Chisholm denies that a genuine entity may lose parts, he must construe organisms and machines as less than genuine, with an identity which is a simulacrum of true identity. Cf. Chisholm, 1976. This is a thought which can be found *inter alia* in Hume and Leibniz, and in a modified form pervades extensional mereology. It is certainly attractive, and more tractable than the Aristotelian alternative, but I am convinced it is wrong.
- ⁶³ § 13, *LUII/1* 258, *LI* 460.
- ⁶⁴ Ginsberg 1929, 112. Cf. also my prefatory note to her paper in this volume.
- ⁶⁵ Wiggins uses this consideration in his 1980 to discredit the idea that a cat can be identical with the mereological sum of its body + its tail, for the cat, but not the sum, could lose the tail.
- ⁶⁶ As suggested in Kripke, 1972, 312 f.
- ⁶⁷ This notation for the proper ancestral is due to Carnap. Cf. his 1954, § 36.
- ⁶⁸ § 14, *LUII/1* 262, *LI* 464.
- ⁶⁹ On Frege’s use of whole-part terminology to describe the phenomenon which he calls ‘unsaturatedness’ of predicates and concepts see his late essays “Die Verneinung” and “Gedankengefüge”, Frege, 1976a. I have expanded elsewhere on the appropriateness of using Husserlian ideas in this connection. Frege’s use of terms like ‘erganzungsbedürftig’, unlike that of Husserl, is not backed by a theory of dependent and independent parts. Indeed, if we are to believe his remarks in Frege, 1895, Frege had a rather low opinion of whole-part theory in general.
- ⁷⁰ § 14, *LUII/1* 263, *LI* 464. We have adjusted the symbolism to our convention.
- ⁷¹ Ginsberg, 1929.
- ⁷² Cf. Meinong, 1899.
- ⁷³ On categorial unities see § 23, for instance. The notion can be found throughout Husserl’s writings.
- ⁷⁴ Ingarden, 1964/5, § 15, n.
- ⁷⁵ Findlay, 1963, 148f.
- ⁷⁶ The idea of separate presentation here derives from Stumpf, 1873. Cf. § 3 for Husserl’s comments, and the historical remarks in Ginsberg’s paper in this volume.

⁷⁷ §25, *LU II/I* 292, *LI* 488.

⁷⁸ Findlay, 1963, 150.

⁷⁹ The Prague and Moscow schools of linguistics were in fact influenced by the third investigation. Cf. Jakobson, 1973, 13–4, Hohenstein, 1975.

⁸⁰ Cf. Pantin, 1968, 37f.

⁸¹ Aristotle, 1930, 292^a 19f.

⁸² § 11, *LU II/I* 253, *LI* 456.

⁸³ *Ibid.*

⁸⁴ As mentioned in n. 46 above.

⁸⁵ Cf. Findlay, 1963, 141f. Husserl in fact makes a very similar distinction between two sorts of relation in Husserl, 1887 and 1891a. His terminology is however more unfortunate, since he calls the relations 'physical' and 'psychical' rather than 'real' and 'ideal'. The term 'psychical' indicates not that the relation is mental, but that it is *of a sort with* the relation between object and content of an idea. Cf. Findlay, 1963, 35, where it is made clear that the mental relation is ideal for Meinong. The terminology of Husserl readily misled Frege into criticising Husserl's theory of number as psychologistic, which it was not. For a clear refutation of the myth of Husserl's early psychologism see Willard, 1974. We have, for obvious reasons, adopted the less misleading terminology of Meinong.

⁸⁶ Cf. the arguments against reflexive relatedness in Vol. 2 of Armstrong, 1978, 91f.

⁸⁷ Hume, 1978, Book I, Pt. IV, § II, p. 200f. Wittgenstein, 1961, 5.53f.

⁸⁸ Cf. Quine, 1960, § 24.

II. Number and Manifolds

§ 1 Introduction: The Philosophy of Number

An adequate philosophical theory of whole numbers has to be able both to give an account of what we accomplish when we make empirical ascriptions of number, for example in answer to “How many . . . ?” questions, and also to provide an account of the content and validity of the propositions of arithmetic. I shall call the theory surrounding the first kind of question the philosophy of *number*, and that surrounding the second kind the philosophy of *arithmetic*. An adequate account must further provide some explanation of the link between the two. As attempts at such philosophical accounts one may take the different philosophies of Frege and Husserl. Frege explained ascriptions of number as assertions about a concept, while he explained arithmetical propositions as concerning certain abstract objects, the whole numbers themselves. The link between the two accounts is provided by his theory of abstraction, in which expressions like ‘the number 0’ are contextually defined. Husserl’s theory on the other hand takes ascriptions of number to concern not concepts but totalities, which are given to the mind in an act of “collective combination”. The transition to abstract numbers is accomplished by a theory of symbolisation, according to which larger numbers are presented to us mediately, through numerical expressions. While there are difficulties in both accounts, they have in common that they present a two-stage theory of number, the first stage dealing with empirical ascriptions of number, the second with the formal validity of arithmetic, with a bridge between the two consisting of a theory of abstraction. Subsequently, interest has shifted almost exclusively towards the philosophy of arithmetic, with various attempts being made, in the wake of Frege’s unsuccessful one, to provide a basis for arithmetic in a formal theory such as the typed logic of Whitehead and Russell or an axiomatic set theory. With this emphasis on deriving arithmetic from some more general theory has gone an increasing willingness, evident already to some extent in Frege, to let numbers be any handy construction with the right formal properties. Since there are many such constructions available, this has led to scepticism that there are such entities as num-

bers at all, over and above the many series of *numerals*, which are expressions suitable for performing a count.¹ I do not share such scepticism.

§ 2 Frege's Criticisms of Manifold Theory

To counteract the tendency to concentrate on arithmetic and ignore the existence of empirical ascriptions of number, I wish to suggest in outline the form which an adequate philosophy of *number* should take. Obviously this cannot be done without an eye as to the likely way in which we arrive at arithmetic, but I shall not stress here the role of a theory of abstraction in any detail. These remarks indicate that I am convinced that a two-stage theory of whole numbers is correct, and I shall be here attempting to say what the first stage should consist in. I maintain that, in a sense to be explained fully below, number is a property of external things of a kind which I call *manifolds*.² In this I shall basically agree with Husserl against Frege, but the theory involved will perforce take account of Frege's objections to such a theory. These objections are the following:

- (1) that if we try to ascribe number to external things, we find we cannot do so consistently, because one and the same thing may be ascribed many different numerical predicates: a pack of cards may number one (pack), or 52 (cards), or 4 (suits) and so on.
- (2) that there is nothing to which we can ascribe the number 0 on such a theory.
- (3) that number is very different from all other properties of external things.
- (4) that number cannot be identified with the way in which a thing may be split up into parts.
- (5) that things do not need to be literally collected together in order to be numbered.
- (6) that the concept of number has a far wider range than the concept of physical thing: we may apply number universally, to non-sensible and abstract things such as the figures of the syllogism as well as to concrete physical things such as boots.³

In defending a theory of number as (in the first stage) a property of external things, I shall take account of all these objections in one way or an-

other. While I shall agree with Frege over (4) I shall nevertheless suggest that there is, for many manifolds, a quite close connection between their numerical properties and their mereological properties, so that my account will be in part a *mereological* theory of number.

My account of number then stays fairly close to that of Husserl, who is by far the most sophisticated "external thing" theorist to date. His theory has answers to all of Frege's objections, but not all these answers are of equal quality or acceptability, e.g. his rejection of 1, as well as 0, as a genuine number. Also, while Husserl approaches number through a consideration of mental acts, ours takes much more linguistic considerations as starting point. So e.g. Husserl's answer to objection (6) is that collective mental acts may consider together objects of any category, while ours is that plural referring expressions may be put together out of terms referring to objects in any category. This difference of starting point by no means rules out the possibility of a more complete unified treatment in which both language and mental acts have their proper place: it is indeed a longer-term desideratum. But my chief disagreement with Husserl is over his contention that pluralities are *constituted* as such by acts of collective combination, and accordingly are higher-order, *categorical* objects. I hold that manifolds are lower-order multiplicities rather than higher-order unities, and that Husserl was here under the pervasive influence of the prejudice in favour of the singular, in a weak but crucial form: weak, because Husserl accepts that we may have mental acts simultaneously directed to many objects at once, but regards number and manifolds as being first constituted in a higher-order act reflecting on such plural consciousness, a move which I hold to be superfluous once the distinctive nature of pluralities is recognised.

One very general form in which we may describe the (possibly unknown) number belonging to a given totality is

the number of *cs* which Φ

where *c* is a common count noun or count noun phrase,⁴ and Φ is an intransitive verb or intransitive verb phrase. For example we have

the number of men in the Red Army

the number of trees in Sherwood Forest

the number of women who have had more than ten children.

Two important variable features of this form are the count noun (phrase) *c* and the intransitive verb (phrase) Φ . Both contribute to deter-

mining what the answer to the question 'How many *cs* are there which Φ ?' The noun tells us what *sort* of thing we have to count: it supplies what I shall call the *counting principle*. In the above examples men, trees and women are what is counted respectively. The verb tells us not what sort of things we should count, but which things among those of that sort given by the noun. It sets limits on the ones which 'count', so I shall say it provides the *delimiting principle*. I distinguish three kinds of delimiting principles:

- (1) restrictive
- (2) limitative
- (3) mixed.

Restrictive principles limit the *cs* which matter to those which possess a certain property, or stand in a certain relation, and so on, where no reference is made to spatio-temporal position. For instance we get the following noun-phrases by restricting a noun in this fashion:

tree which is taller than 50 metres
man who admires Cleopatra
woman who has had ten or more children.

Limitative principles set spatio-temporal limits within which the counted *cs* must fall, such as

Public House within two miles of Trafalgar Square
woman in England on January 1st 1979

and *mixed principles*, as their name implies, are neither purely restrictive nor purely limitative, for example

woman in England of January 1st 1979 who has had ten or more children.

It seems plausible that mixed principles are the most common.

Frege called count common noun phrases of the kind we have been considering *concept-words*. His use of the term 'concept' was and is deviant in two respects. Firstly, whereas traditionally concepts would have been regarded as the *senses* of predicate or noun-expressions, for Frege they were their *referents*. Frege was aware of this and pointed it out in correspondence with Husserl.⁵ Secondly, since Frege intended to interpret numbers as 'concerning' concepts, he allowed expressions with de-

limiting clauses to stand for concepts. Those which had variable aspects, such as a tensed verb, could have different numbers at different times, but the clearest cases were those which contained no variable elements, whose number 'is the same for all eternity'.⁶

Now the second point, that Frege extended the term 'concept' to include those concepts belonging to expressions containing delimiting phrases, as in our examples, does not appear to me to be a serious one. There are precedents for even unitarily-lexicalized expressions involving some reference to particular individuals, or to some place or time, e.g. 'Aristotelian', 'mediaeval', 'Scandinavian'. The first point, that it would be happier to call the senses of predicate or noun-expressions 'concepts' seems to me, at least so far as it is a dispute about terminology, to be one where it is better to side against Frege for the sake of clarity. This does not prejudice one's attitude to Frege's position about functions and objects, since one could simply adopt a new word for those functions which take objects as arguments and yield truth-values as values, instead of Frege's 'concept'. Since however I have no intention of defending a Fregean ontology or philosophy of number, but wish to provide an alternative, it is worth pointing out that Frege wanted concepts to be something objective, so that numerical predications could be objectively true. But this objectivity would equally be guaranteed were we to interpret concepts as senses. The role of the concept was to unify or collect individuals in a way which does not involve physical displacement. Now of course no literal collection goes on at all here, so it would be better to replace talk about collection and unification altogether. The role of concepts (in the traditional sense) in enabling us to refer to manifolds will be set out below.

§ 3 Plural Terms and their Designata: Manifolds

Frege used the term *Eigenname* for both proper names like 'Aristotle' and definite descriptions. These have often been classed together as singular *terms*.⁷ The rationale for this is that both kinds of expression perform a similar role: that of making a definite reference to something. Of course they work in different ways, but this does not make it appropriate to put them into completely different categories. They are syntactically intersubstitutable. This is not sufficient to mark out the category of *terms*, which I shall be interested in, since terms are also intersubstitut-

ible *salva congruitate* with *quantifier phrases* such as 'some man', 'several days', 'all rabbits' etc.⁸ These do not serve to make reference to things, as can be seen by their different behaviour, in connection with negation, from terms.⁹ I wish to consider among terms two other great categories of expression: firstly those which refer only in a specific context, so-called *indexical expressions*. These are a rag-bag of assorted expressions which share only this feature of reference only within a concrete context, and include personal pronouns and expressions with demonstrative pronouns, e.g. 'we', 'that man', 'my friend', etc. I am not particularly interested in such terms in the present context, and include them mainly for the sake of completeness. Much more important for present purposes are *plural terms*: the sort of expression which can be used to refer to more than one thing at once, e.g. 'my friends', 'the men in this room', 'Jack and Jill'. Plural terms are the Cinderellas of philosophical grammar, and very few philosophers have recognized them. One notable exception is the early Russell.¹⁰ Otherwise there has subsisted a remarkable prejudice in favour of the singular, which has not been without its deleterious effect on the philosophy of number.

Plural terms comprise plural descriptions, plural proper names, if there are any, and conjunctive lists of terms, singular and plural. Here are samples of various kinds of plural term to set alongside the more familiar kinds of singular term.

Plural proper name	Benelux
Plural definite description	the fishermen of England
Plural demonstrative	these books
Plural personal pronoun	they
Name list	Tom, Dick and Harry
Mixed term list	Jason and the Argonauts

Whether an expression actually manages to designate something on a particular occasion of its use is immaterial to its status of being a term. For instance the expression 'the greatest prime number' necessarily does not designate anything, but we can recognize it as a term in virtue of its syntactic structure and the syntactic categories of its components: it comes from the same syntactic bag as an expression like 'the tallest man in Finland', and the expressions 'greatest' and 'prime number' themselves occur within terms which may designate something.

Since I take number to be a property of manifolds, and manifolds to stand to plural terms as individuals stand to singular, it will be very im-

portant to get clear how plural reference works. We might say that 'manifold' is the plural of 'individual'. Whether or not a term actually picks out a manifold on a particular occasion of its use is, as with terms in general, not the main issue. Just as individuals are what *could be* designated by a singular term, so manifolds are what *could be* designated by a plural term. Just as one and the same expression which is a singular term may on different occasions of its use denote different individuals, so one plural term may also on different occasions designate different manifolds. Just as 'the President of the United States' denotes different men at different times, so 'Farmer Brown's prize herd of Friesians' may on different occasions designate different manifolds of beasts. Similarly, just as two terms with different meanings may yet have the same referent when singular, so two plural terms with different meanings may yet have the same referents. For instance, if the men in a certain car are Tom, Dick and Harry Jones, the sons of Donald and Edna, the following three plural terms may all designate the same three men:

Tom, Dick and Harry
the men in the car
the sons of Donald and Edna Jones

Hence Frege's sense/reference distinction carries across without difficulty to plural terms.

For an expression to designate a manifold is simply for it to designate each of a number of individuals. There is no difference between the manifold, and the several individuals, despite the fact that we can talk about *a* manifold, and indeed can count manifolds to some extent as though they were individuals. So when an expression designates A and B and C . . . , where these are individuals, this is to say no more than that it designates A and designates B and designates C . . . Russell at one time thought that he could discern these two ways of "denoting",¹¹ but the attempt to do so landed him in the most dreadful muddles about one and many. There is indeed a genuine one/many problem to be laid to rest here, and Russell is not to be deprecated for appreciating this. Commenting on Russell's problem, Quine finds it difficult to see why there should be any difficulty in a set's having many members, any more than there is a difficulty in a single attribute's applying to many things.¹² But Russell's problem was not how something may have many things related to it, e.g. how a man may have many brothers. It is rather the problem of how one thing can also *be* many. It is Quine's insouciance rather than

Russell's difficulty which I find appalling here. I shall return to the one/many question later in discussing whether there can be manifolds of manifolds.

It is not too surprising that Russell's view of denoting as in every case being the same kind of relation, coupled with the view that even quantifier phrases denote, should give rise to difficulties. It might be objected however that in *my* use of the idea I have gone too far: while it may be acceptable to say that 'the man in this room' denotes Henry on a certain occasion, surely it can't be true that 'the men in this room' could denote John and also denote Henry? Surely it would then be ambiguous – which one does it denote? Ought we not to consider either that it denotes some third thing, such as a class, or that it had better be paraphrased away in favour of singular terms only? This attitude shows clearly the hold of the prejudice in favour of the singular. While in many contexts plurals can be paraphrased away, it is not certain that this applies in all cases, and furthermore there appears to be no good reason to seek such a paraphrase, once the harmlessness of plural terms has been recognized. There is further no reason to suppose that all the familiar properties of singular terms carry across to plural terms. If it is objected that a term cannot denote two things equally without ambiguity, since denoting *means* denoting one thing, then I can simply hand the objector the term 'denote' and set aside 'designate' for my own ends. Though I do not believe that 'denote', especially in the liberal hands of Russell, started out as being confined to singular denotation, it has perhaps, through familiarity, come to have that connotation, and I shall for the sake of clarity, distinguish between denoting, which is the special case of a term's designating one individual, and designating in general. Now a singular term, that is a term which is syntactically singular and is not a collective term, is indeed defective if it could equally well denote two individuals. In such a case we are either charitable about the grammatical number or are strict and declare the term thereby empty. In a converse fashion, a plural term which fails to designate more than one thing ought strictly to be taken as empty, though frequently charity prevails, where there is just one individual which, but for the difference in grammatical number, would be denoted by the term.

The plausibility of taking a plural term to designate each of the several things it does, appears to vary according to the kind of term involved. Let us see how this is so. With any compound term there may or may not be components which can stand on their own as terms. This is already

apparent in singular descriptions, e.g. ‘the present King of France’ contains the term ‘France’. I shall call these *subterms* of the term in question. The same applies, even more so, to plural terms. Any plural term obtained by conjunctively listing other terms obviously contains some subterms. Let any individual denoted by a subterm be said to be *subdenoted* by the whole term, and let any individual designated by a subterm be said to be *subdesignated* by the whole term. Thus e.g. ‘John and Henry’ subdenotes John and also Henry, while ‘Jason and the Argonauts’ would subdenote Jason and subdesignate each argonaut, if there were any.¹³ Something may be subdesignated by a term and designated by it also, as the above examples show. More surprisingly, perhaps, one individual may be both denoted and subdenoted by the same term, as e.g. Arthur may be by the term ‘Arthur’s favourite person’ (since he happens to be a narcissist). An individual may be designated without being subdesignated, and vice versa, as a term like ‘Arthur’s mother’ can show. An individual may be denoted or designated, subdenoted or subdesignated more than once by a term. The idea that plural designation is more plausible in some cases than others stems from the differences between those cases where the individuals designated by a plural term are also subdenoted by it, and those where this does not happen. So e.g. ‘John and Henry’ both designates and subdenotes John and Henry, whereas ‘the men in this room’, if it designates them, subdenotes neither and subdesignates neither, while it subdenotes this room. But once again, a liking for subdenotation seems to be simply a further manifestation of a preference for singular terms. Certainly a plural term doesn’t designate *just* one individual – but then if it did it would not be a plural term! In the case in question, John and Henry both bear the same kind of relation to the term ‘the men in this room’: it is simply this that I am calling ‘designation’.

§ 4 Against the Group Theory of Number

I shall return later to a discussion of what sorts of term designate manifolds under what conditions. First I wish to reject two other possible candidates which have been suggested as bearers of number-properties, but which, unlike Frege’s ‘concepts’, are also ‘external things’.¹⁴ I shall call these, respectively, *aggregates* and *groups*. Aggregates are defined mereologically. The aggregate of A and B is that individual all of whose

parts have some part in common with A or B; the aggregate of the *c*s is that individual all of whose parts have some part in common with at least one *c*. In general, the aggregate of a number of individuals is the smallest individual all of whose parts overlap some of those individuals.¹⁵ There are attractions to having aggregates as bearers of number-properties. Firstly they are reassuringly concrete. The aggregate of a number of things which take up space itself takes up space, just the space they take up. Secondly they provide a unitary bearer for the property. It is no detriment to an individual's status as such that it has parts discontinuous from one another. Denmark is no less a country than any other for being rather scattered. Certainly some aggregates are more natural than others: an organism is much more a whole than the aggregate consisting of my pet hamster's left ear and the Isle of Mull. It would however be unwise to rule out such bizarre individuals simply because they are bizarre. In bizarre circumstances there could be reason to regard the aggregate as less unnatural, e.g. if I owned the island as well as the hamster and left the island with the hamster's left ear as a bequest. The aggregate would even have a market value, should anyone feel like buying it from me. The reason for liberality in admitting individuals is not the capricious one of giving lawyers or philosophers silly tasks, however. It is because we have no clear idea where to draw a line between natural and unnatural wholes that we cannot afford to be too dogmatic about what is to count as an individual: the bizarre aggregates are a whimsical side-effect of this conceptual caution. So, since all aggregates are unitary, though not all naturally so, aggregates provide unitary bearers for number-properties.

This however leaves the original objection of Frege against "external things" unanswered. One and the same aggregate may have many different numbers ascribed to it: the pack of cards example will serve. It is one pack, but also the aggregate of fifty-two cards, etc. It is for this reason that Russell, for instance, took numbers to be properties of classes, rather than what he called "wholes", since a whole is essentially one rather than many.¹⁶ Strictly, this does not rule aggregates out as bearers of number-properties, but it does rule them out as bearers of number-properties other than the first, which would cripple a theory of number so based.

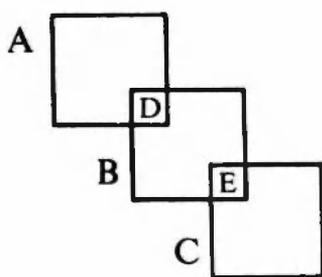
However, it may be objected that Frege's position depends upon a hidden assumption which may itself be questioned, namely that different number-properties are mutually exclusive, so that one thing cannot

have more than one number correctly ascribed to it. Armstrong has recently defended the view that one particular may be ascribable many numbers.¹⁷ The pack of cards has the formal properties of being fifty-two-parted (having fifty-two parts), being four-parted etc. Whatever is n -parted is also m -parted, where $m < n$. To see this, we can take the original n parts and consider $(m-1)$ of them left unaltered, taken together with the remaining $(n-m+1)$ aggregated as a single part. Armstrong takes this relation between these formal properties itself to be a part-whole relation. The difference between the pack as fifty-two cards and as four suits is easy to account for, since each of the requisite parts falls under the predicate 'is a card', while no proper part of it does, and it is not a proper part of anything that does either. Armstrong sketches the transition to arithmetic by taking the numbers to be the logically possible set of properties, *being-two-parted*, *being three-parted*, . . . etc. Number, for Armstrong, attaches to a class as one, i.e. an aggregate, rather than a class as many. But number attaches to the aggregate not merely taken as a heap, but taken as exemplifying certain properties which divide it into parts.

Now it seems to me that, while there is much to be said for this view, in particular that Armstrong recognises the importance of both mereological considerations and plural reference, it cannot be the final word on the subject. Frege's objection is based on the view that, at some level, number-predicates are mutually exclusive, and he is surely right on this, otherwise there would be no correct answers to "How many . . .?" questions, or rather, there would be many correct answers, and this does not accord with our practice. It looks as though Armstrong may avoid this problem by having recourse to the case not of formal properties like *being three-parted*, but material number-properties like *being an aggregate of three apples*. Nothing can at the same time be an aggregate of three apples and an aggregate of some other number of apples. This sort of consideration, which obviously relates closely to the sortal noun 'apple', is presumably what facilitates our normal practice and prevents us from having to ascribe different number-predicates to one and the same aggregate: we think of it not merely as an aggregate, but as an aggregate of *cs*, where *c* goes proxy for some suitable common noun (phrase). This accounts for the necessity of importing a common noun, what Frege called a concept-word.

However this account still does not separate the number-properties so that they are mutually exclusive. In the following figure, let 'square'

mean 'area bounded by and including a square figure'. Then the figure as a whole



may be taken as the aggregate of three, four or five squares, that is, either as $A + B + C$, or $A + B + C + D$, or $A + B + C + E$, or $A + B + C + D + E$, where '+' denotes mereological fusion or summation. The reason is that a square, unlike an apple, can be a proper part of one of its own kind.

For this reason we might try a second, related suggestion, that numbers attach not to aggregates *per se*, but to *groups*, which are aggregates *qua* composed in a certain way. The number three attaches to the above aggregate only *qua* the aggregate of A, B and C, and not *qua* the aggregate of A, B, C and D, etc. Number-properties are then not categorematic, but syncategorematic. What is true of an aggregate *qua* composed in one way need not be true of it *qua* composed in another. This version of the concept of group I have taken from Sprigge,¹⁸ who derives the word from McTaggart.¹⁹ McTaggart seems to me to use the word much more as I would use the word 'manifold', since while he allows that two groups may have what he calls the same 'content', as e.g. the groups A, B, C and A, B, C, D do, he regards them still as different groups, since they have different members. Groups are determined by their members for McTaggart, so I think that there is reason to suppose that *his* group is a class taken in extension, or what I call a manifold.²⁰ Sprigge on the other hand regards two different groups with the same content as absolutely identical, though not the same group. This is because the group is simply the aggregate *qua* composed thus and so, whereas 'is the same group as' is not for Sprigge a genuinely relational predicate. 'The group of F's is the same group as the group of G's' means for Sprigge 'The aggregate of F's *qua* being the aggregate of F's has the same members as the group of G's'. Now there is no need to relegate the predicate 'is the same group as' to such a lowly status if one accepts, as Sprigge does not, that identity

is relative, or at least that there can be relative identity predicates, of which 'is the same group as' and 'is the same aggregate as' are two. One and the same entity may be the same aggregate as something but not the same group as it. The aggregate is then the same aggregate as, but a different group from, other groups with the same underlying content.

I wish, so far as is here possible, to skirt the vexed question of relative identity, which obviously bears on our issues quite closely, but which can I think be partly set aside at least in regard to the problem of what it is that bears number-properties.²¹ For whether we take Sprigge's view of groups, or a relativist view, there is still, I believe, a serious objection to taking groups as the bearers of such properties. This is that there are certain pluralities which can never form groups, since their members could never be considered to compose an aggregate. Obviously anyone who is unhappy with the "bizarre" aggregates previously mentioned will feel this objection to be even stronger than I find it. Even if we are happy with aggregating entities of different categories, e.g. events and continuants, as e.g. when we consider that aggregate which is the sum of a man and all the events befalling him in his life, some cross-categorical aggregates are just too incredible. A sigh, Chairman Mao and the number five could never be considered to form a whole, if only because the senses in which we talk about their *parts* differs from one to the other. They can of course be *considered together*, as we have just done, and they may be the extension of the concept 'things I have thought about in the last minute': to *that* extent they may be unified. But this unification is purely extrinsic to them, and lies in the acts of mind of the reader, and in the occurrence together of expressions denoting them within the compass of a single term, or in their all satisfying a certain predicate. I confess to being unable to decide where to draw the line between merely bizarre aggregates and pluralities to which no aggregate corresponds. However, there is a still stronger reason for regarding some pluralities as unable to form an aggregate. The examples I have in mind are those where we have to do with a number of mutually exclusive possible states of affairs. Suppose, for instance that I wish to calculate how many different ways the first two cards in a pack may turn up when I next deal. Then since the cards will, on this next deal, only turn up in one of these ways, to the exclusion of all the others, I cannot be counting actual events, since the same answer holds whether I run through the gamut or not. Even if I do run through all the different ways, I cannot run through them all on the *next* deal, since this can occur only once. So what is counted or calculated

must be a number of possibilities, possible events, say. Each such event, were it to happen, would thereby exclude all the others from happening. Now these many mutually exclusive possible kinds of event could never form a whole, since they are all, so to speak, denizens of different possible worlds. It would be a horrendous ontology which allowed aggregates composed of events in as many different possible worlds to form an aggregate across worlds, even supposing one were happy with this view of possibility anyway. The mutually exclusive possibilities can of course be *considered together*, but this is something quite different from their forming a whole. The very least we can ask of an aggregate is that its parts can mutually co-exist in one world, even if they do not all exist at the same time. This requirement also would disallow aggregates composed of parts from different ontological realms, if there is more than one, e.g. a man and a Platonic universal. It is noticeable that the defenders of aggregates and groups tend to pick their examples from a single category and a single realm. Armstrong, for instance, is a physicalist, so there is some plausibility in his espousal of aggregates as bearers of number-properties. I do not wish my theory of number to be so tied to a particular ontological doctrine, so that if the ontology fails, so does the theory of number. This is a good reason for avoiding Frege's view of numbers as the properties of concepts (at least for the first stage of a philosophy of number): Frege's objective concepts, referents of predicate-expressions, are ontologically dubious.²²

The aggregate and group are attractive as bearers of number-properties because they are unitary. This is an aspect of the prejudice in favour of the singular: it is deemed that whatever has a property must be *one* thing, so whatever has number-properties must also, in some sense, be one thing. It seems to me, on the contrary, that some properties of their very nature are borne by more than one thing. This is, I think, Armstrong's reason for ascribing number-properties to aggregates rather than manifolds, even though he is aware that plural reference may make possible a non-platonistic theory of classes. This is because Armstrong believes plural reference to be essentially eliminable: where it is easily eliminable, as in

Tom, Dick and Harry went to a party,

in favour of a triple conjunction, Armstrong is happy, but where elimination is not straightforward, as in

Armstrong balks at plural reference, and prefers to say that the plural term refers to a single entity, namely the girder-lifting *team*.²³ My point, to which I shall return later, is that a word like 'team' is itself already a collective noun, which may be true of a plurality of individuals without being true of the individual members. Armstrong shows that he accepts plural reference as such only where the predicate satisfied by the entities designated by the plural subject is satisfied by the entities separately. Such a case I shall call *perfect distribution* of the predicate.²⁴ There are many predicates, of which numerical ones offer a clear example, which are not thus perfectly distributive, e.g. 'played a competitive game of chess', 'can speak seventeen languages (between them)'. It may be that plural reference is eliminable in these cases. In the case of number-properties I am not so sure. In any case, eliminability is not in itself something to be held against a certain kind of expression.

We have grown accustomed to the idea of relations being true of more than one thing, and not wishing to reduce relational predications to attributive. The same considerations apply, *mutatis mutandis*, to predicates having plural subjects. Some of them may find their foundation in singulars, just as some relational predications are true in virtue of properties of the individuals involved. There is in fact little to choose in many instances between describing something as a relation holding among n different individuals and as a property of these n individuals which does not distribute to them or to any submanifold of them. This is so where the relation contains no asymmetry, i.e. if ' $Ra_1 \dots a_n$ ' is true, then so is the predication obtained by permuting the terms a_i in any order. This is shown even in ordinary English, where symmetrical relations are as often as not expressed with a plural, conjoined subject, e.g. We may express the predicate ' ξ is playing chess against ζ ', which is symmetric, by the predicate ' ξ and ζ are playing chess', or we may say 'John and Henry are the same age', 'The Jones brothers all sleep in one bed'. The last example shows an aspect of the utility of plural reference: it remains true and expressible even when we don't know how many Jones brothers there are.

We may indeed have arrived at the numerical properties simply by turning certain relational predications into the requisite form with a plural subject. 'A is different from B' is certainly logically equivalent to 'A and B are two (different things)', so we may proceed to 'A is different

from B, and B from C, and C from A' for 'A, B and C are three', etc. We get the general *form* of such predicates by considering the variable predicate '... are all different'. This may take plural subjects designating any number of individuals from two upwards: how many individuals are so designated determines which of the infinitely many number properties is picked out. This account presupposes the applicability of the concept of absolute identity. For those unhappy with this, and for those who believe that there are additional relative identity predicates, we may offer a sortalised version:

'A is a different *c* from B' for 'A and B are two *cs*'

'A is a different *c* from B, and B from C, and C from A' for 'A, B and C are three *cs*'.

Here it is assumed that the truth of 'A is a different *c* from B' ensures the truth of both 'A is a *c*' and 'B is a *c*'. As before, we have the general form of the sortalised numerical predicate: '... are all different *cs*'. It seems to me that there is nothing to choose between seeing the numerical predicate as true in virtue of a multiply-adic relation among individuals, and true in virtue of a non-distributive property of a manifold, save that in the latter case we have a general recipe for constructing one predicate from the next, namely lengthening the conjunctive subject by one sub-term.

This then accounts for the ubiquity of number. It is as ubiquitous as identity and difference.²⁵ Manifolds, that is, 'more-than-ones', are just as "external" as individuals. They do not need to be regarded as having a unity which is "just so much . . . as is required to make them many, and not enough to prevent them from being many". The unity of a manifold is no less than that of an individual: it consists in the manifold's being *just these* individuals and no others. Of course it may require a feat of mind to *consider* the many things together, especially if there are rather many of them and we have no handy predicate true just of them. But it may equally require a feat of mind to think of certain individuals. They are no less 'external' and objective for that. We may have access to certain manifolds only because we have plural expressions which we can concoct. But the same problem of remoteness affects individuals: some of these we can only consider through the offices of language. But considering and being are two different things. A manifold exists in so far as its members exist: it is 'external' and 'objective' in so far as its members are.

It is worth pausing at this stage to consider how far we have answered Frege's objections to an 'external things' theory of number. The first problem has been successfully taken care of. Frege's objection applies only to a crass view of external things according to which they are all aggregates, heaps of matter or 'chunks of reality'. What is the relation between aggregates and manifolds on my account? A manifold has the same kind of being as its members. If these are homogeneous enough to be able to constitute an aggregate, then the relation between the aggregate of A_1, A_2, \dots and A_1, A_2, \dots is that they *compose* it, since if we call the aggregate A , we have that A is identical with $A_1 + A_2 + \dots$, the mereological sum of the many individuals in the manifold. The manifold and its aggregate *coincide*, that is, they occupy the same spatio-temporal region, but they are different, for the aggregate is one, and the manifold is many. The aggregate could be referred to by a singular term which does not presuppose plural reference, whereas the manifold must be referred to plurally. The aggregate has the numerical properties *indirectly*, through being the fusion of the members of manifolds which have them directly. There is no problem at this level of the numerical predicates being mutually exclusive: one and the same thing may be composed of three squares as well as of five squares. However, the same manifold cannot at the same time *be* three squares and *be* five squares: Frege's insight that the numerical predicates are, strictly interpreted, mutually exclusive, is upheld. The problem of the number zero has not yet been solved. I shall return to it, and also the number one, later on.

Number is certainly different from many other properties of things. We have seen that number-predications require plural subjects (for numbers greater than one) if they are to be true. Further, number-properties do not distribute to the members of manifolds, whereas the sorts of predicate Frege had in mind as paradigmatic 'physical' property-predicates, e.g. 'green', typically are perfectly distributive. My theory recognizes that number-properties cannot be simply identified with the way an individual can be split into parts. In this, the theory is more sophisticated than the somewhat naive one adopted by Armstrong. The idea that things literally have to be brought together into one place in order to be enumerable receives no credence at all on the present theory. Finally, we can account as well as Frege for the universality of number, without his dubious doctrine of concepts. This universality is not dependent upon a prior decision as to what there is, as troubles Armstrong's account. For, even more than the relations of part and whole,

the relations of identity and difference are ubiquitous and 'topic-neutral'.

§ 5 Counting and the Mereological Properties of Aggregates

There is, however, a group of interesting considerations which cluster around what we might call *homogenous* manifolds, those which can without strain be thought of as composing an aggregate²⁶. I am interested in the conditions under which such manifolds exist, i. e. when an aggregate, together with a suitable count noun, yields a manifold by division. This will then provide a partial justification for including a paper on the philosophy of number in a collection devoted principally to questions of mereology.

Let us call an individual an *aggregate of cs* when it is composed of a whole number of *cs* and nothing else besides. More explicitly, *A* is an aggregate of *cs* iff every part of *A* overlaps some *c*, and every *c* which overlaps *A* is part of *A*. Now on the naivest view of how aggregates of *cs* fall into kinds according to the way in which they divide into manifolds of different numbers, we could divide aggregates of *cs* by the following definitions:

A is a one-aggregate of *cs* iff *A* is a *c*

A is a two-aggregate of *cs* iff *A* is the disjoint sum of a *c* and a one-aggregate of *cs*

A is a three-aggregate of *cs* iff *A* is the disjoint sum of a *c* and a two-aggregate of *cs*.

In general

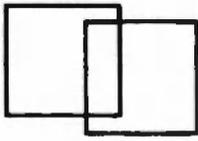
A is an $(n + 1)$ -aggregate of *cs* iff *A* is the disjoint sum of a *c* and an *n*-aggregate of *cs*.²⁷

This recursive specification of these various kinds of aggregate of *cs* only works for those finitely divisible into *cs*. We may cover all sizes of infinite aggregates by the stipulation that an aggregate of *cs* is infinite iff it contains as proper parts aggregates of *cs* of all finite adicities. It then turns out that an aggregate of *cs* is infinite iff it is the disjoint sum of a *c* and an infinite aggregate of *cs*.

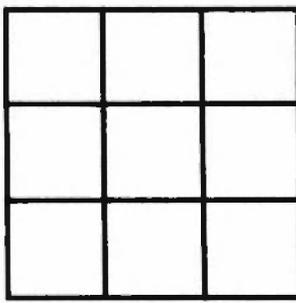
The reason this naive account of when we can divide an aggregate countably into parts which comprise a manifold is not sufficient may be

seen from the previous figure. This is an aggregate of squares, and it does not fall into any of the kinds given by the recursive definition, which is just as well, since it is an aggregate equally well of three, four (two ways) or five squares. The account must be broadened to explain how we may count such squares even when they overlap.

Suppose different *cs* have no common part, are disjoint. Then *cs* will be said to be *absolutely discrete*. *cs* which are such that no common part of two *cs* is ever a *c* will be called *relatively discrete*. If *cs* have no proper parts, they will be called *absolutely atomic*: if no proper part of a *c* is ever a *c*, *cs* will be called *relatively atomic*. Absolute discreteness and absolute atomicity entail respectively relative discreteness and relative atomicity. Relative discreteness and relative atomicity come to the same thing: one *c* is a proper part of another iff the two different *cs* overlap to the extent of the smaller. On the other hand absolute discreteness and absolute atomicity are different properties, the latter entailing the former, but not vice versa. Apples, to take our previous example, may be counted as absolutely discrete, but they would hardly provide much nourishment if they were absolutely atomic. Relative atomicity (or discreteness) does not entail absolute discreteness, since in the figure given below no proper part of a square in the figure is a square in the figure, but the two squares overlap.

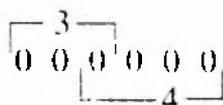


Consider the following linear figure. How many distinct subfigures

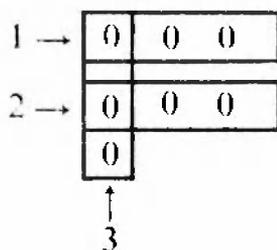


does it have which are squares? The answer is fourteen. These square subfigures are not all discrete from one another: some share vertices or

edges. The square figures are in fact relatively atomic. Were we to consider instead the filled-in squares, i.e. the square figures together with their interiors, we should lose this property, but we still get the answer fourteen, so I shall for brevity consider only the linear case. We can therefore settle the number belonging to the manifold of square figures despite the aggregate of these figures not being, in the sense given by the previous recursive definition, an aggregate of disjoint figures. Yet disjoint collections are clearly important in the philosophy of number, and this importance carries over to the philosophy of arithmetic. We do not illustrate the sum of four and three in the following way:



nor do we illustrate the product of three with itself in this fashion:



but rather take the sum to be of two disjoint manifolds, the product also a certain number of disjoint manifolds all of the same number. I venture to suggest that we should be unable to handle overlapping manifolds like the squares unless we had first built up the ability to recognise manifolds of different number in the disjoint case.²⁸

We may split the linear figure into discrete elements consisting of 16 vertices and 24 open sides (i.e. edges between vertices, lacking their endpoints). Were we dealing with the filled-in squares we could add the nine open interiors. Now various relations of contiguity hold between vertices and open sides, and relations of being parallel and being perpendicular hold among the open sides. Were we to give each of the 40 elements a name these relations could be spelled out in a finite number of sentences, but I shall not go to this length.

We now define a *line* as the mereological fusion of an open side with its two contiguous vertices. Lines, unlike the elements, are not disjoint,

overlapping in vertices. A *linear figure* is then any mereological fusion of lines as defined. The whole figure itself is a linear figure.

A linear figure has a *free* vertex iff the vertex is in (i. e. is part of) the linear figure, and is part of only one line in the figure.

A linear figure has a *junction* vertex iff the vertex is in the figure and is part of more than two lines of the linear figure.

Vertices in a linear figure which are part of exactly two lines are called *corner* or *straight* vertices, according as the lines they are part of are perpendicular or parallel respectively.

A *square* is then a linear figure containing no free vertices, no junction vertices and four corner vertices, such that between any two adjacent corner vertices there are the same number of straight vertices. Squares are thus uniquely determined by their corners.

I contend that we can count or calculate the number of squares in the figure because we can break it up into disjoint elements in this way, and determine which aggregates of these elements are squares. It might be objected that as a general account of when an aggregate can be divided to yield manifolds this is circular, since it depends on the ability to settle the number of certain manifolds in the figure, such as the four corner vertices of a square etc. The objection is misplaced however, since what we are using here is not the general account but the more limited one already given for settling the adicity of manifolds whose members are absolutely discrete. Here we are determining numbers of open sides and vertices, which are absolutely discrete, by construction. This is why they were called *elements*. There is no need for the elements to be all alike, as this example shows. We here have both vertices and open sides as elements.

We now define relative discreteness in a new sense. A manifold of *cs* will be said to be *discrete relative to its ds* iff:

- (1) every *c* is an aggregate of *ds*
- (2) the *ds* are absolutely discrete
- (3) two *cs* are different iff they contain a different manifold of *ds*.

Where *ds* are absolutely discrete they may be counted or calculated by the simplistic way given before. Thus, in our example, linear figures in general, and squares in particular, are discrete relative to their elements (vertices and open sides). When we actually decide for ourselves how many square subfigures there are in the figure, we do not need to go through the rigmarole of dividing the figure into elements and determin-

ing the conditions for when we have the same square and when we have a different one. But it is my suggestion that the much more intuitive method of 'seeing' square *Gestalten* in the figure has its basis in the fact that such a rigorous method is possible. My suggestion, for what it is worth, is that we can only divide an aggregate into a manifold in a certain way if the manifold is of a kind of thing which is discrete relative to some constituent elements. Of course if *cs* are absolutely discrete, they are thereby relatively discrete, being their own elements. This condition of relative discreteness goes some way to clarifying a remark of Frege's:

Only a concept which marks off what falls under it in a determinate manner, *and which does not permit arbitrary division of it into parts*, can be a unit with respect to a finite number.²⁹

The unwanted arbitrary divisibility is excluded here precisely by the condition that whatever falls under the concept *c* must be an aggregate of elements which are absolutely discrete. It would not matter for this requirement's being met if there were infinitely many such elements in a *c* so long as there were some effective way of deciding which *c* is which.

It seems to me that this requirement of relative discreteness (in the second sense) shows that there is a much closer connection between mereological considerations and those pertaining to what kinds of common nouns are sortals. Griffin, for instance, explicitly rejects all mereological criteria for sortality, in favour of one connected with countability:

A term 'A' is a sortal iff there *can* be cases in which 'A' provides, without further conceptual decision and without borrowing other principles of individuation, principles adequate for counting As.³⁰

However, since it is a requirement for being able to count, or, at least, for a common noun as applied to an aggregate to yield a manifold with a definite number, that the noun satisfy the mereological condition of relative discreteness, countability itself in such cases presupposes mereological principles. Indeed it would seem that the only region in which sortal terms do not effect a division of reference is one where we should have doubts about there being any antecedently given 'material' upon which the sortal terms work anyway, namely the abstract realm of mathematical objects. It is decidedly odd to think that numbers, for instance, are given to the mind firstly as an 'amorphous lump' or undivided aggregate, which the mind, with the use of the reference-dividing sortal noun

'number', then proceeds to slice up into individuals.³¹ The idea of an aggregate of numbers, i.e. an individual which contains numbers as its (perhaps atomic) *parts*, is a strange one. There may indeed be ways of making it work. But I cannot escape the conviction that this is a *transference* of the idea of whole, part and aggregate, to a region where it is not ultimately at home: that numbers, in so far as they are given to us at all, are given *already* as individuals. We could only gain access to an aggregate consisting of numbers *via* the sortal noun 'number', i.e. *via* the individuals comprising the aggregate. Hence it seems to me that where sortals serve genuinely to divide reference, to articulate for us the world into individuals, mereological criteria for sortality apply.

The first part of Frege's passage quoted above speaks of a concept's marking off what falls under it in a determinate manner. This yields a further condition which a sortal noun (phrase) must satisfy if it is to be able in a given context to determine a manifold: it must not be vague. Vagueness is not necessarily determined simply by the noun alone. A noun or noun phrase may easily determine a manifold in one context and not in another. For instance, while watching a standing wave experiment in a hydrodynamics laboratory I may observe 'These waves have a clearly sinusoidal shape', and the plural expression 'these waves' may pick out a precise number of waves. But if, say, I am standing on the deck of a ship watching a heavy sea, and I observe 'These waves are over twenty feet high', then while what I say may be perfectly true, it is very unlikely that there could be any answer to the question 'How many waves were referred to by the expression "these waves"?' The same is more obviously true of an expression like 'the waves now in the Atlantic Ocean', which has at least three areas of vagueness attached. Firstly there is the problem of just how far the Ocean extends, so we know which waves are in it and which are not. Then there is the problem of which particular watery disturbances are to count as waves and which are just regions of general turbulence, etc. Finally there is the problem that there is no general recipe for deciding where one wave ends and another begins. This arises where wavefronts converge, catch one another up, etc. For all that, we may make true predications about the waves in the Atlantic Ocean: We may say, e.g., on an abnormally stormy day, that the waves in the Atlantic Ocean are overall larger than they usually are.

The problem of whether a noun brings with its meaning a general recipe for dividing reference, settling the boundaries of the individuals falling under it, has been treated at length in recent philosophy under

the heading of 'criteria of identity'. We shall not here dwell long on this topic. Suffice it to say that the wave example shows that it is not a necessary condition of a sortal noun's being able to determine a manifold that it should possess an associated criterion of identity which applies in *all* circumstances. We require only that it should present one with such a criterion which may work in *some* circumstances.³²

Mass nouns may sit within terms which do not designate individuals or manifolds, and they may also sit within terms which do. For instance, the plural term 'the slices of beef on my plate' may determine a manifold, but not because the noun 'beef' does anything towards this: rather the noun phrase 'slice of beef' divides reference here. By contrast, expressions such as 'the beef on my plate' or 'the gold in this ring' do not designate individuals or manifolds, except in so far as each expression determines, or, as I shall say, *delineates*, a certain individual which is the aggregate of all the beef on my plate or all the gold in this ring. Indeed this is the way in which Quine accounts for the use of mass nouns in subject position, as e.g. in 'Snow is white': the term 'snow' here denotes, according to him, the single scattered individual which is the heap or aggregate of all snow.³³ Whether this proposal is right or not I cannot tell: there appear to be few considerations urging either an acceptance or a rejection at present. However, not all uses of terms containing mass nouns are to be so explained: in particular those which already parcel the stuff up in some way, e.g. 'the ingots of gold in Fort Knox'.³⁴ The ability to make true predications using terms containing mass nouns, even without it being necessarily the case that such a use needs to designate an individual (as distinct from delineating one) is closely parallel to the ability to use plural count nouns in a similar fashion, as mentioned above. For on such occasions we can regard the plural count noun as performing a role just like that of a mass noun: it is applicable to a more or less vaguely delineated sector of reality, but does not determine a manifold. We could imagine a suitable mass noun being used with equal facility, e.g. rather than talking about waves more than twenty feet high, one could talk of a twenty-foot *sea*. Here the question 'How many seas . . .?' is not only without answer: it is patently the wrong one. The noun 'sea' is here being used as a *mass* noun, albeit that it can also be used also as a count noun. The question is as ill-formed as the question 'How many golds are there in this ring?' The 'How many . . .?' question simply has no application in connection with a noun used as a mass noun. The same consideration therefore applies to those uses of plural

count nouns where a mass noun would be equally appropriate, even though the question 'How many...?' is not here *grammatically* wrong.³⁵

I spoke of a term incorporating a mass noun as *delineating* an individual (whether or not it also *denoted* it). The same can be applied to those plural terms which designate a manifold which can be heaped to form an aggregate. We may describe the aggregate as that one *delineated* by the plural term. Two plural terms designating different manifolds may yet delineate one and the same aggregate, as we know from various examples. In such a case we may say that the manifolds *coincide*. The aggregate as such could also be delineated by a term using a mass noun. For example 'the aggregate of the slices of beef on my plate' could delineate the same aggregate as 'the aggregate of beef on my plate' (provided all the beef on my plate is in slices). This provides another reason for thinking that sortal nouns applying to abstract regions do not effect a division of reference, since any mass noun applying in such a realm would surely have to be defined *via* the sortal terms, whereas in other, more concrete regions, the sortal noun effects a division of reference on an aggregate to which we may have access other than as the aggregate obtained by heaping together as an individual the members of a manifold. In the development of cognition, division seems to be more primitive than composition, except in the atomistic psychologies of the empiricists. In examples drawn from everyday life, however, both division and composition may play a part in determining a manifold designated by a plural term. For example, a settler arriving in virgin territory determines his land by dividing it from the rest, and he may then determine the estates of his various sons by further dividing the land which is his. But he may also determine these estates by composing or putting together plots which have already been divided, such as fields or strips. This example suggests that the manifold, which is, as it were, something which has both *internal* and *external* boundaries (external ones which divide the aggregate so delineated from the rest of the world, and internal ones which divide the aggregate up into the members of the manifold) is no more and no less a product of human thought and action than the aggregate itself. If we regard the aggregate as already existing before the settler arrived, and think that he merely picked it out from among all the other aggregates of land there, there is no reason to suppose that more boundary-fixing will bring something new into existence when the first episode of boundary-fixing did not. If on the other hand one wishes to

say that the settler brought the various plots into existence when he fixed their boundaries, which are internal to those of the original plot, then one must say the same about the original plot itself. To suppose that while the sub-plots did not exist before the act of demarcation while the large original plot did so exist is to manifest prejudice in favour of the singular again: manifolds are no more *and no less* objective and mind-independent than individuals.

Spatio-temporal coincidence has often been held to be a sufficient criterion of identity for material objects. This may go some way towards explaining the temptation to identify a manifold, especially a material manifold, with the aggregate delineated by the plural term designating the manifold, and hence to overlook manifolds and plural reference. Indeed many such manifolds share many of the properties of their associated aggregates. A manifold of things which are red may itself be regarded as red, just as its aggregate is red. A manifold is not to be regarded as identical with a complex individual, however. A complex individual is one thing with many parts. It is the aggregate, not the manifold, of its parts. Of course, it is not *merely* the aggregate of them, since its parts are in various determinate relations to one another. A manifold, on the other hand, has members rather than parts. It is many individuals rather than one. It is wrong to suppose that 'being' and 'being one' mean the same thing, as Plato knew.³⁶ This fact has however got lost with the tendency to concentrate exclusively upon the singular. One good reason for refusing to identify an aggregate with its coincident manifolds is that we should then identify the manifolds, and lose the desired mutual exclusivity of number-predicates. Another good reason is that the aggregate and a manifold which coincides with it may exist at different times. For instance, a certain aggregate of clay may exist long before it is fashioned into a number of clay vessels, and may still exist after they have been broken up. Conversely where individuals may survive despite part-replacement, as organisms may do, an individual may exist long after the matter which made it up has become widely dispersed, perhaps even partly annihilated.

§ 6 One and Zero

There is still the question as to what we can do with the numbers 0 and 1 on the present theory. The number one is easy to accommodate. We may

regard an individual as a degenerate case of a manifold. It is not many, but one. However, we need not discriminate against it on those grounds. As mentioned above, it is not an empty predication to say of something that it is one thing, since there are true predications with plural subjects. Indeed in certain cases a plural expression may succeed in designating only one thing. For instance, someone with a shaky grasp of Roman history may believe that Livia married both Octavian and Augustus. Here the expression 'Octavian and Augustus', which looks plural, turns out to be 'logically', i.e. semantically, singular, in much the same way as an expression which is grammatically singular, e.g. 'Queen', may designate a plurality (in this case a rock band). Any individual is as such one thing, just as any pair is as such two things.³⁷

The number 0 presents a greater difficulty. It was Frege's boast that his own theory of numbers as (in the first stage of the theory) properties of concepts was the only one which could account smoothly for the fact that there is a number nought. For a concept may have no things falling under it as easily as it may have some things falling under it. But in the case of a manifold of no things – there is no such manifold, so there is nothing to which we can ascribe the number zero.

Firstly we must ask whether it is such a grievous defect if a theory of number makes a difference between zero and all the other 'positive' numbers. We all recognize the difference between a positive answer and a negative one to a question such as 'Are there any biscuits left in that tin?' The answer 'There are none', while not itself to be regarded as a rejection of the question (as e.g. we should have to make if there were no tin there referred to) is a very different answer from 'Yes, there are just three left'. We all know that the sign for zero, and the concept that goes with it, are later inventions than signs for the other numbers, even on the advanced Indian-Arabic numbering system. The number nought is and was always felt to be something of an invention by comparison with the 'positive' whole numbers, more on a par with negative numbers than with the positive integers. Why is it a defect of a philosophy of number that it should explain this difference? Of course Frege too had an explanation of the difference, but this explanation makes rather too light of the conceptual jump required to arrive at the number 0. In our view it is precisely because there is no manifold or individual denoted or designated by an empty term that we must look for a different explanation of the number 0 from the others. Of course a concept may have no objects falling under it, but cannot have – 3 objects falling under it. But we have

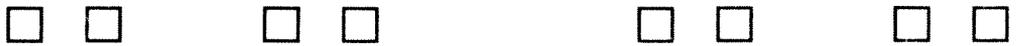
an explanation of that also. We may accept that it is true that there are no *cs* without being forced to Frege's conclusion that a philosophy of number is inadequate unless this is taken into account. For if there are no *cs*, the term 'the *cs*' is empty, i.e. designates nothing, i.e. *does not designate*. Therein lies the difference between zero-predications and positive predications of number. We do not need to conjure a null manifold or object into being to bear this property: with a robust sense of reality we maintain that there is nothing that is nothing. That is not to say that we have no explanation as to what is going on when we deny that there are any *cs*: we are simply doing just that. Only by a later convention do we take that to be a *numerical* predication. In Chapter VIII of *Philosophie der Arithmetik* Husserl draws an illuminating analogy which enables him, so it seems to me, to win this point hands down against Frege. If we accept that numbers correspond to answers to "How many?" questions, even the negative answer "None", we could similarly treat places and times as corresponding to answers to "Where?" and "When?" questions respectively. But if we follow Frege's argument, we should by parity of reasoning have to accept that there is a special place called "Nowhere" and a special time called "Never". The absurdity of this position ought to reflect back on Frege's position on the number zero. If it is felt that the case is here weaker, I would suggest two reasons: firstly the avowed *utility* of zero in arithmetic, and secondly the dulling of the senses brought about by generations of passive acceptance that Frege was right.

If we wish to establish a convention about what a null manifold is, we can only do so within the scope of a theory of manifolds as determined by sortal terms. It will not work for a general theory. In certain circumstances we can say that individuals or manifolds are null-manifolds. An apple, for instance, or several sheep, are all null-manifolds-of-insects. That is to say, they are none of them insects. But it seems unwarrantedly artificial to take such a step simply to have the satisfaction of providing a bearer for a special null property. In any case, *being neither an insect nor many insects* seems to be a predicate corresponding to no property.

§ 7 There are no Second-Order Manifolds

There appears to me to be just one obstacle to be removed before we can accept the theory of numbers as properties of manifolds (and degen-

erately, in the case of 1, individuals). It is not one which we find among Frege's list as such, though it arises from a special case of considering the ubiquity of number. The problem is that we can count, not merely individuals, but also manifolds of individuals. Suppose that in looking along a wall I see four pairs of chairs arranged as shown. Then I may count the pairs just as easily as I may count



the individual chairs. Indeed, because of the peculiar arrangement of the chairs I may even count pairs of pairs. This makes it look as though I am counting manifolds of manifolds.

At first sight it looks as though there is an easy way out of this. It is to suppose that in indulging in such counts we are in fact counting aggregates which consist of chairs, rather than manifolds. We count first the aggregates which have as parts two chairs close together, and then we count the aggregates which consist of two of these two-chair aggregates close together. There is indeed some plausibility in the suggestion in this particular case, the more so as both the pair-aggregates and paired-pair-aggregates are here absolutely discrete. The suggestion amounts to taking the bearers of the number-properties so ascribed to be not manifolds of chairs but aggregates of chairs of a certain kind, i.e. certain complex individuals with chairs as parts. While the suggestion appears both harmless and profitable in the present case, we cannot apply it in all cases because not all manifolds have the nice discreteness and atomicity properties we have here. Take the case of the three overlapping squares with two smaller squares at the overlaps, considered before. We may remember that there were precisely four different manifolds of squares all delineating the same aggregate, one of three squares, two of four and one of five. But precisely because they do all correspond to the same aggregate we cannot say that we count these manifolds by counting the associated aggregates, since there is only one such, whereas the number of manifolds is four.

It is worth having recourse here to the concept of relative discreteness mentioned before. We may count *cs* even though they are not absolutely discrete, so long as they are aggregates of *ds*, which are. Now we might apply parity of consideration to manifolds. A manifold is determined by its individuals. "Two" manifolds are the same which have the same

members, i.e. comprise the same individuals. It is for this reason that we may have such things as true plural identity predications. This means that the identity of a manifold is determined by the several identities of its members. So long as we can discern the individuals of a manifold we can tell whether or not it is identical with a given manifold. Two manifolds may be distinct but overlap, because they have some but not all members in common. One may indeed be a submanifold of the other. It is important to recognize the difference between the sense in which individuals may be said to overlap, namely when they have a common part, and that in which manifolds may be said to overlap, namely when they have a common member. (There are connections between the two notions of course.) If we allow individuals, i.e. members, of manifolds as submanifolds, then every manifold of n members has $2^n - 1$ submanifolds, including the manifold as a submanifold of itself. The difference between this and the number of subsets of a Cantorian set (2^n) lies in the fact that there is no empty manifold. It is because manifolds are determined by their members that we can count them. Once again, the more complex ability, that of counting manifolds, rests on and presupposes the ability to count individuals, just as the ability to count overlapping individuals depended on the ability to count discrete ones. The difference between the chairs and the squares lies in the fact that the squares in the figure were aggregates of their elements, while the pairs are manifolds of their individuals.

But now two questions arise. If we can thus count manifolds, are not the manifolds new unities in their own right, distinct from the several individuals which make them up, contrary to what I have maintained? We can after all speak of four pairs of chairs, two pairs of pairs of chairs etc. The expression 'this pair of chairs' is singular, as is 'this pair of pairs of chairs'. Here I wish to maintain the deflationary position that manifolds are not new, higher-order, unities distinct from their several members. Of course, each manifold is distinct from each of its members; provided it has more than one, that is. But that tells us nothing new. The ability to count manifolds I have already explained. The fact that we have a singular expression 'pair of chairs' may be taken as a mere slip of syntax. We do after all need to know how many individuals make up a pair in order to be able to understand the word. Of course, some senses of 'pair' mean more than just 'two of a kind'. They mean two of a kind which are matched by a certain relation, as e.g. a pair of shoes are matched in style, size, colour, and by being one left and one right etc. The same remarks

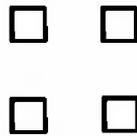
go for e.g. pairs of gloves, duelling pistols etc., and *mutatis mutandis* for other sets of things which are meant to 'go together' in a certain way. Such sets are likely to be discrete from one another, so there is no difficulty in accounting for our ability to count them in either of the two ways mentioned above. Languages are well-endowed furthermore with *collective* nouns which go in the singular but which, when attached to count nouns in the plural, serve frequently to help in designating manifolds. The fact that the term itself, e.g. 'this collection of stamps', 'this group of people' is in the singular does not divert us from recognizing that the expression designates a plurality of individuals. Of course what makes us say certain pluralities are a collection, a group, a set etc. is frequently much more than their being simply several of a kind. Not just any plurality of stamps constitutes a *collection*. For that, common ownership is required. It is frequently appropriate to regard such multiplicities as constituting higher-order objects in their own right. A stamp collection may grow and still remain the same collection, but contain a different lot of stamps. We might say that it is constituted or made up by different manifolds of stamps at different times. It cannot be identical with any such manifold, since they, but not it, are determined by membership. All the same, an expression like 'Uncle Harry's collection of stamps' may at different times designate different manifolds of stamps, *per accidens*, through denoting a collection with a certain membership, namely those stamps making up that manifold.

There are several ways in which manifolds get counted, not just one. This fact has become obscured by the blanket tendency to treat all pluralities or higher-order entities alike as sets, which are understood in an abstract, Cantorian fashion, as individuals distinct from their members, and distinct even from their members taken together. Such entities constitute Russell's 'sets-as-one', as distinct from our manifolds, which are his 'sets-as-many'.³⁸ Some counts which we might construe as counting manifolds are in fact counts of institutional and higher-order objects, which may indeed be regarded as individuals. For instance we might count the orchestras playing in a certain city. There might be, say, three such orchestras, even though one of them contains just the same players as another. One and the same manifold of players gets counted twice over, without error, in this case, because orchestras, unlike manifolds, are not determined by their members. Other counts of manifolds do treat them just as many individuals rather than as higher-order objects, after the fashion described above.

This raises a serious problem. Is there a different sense in which several individuals are, say, three, and several manifolds are three? We have to answer both yes and no. The answer is negative in so far as the concept of identity is applicable to manifolds as well as to individuals, without that making manifolds new individuals. If we allow a neutral identity predicate, which applies as well between plural terms as between singulars, then this same predicate applies to predications of the form 'A and B are two', i.e. 'A and B are different', and so on through the other forms of number-predication. The answer is positive in so far as we *may* distinguish between a singular identity predicate and a plural or neutral identity predicate. If we count three manifolds, then that manifold which is their union is three in a different sense from that in which it is however many individuals it is. That sense in which a manifold is many individuals must be regarded as the primary sense, and the sense in which a manifold may be many when we are considering submanifolds of it, as secondary. This implies that we are rejecting the concept of a second-level manifold, that is, a manifold of manifolds which is distinct from any manifold of individuals. Just because we can identify and count manifolds does not mean that we can make manifolds of them. Of course, given several manifolds, we may consider that manifold which is their union, i.e. the manifold comprising just those individuals which are members of any of the several manifolds. But this is once again a manifold of individuals. The several manifolds from which it is formed are not its members, unless these several manifolds already happen to be individuals, i.e. unit manifolds. For a manifold is just the one or many individuals designated by a term. When we consider a plural term with other plural terms as subterms, e.g. 'the Smiths and the Browns' then the individuals designated by this term are just those which are designated by at least one of the conjoined plural subterms. If on the other hand we count *families*, then, in so far as we may consider families as higher-order entities, we can count here just the two. (The problem with families is settling a criterion of identity which will account for the possible intricacies of intermarriage, but in this case it might be that no question of close relation arises.)

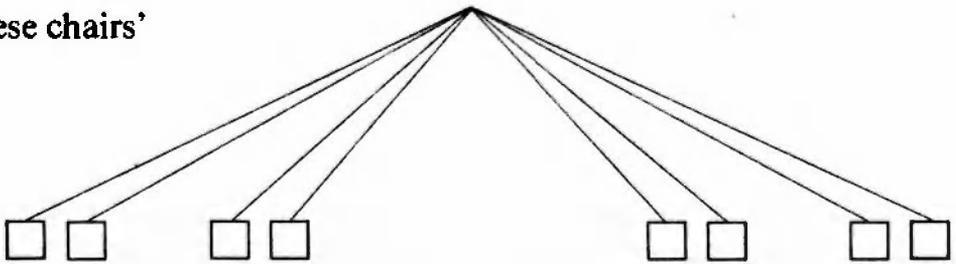
It has frequently been maintained that mathematics would be crippled if we did not have classes of classes, classes of classes of classes etc. While much of this dependence has come about through the reinterpretation of a great many mathematical entities as sets of various other entities, including also sets, it does look as though there are areas where sec-

ond and third-level classes are called for, e.g. in combinatory problems like 'How many ways can I pair socks in a drawer containing twenty-four socks of which twelve are blue and twelve are brown?' It seems to me that the apparent requirement for manifolds of manifolds, in order to make sense of questions like this, has been overrated. We grant that there is a type distinction between individuals and the manifolds of which they are members. So the sense in which *the four manifolds of overlapping squares form the same aggregate is a different sense from that in which these four squares*

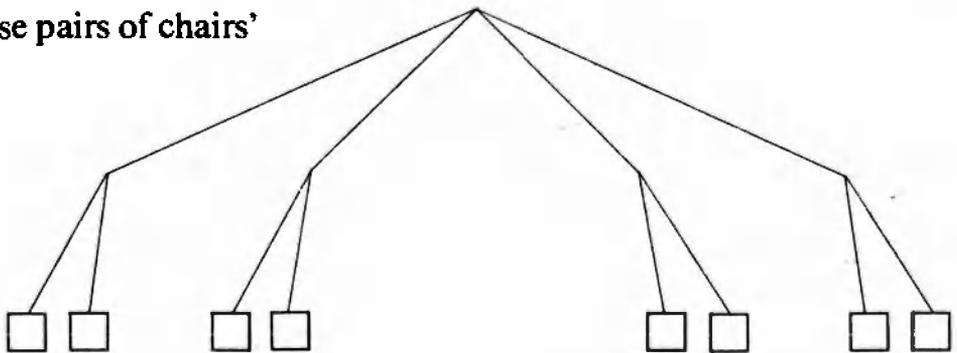


form a manifold. It is usually supposed without question that this is sufficient to motivate an infinite hierarchy of types. This I believe is not so. There is scant need if any to ever consider *manifolds* of manifolds, though we may often wish to consider together a *number* of manifolds. Of course we may regard our considering-together as showing up the various possibilities. We might look upon different plural expressions as relating to the objects in a different way. Consider the case of the chairs against the wall. It might be that we should wish to look upon the role of different referring expressions like this:

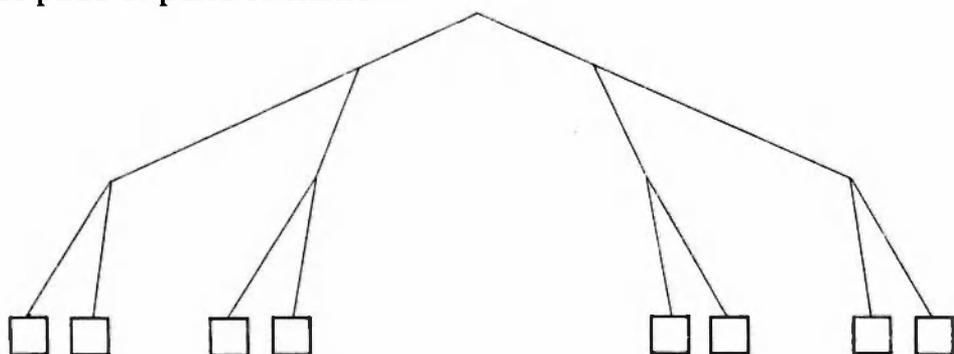
'these chairs'



'these pairs of chairs'



'these pairs of pairs of chairs'



It is one thing, however, to draw diagrams like this showing how we may group and subgroup individuals into larger or smaller groups: it is quite another to think that we have made any semantic sense of these diagrams in terms of higher-order manifolds. The individuals designated by a term are represented by squares. We have represented subterms by nodes in the tree. To take there to be manifolds as entities distinct from the many individuals comprising them is to treat higher nodes just as if they were nodes immediately above the individuals. But the diagrams here carry a little plausibility because they begin to treat nodes or their associated manifolds just as new individuals alongside, but slightly different from, the original ones. We can of course treat nodes, expressions, or any other individuals for that matter, as *representatives* of manifolds. In this way axiomatic set theory may be regarded as a theory of individuals (sets-as-one) representing manifolds (sets-as-many).³⁹ There is then no problem about allowing that there are sets of representatives of sets, which are equivalent to sets of sets in the usual interpretation. In the diagrams, a node is treated as though it were an individual, whereas the semantics of the situation tells us that it is not. If we wish to represent e.g. a pair of individuals, the best way is not to draw a pair connected to a node, but simply to draw a pair of dots. These *are* a pair. When we refer to the manifold consisting of A and B on the one hand, together with B and C on the other, where A, B and C are all individuals, then we have referred to simply the manifold of A, B and C, albeit in a somewhat redundant way.

If we look at combinatorial problems which are usually interpreted as involving counting classes of classes etc., we find that what is actually asked for in the count is often something like a number of possible arrangements, orders, selections, combinations, partitions and the like.

We may easily count or calculate such numbers without appeal to higher-order classes, which are introduced only for the sake of uniformity. The counting or calculating procedure very often takes the form of *proxy-counting*, that is, letting some order, arrangement or whatever be represented by something else, such as a sequence of symbols, and counting or calculating the different orders or kinds of such sequences, modulo some equivalence relation such as equiformedness in many cases.

While this is no more than a sketch of the intricacies involved in counts which proceed by proxy, it is of course clear that there must be some general rule enabling us to get from the proxy objects to their originals. This need not be a simple one-one correspondence. By setting up such rules we can enumerate manifolds of entities which could not be presented to us one by one for a more basic kind of count, where we tell off the objects in turn as they are presented. We are thus enabled to settle the number of manifolds of entities which cannot be given to the senses, or which cannot all be presented together, such as populations of a country, or ways of drawing two million pairs of socks out of a drawer . . . , or mutually exclusive possible events or states of affairs.

In this way we hope to have shown the plausibility of the idea that individuals and pluralities belong together in the lowest ontological type, albeit not a type in the usual sense consisting only of individuals, but lowest in Frege's sense of being objects rather than properties or concepts. In a sense, it is wrong even to speak of 'individuals and pluralities' as though, like cats and dogs, they could exist without one another. Certainly a plurality is never *identical* with any individual, but they are ontologically inseparable: whoever admits the existence of at least two individuals admits that of at least one plurality, even if, like the man who had talked prose all his life, he had never realised it. While one can, it seems to me, consistently affirm individuals and deny sets (indeed some, e.g. Goodman, have done just this), one cannot likewise affirm at least two individuals but deny pluralities, for the plurality of two objects just is *them*. It would be like affirming that something is red and also affirming that it is round, while denying that it is both red and round. The penalty in each case is the same: formal contradiction.

Thus predications of number attribute properties to manifolds, in the basic sense when we consider the number of individuals, in a derived sense when we consider a number of manifolds which are not all singletons, i.e. individuals. The analogy between the two senses is provided by

the fact that we can express both using an identity predicate (whether absolute or sortal-relative) which is neutral as between singular and plural identity. In very many cases where we may consider the individuals in a manifold as heapable together in an aggregate, the possibility of counting or calculating how many such individuals the said aggregate divides into is secured by the individuals' falling under some sortal noun which is discrete relative to some other noun giving elements of these individuals. Where mathematics has erected higher-order manifolds or sets, this is an artificial construction which has been introduced to unify the treatment of various branches of the subject. Sometimes, as for instance in geometry, a set-theoretic treatment can be replaced by a mereological one without detriment, and with added intuitive content. I hope therefore to have shown that Frege's objections to making number a property of 'external things' can be met. Clearly however before we can say that a rounded philosophy of number has been provided, more work needs to be done on the various conditions which enable terms to designate manifolds upon occasions of their use, and on the various ways in which we are able to settle the number of such manifolds. The essential task before us is to build a bridge which will connect, by means of an adequate theory of number, the philosophy of number with the philosophy of arithmetic.

Notes

References in these footnotes are to the works listed in the bibliography at the end of these essays. Works are cited under the name and year in which they are listed there.

For this essay in particular the following abbreviations are used:

Pr Russell, Principles of Mathematics (*Russell*, 1903)

Grl Frege, Grundlagen der Arithmetik (*Frege*, 1884) (Page references are to the 1953 German/English edition.)

¹ See Benacerraf, 1965.

² I should here make some remarks on the word 'manifold' itself. It is sufficiently uncommon in modern usage for me to be able to annex it for my own purposes. The word was more common in nineteenth century books, where it was roughly synonymous with 'set', 'class', 'aggregate' or 'collection'. Cantor for instance used the German equivalent *Mannigfaltigkeit* before later changing to *Menge*: cf. Cantor, 1932. I prefer to reserve the word 'aggregate' for mereological sums or heaps. I have avoided the terms 'set' and 'class' because they have too close a connection to the tradition stemming from Peano and Frege which I oppose. For more detailed discussion cf. the third essay below. The

word *Mannigfaltigkeit* as used by Husserl is translated variously as 'manifold', e.g. by Findlay, and 'multiplicity', by Cairns. Given Husserl's avowed intention to broaden the original mathematical concept of Riemann (cf. Husserl, 1929, Ch. 3) and the practice of using the word 'manifold' in English-language mathematics for the Riemannian concept, it might seem that this is the better translation in Husserl. However Husserl's concept is far wider than the mathematical one, and most closely approximates the modern concept of *model*, in which sense the term 'manifold' is used by Null and Simons in their essay in this volume. For this reason I prefer Cairns' translation. This still allows the unequivocal alternative 'plurality' (*Vielheit, Mehrheit*) for my concept. The relation between my concept and Husserl's is this: both are generalisations of the idea of an object or referent of a mental act or linguistic expression. Whereas Husserl's *Mannigfaltigkeiten* are the referents of formal theories, mine are the referents of plural referring expressions.

³ *Grl* §§22–4.

⁴ When I mention common nouns I shall also mean common noun phrases. A common noun phrase of the form c (c which Φ s) is + count iff it is grammatically congruous to say something such as 'there are three c s (which Φ) in Mongolia'. Cf. Griffin, 1977, 23. I shall refrain from using the word 'noun' for proper names, which are among what I call *terms*.

⁵ Frege, in a letter to Husserl of 24. 5. 1891. Frege, 1976b, 96–7.

⁶ *Grl* § 46, p. 60.

⁷ See e.g. Quine, 1960, 90f.

⁸ For different treatments of quantifier phrases see my 1978 and Evans 1977. My use of the word 'term' differs sharply from that of Russell in *Pr*. For me a term is always an expression, whereas for Russell it is the object or referent of an expression.

⁹ See e.g. Geach, 1962, 31 f.

¹⁰ *Pr* § 70, p. 69, n., where it is asserted that propositions with a plural subject have not one, but many logical subjects. Cf. further the third essay below.

¹¹ *Pr* § 61, p. 59, where Russell takes '*all as*' to denote a_1 and . . . and a_n , whereas '*every a*' denotes a_1 and . . . and denotes a_n (n always finite). There is surely here a distinction without a difference. What perhaps leads Russell to over-subtlety is the grammatical difference that the first is singular while the second is plural. Since the distinction which I make between singular and plural reference does not pertain at all to such quantifier phrases I need not even consider Russell's position. It is part of the side of *Pr* that I find unacceptable that Russell did not distinguish reference from quantification. But there is more to Russell's extension of 'denote' than its illegitimate use with regard to quantifiers.

¹² Quine, 1966, reprinted in Klemke, 1970. The passage mentioned is on p. 6 of the reprint.

¹³ In Appendix B of *Pr* Russell is forced, because of type theory, to deny that 'Heine and the French' denotes something of the same type as 'the French'. Russell admits that this is against common sense. We can accommodate common sense here: both designate manifolds.

¹⁴ Frege's concepts are *objective* but are not *objects*: they cannot be named. Cf. Frege, 1952, 45.

¹⁵ In Leonard and Goodman, 1940 aggregates are called *fusions*.

¹⁶ *Pr* § 70, pp. 68–9.

¹⁷ Armstrong 1978, Vol. II, Ch. 18, pp. 71–4.

¹⁸ Sprigge 1970, pp. 112–30. The diagram of overlapping squares is taken from p. 116.

¹⁹ McTaggart 1921–7, Ch. 15. My manifolds differ from McTaggart's groups in that I do not allow what he calls *repeating groups*, such as A, B, A, B. For me this is identical with A, B. Cf. the third essay below. Moreover McTaggart allows that groups may have

parts, whereas I hold that only where the members of a manifold admit of a sum or aggregate can we talk of parts, and these are not parts of the manifold itself but its sum, except when the manifold is a singleton.

²⁰ My manifolds are like Russell's classes as many, Cf. essay 3 below.

²¹ There is more than one doctrine of the relativity of identity. Cf. Wiggins 1967, Griffin, 1977.

²² It is the weight of such considerations which has dissuaded me of the group view of numbers which I originally held. I am grateful to Barry Smith for so forcing me to stretch my concept of aggregate in order to maintain this position that the concept eventually broke under the strain, and led through to the position of this and the next essay. Another objection to the group view is that it slurs the sense/reference distinction. An aggregate or a manifold is the referent of an expression, but an aggregate *qua* composed thus and so seems to straddle the divide between the thing considered and our manner of considering it. Cf. the remarks in the previous essay on the inducement of opacity by *qua*.

²³ Armstrong, 1978, Vol. 1, 32.

²⁴ Black 1971 discusses in outline the various possibilities for distribution of the predicate.

²⁵ The close relation between number and diversity was recognised by Jevons, and by Descartes before him. Cf. the remarks in *Gr*, p. 46. Frege's justified criticisms of Jevons are so justified only as an attack on his theory of abstraction. Jevon's first-stage philosophy of number is preferable in my view to Frege's own, though he did not recognise any difference between philosophy of number and of arithmetic.

²⁶ It is of course for homogeneous manifolds that the group theory of number is most attractive. For sparse ontologies, all manifolds are homogeneous. Rejection of the group theory takes Frege's sixth objection to an 'external things' theory more seriously.

²⁷ C is the disjoint sum of A and B iff: $C = A + B$ and A is disjoint from B , i.e. A and B have no common part. ($A \perp B$ in the notation of Leonard and Goodman, 1940). Here '+' is the mereological, not the arithmetical operator. '+' must also be differentiated from the use of 'and' to form plural terms.

²⁸ For similar considerations and diagrams, see Fogelin, 1976, Ch. XV.

²⁹ *Gr*, p. 66. (My emphasis.)

³⁰ Griffin, 1977, 43.

³¹ Cf. the picture sketched by Dummett in his 1973, p. 563 f. Dummett explicitly recognises that relative atomicity is not necessary for countability in a diagram of overlapping rectangles on p. 549.

³² It follows that the distinction in Geach, 1962, 39, between *substantival* and *adjectival* general terms is somewhat skew to our purposes. It may be that many sortal nouns possess open texture, so that we are not in possession of an infallible recipe for counting their instances. Cf. Zemach, 1974. But the open texture of sortals does not prevent us from sometimes or often counting under them, though it makes it harder to say in advance of any given noun whether it is relatively atomic, absolutely discrete, etc.

³³ Quine 1960, 98.

³⁴ Words like 'piece', 'heap', 'ingot' etc. which when used with a mass noun yield a count noun (phrase) have been called 'parcel words'. Cf. Griffin, 1977, 61.

³⁵ That plural count nouns behave very much like mass nouns has been forcefully argued by Laycock in his 1972, and is supported in Griffin, 1977, 33; 61. I cannot however agree with Griffin's statement (p. 61 n.) that plural count nouns convey no criterion of identity. This may be true for an example like 'waves', but where there is a good criterion for the noun in the singular its plural inherits the criterion in the plural, so to speak: if I can tell by it when I have the same c twice I can tell also by it when I have the same cs twice.

³⁶ Plato actually contrasts unity with identity, but identity and being are related by the formula: to be is to be identical with something. Cf. the following passage from the *Parmenides*:

PARMENIDES: Well, I think you will admit that the nature of unity is one thing, and that of sameness another.

ARISTOTELES: Why?

PARMENIDES: Because a thing does not always become one when it becomes the same as something.

ARISTOTELES: But why not?

PARMENIDES: If it becomes the same as the many, it must thereby become many and not one.

(Plato, 1953, 139d.)

³⁷ Even the status of 1 as a number is something of a modern convention. Both Descartes and Hume contrasted unity with number, i.e. plurality. Husserl continues the older usage: cf. p. 12 of his preface to his 1931, where he cites the number series as 2,3,4 . . . This appears to me to be going literally one too far. In *Philosophie der Arithmetik*, p. 129 ff., where Husserl argues his case against 1, he lays far too much stress on the idea that an answer to 'How many?' must be of the form 'so many', where this excludes just one. He is then left in the invidious position of having two *quite different* sorts of negative answer to such questions. The whole tenor of my following paper is both to draw attention to and show how, for scientific purposes, to minimise the awkward grammatical distinction between singular and plural. I would agree here with Frege that 'One' is a perfectly satisfactory positive, one might say singularly positive, answer to a 'How many?' question.

³⁸ *Pr* § 70. For Russell a class-as-one is the aggregate of the corresponding class-as-many. Cf. the next essay.

³⁹ For a more fully developed account of such representation see § 6 of the next essay.

III. Plural Reference and Set Theory

Most mathematicians do not perceive the problem which is posed by the abstractness of set theory. They prefer to take an aloof attitude and pretend not to be interested in philosophical (as opposed to purely mathematical) questions. In practice this means that they limit themselves to deducing theorems from axioms which were proposed by some authorities . . . the writings of contemporary set theorists and logicians do not offer very much which could help us in solving these problems.

Mostowski, 1966, 140f.

This essay has three aims, only one of which is furthered in detail. The first, and basic one, is to criticise the conventional interpretation of axiomatic set theories as alternatives in a programme of formalising the 'naive' concept of set, collection or class. The polemic which needs to be directed at the various conceptions of set used in defence of this view has already been convincingly accomplished by Max Black and Erik Stenius, so I need not carry that through here.¹ I shall be more concerned with developing a positive account of what I take the naive conception to amount to. The principal idea, which is Black's, is that sets are to plural terms as individuals are to singular terms. In the previous essay I called such entities manifolds. They entered in the context of the philosophy of number, as bearers of number-properties, whereas in this essay I shall consider them for their own sake and in greater detail. Cantor himself was led to abstract set theory through consideration of number, in particular transfinite numbers. It was he who first showed clearly what it means for one infinite collection to have more members than another infinite collection, and showed that there could be collections with different transfinite cardinality.

The positive theory of manifolds will be treated in § 4. §§ 2–3 prepare the way for this. In § 1 I shall suggest that the basic idea of a manifold, or class as many, has a nobler and longer history than Black and Stenius might suggest, and that echoes of this conception still inform some systems of axiomatic set theory.

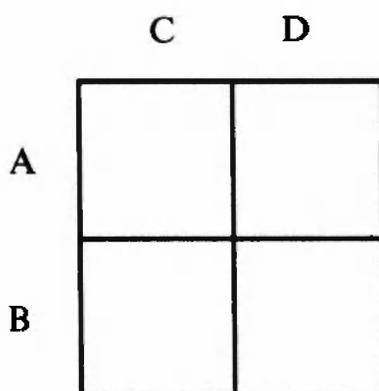
The second aim is reinterpreted. If axiomatic set theory is not a theory of manifolds, then what is it a theory of? The key notion here, that of an individual which is a *representative* of a manifold, is also suggested by Stenius, but again the idea goes back further. This aim will not be pursued in any great detail, though outlines of a theory embodying such a reinterpretation are sketched in § 6.

The third aim arises out of the other two. Because of the power of most systems of axiomatic set theory, sufficient power in most cases to serve as a foundation for finite and transfinite arithmetic and almost all of the rest of mathematics, sets have been massively over-used by logicians and philosophers in ontological investigations, and made to do service for such diverse entities as numbers, properties, relations, orderings, functions, propositions, facts, theories, worlds, persons, material bodies, higher-order objects, and so on. If, as I believe, a theory of manifolds serves to outline the ontology of nothing but manifolds (whatever they are manifolds of), then much of the set-based ontology of modern philosophy represents theft rather than honest toil, and the work for the most part remains to be done. The third aim, which is accomplished if the first two are, is not to *do* this work but to clear the decks for it. The substantive work left to be done is formal ontology, of which manifold theory comprises a small but not insignificant part.

§ 1 Classes as Many and as One: Historical Remarks

Introductory textbooks on set theory usually contain on page 1 a sentence like this: 'A set is a collection of things regarded as a single object', with a warning not to take 'collection' to imply any kind of physical bringing-together of the things in question. Such a conception raises in extreme form the ancient problem of the one and the many. Something which is a collection, i. e. many, is also one. It is completely specified by its members but is distinct from them, even when it has only one. The intelligibility of this kind of stipulation has in recent years been questioned, above all by Black and Stenius. This essay is in large part a development of the line of thought opened up in particular by Black, who first formulated with clarity the view that sets are to plural terms as individuals are to singular terms. The one-many problem cannot be avoided by taking a set to be the whole comprised of its members, their mereological sum. In the first place, such a sum does not in every case exist, or at least

it is not clear that it must.² In the second, even when such a whole does exist, it will not usually satisfy the fundamental principle of sets, the principle of extensionality: that sets are the same if, *and only if*, they have the same members. For two different collections may comprise the same whole when summed: this divided square



is the sum of the top half and the bottom half as well as the sum of the left half and the right half. We must accordingly distinguish the sum $A + B$ from the set $\{A, B\}$, for $A + B = C + D$ but $\{A, B\} \neq \{C, D\}$.³ Sums are wholes, and thus also individuals, whereas manifolds with more than one member are not individuals but pluralities.⁴ The whole-part relation $<$ is a relation between individuals, and must therefore be distinguished from the membership relation \in .^{5, 6} A mereological approach to classes has always held attractions for those of an anti-Platonist turn of mind. Goodman indeed *defined* Platonism, somewhat idiosyncratically, as the acceptance of sets. I would suggest that nominalist scruples about sets as abstract entities, 'high-brow' sets, might be to some extent assuaged by the use of manifolds, which are 'low-brow' sets, no more abstract than their members.

In the face of the successful advances of axiomatic set theory since Zermelo's first axiomatization of 1908, logicians have for the most part simply put aside or ignored the problem of one and many.⁷ If we look back, however, to the origins of set theory, when intuitions were perhaps fresher and less apt to be moulded by a tradition, we find a much greater awareness of the issue. In particular I wish to show how the problem made itself felt to three great set theorists: Cantor, Russell and, more recently, Bernays.

§ 1.1 Cantor

Cantor explicitly regarded a set (*Menge*) as a comprehension into a whole (*Zusammenfassung zu einem Ganzen*) of a plurality (*Vielheit*) of different objects. After the appearance of Burali-Forti's paradox, and in view of his own proof that since a set has more subsets than members, the impossibility of there being a universal set (sometimes called Cantor's paradox), he came to realize that it cannot be the case that to all pluralities (*Vielheiten*) there should belong a set (*Menge*). Those pluralities which can be comprehended into wholes he called *consistent*, those which cannot he called *inconsistent*.⁸ That Cantor can accept contradictions with such remarkable equanimity is due not to his being a working mathematician with better things to do⁹ but to his having on hand the distinction between sets and pluralities. He even went so far as to outline principles for deciding when pluralities can be comprehended and when they cannot, foreshadowing later developments.¹⁰ It is unfortunately not clear what the nature of this 'comprehension' is, but the important point for our purposes is that Cantor apprehended a distinction between sets and other individuals on the one hand and pluralities on the other, turning it to good use when the paradoxes were discovered.

§ 1.2 Russell

A distinction analogous to that between *Mengen* and *Vielheiten* is to be found, independently of Cantor, in Russell's early work of genius, *The Principles of Mathematics*. The importance of this work for our purposes lies in the circumstance that Russell, in the first flush of his enthusiasm for realism, was more sensitive to fine distinctions than he was to be later, after the success of the theory of descriptions in depopulating much of his universe spurred him to further reductions. In § 70 of this work, Russell distinguishes between a *class as one* and a *class as many*. He regards this as an 'ultimate distinction'.¹¹ What is especially interesting and important is that Russell, like Cantor, does not introduce the distinction for the express purpose of providing a way out of the antinomies, although, like Cantor, he does thereafter avail himself of the distinction for this purpose.¹² The immediate need for the distinction arises rather in connection with the argument put forward by Peano and Frege for distinguishing singleton sets from their members. The argument

goes thus: suppose we invariably identify x with $\{x\}$. In the case where x is itself a class with more than one member, since $\{x\}$ has just one member, and $x = \{x\}$, it follows that x both has just one member and more than one, a contradiction. Russell takes this argument to establish rather that we should not be tempted to identify classes as one with classes as many: 'the many are only many and are not also one.'¹³ For $\{x\}$ can only have one member which is itself a class if ' x ' denotes a class as one, while x can only have many members if ' x ' denotes a class as many. The Peano-Frege argument turns on an ambiguity and so founders: there can be, from Russell's point of view, no case where a class as many is a *member* of another class, since only *individuals* (Russell's *terms*) can be members. The difference between individuals (including classes as one) and classes (as many) is one of *type*.¹⁴

The distinction blocks Russell's paradox in that the non-self-membered classes comprise only a class as many: there is no corresponding class as one.¹⁵ This is essentially the same as Cantor's approach.

Russell even anticipates, though somewhat unclearly, Black's view on the crucial role of plural reference:

In such a proposition as 'A and B are two' there is no logical subject: the assertion is not about A, nor about B, nor about the whole composed of both, but strictly and only about A and B. Thus it would seem that assertions are not necessarily *about* single subjects, but may be about many subjects.¹⁶

Russell adverts to the use of 'and' to form what he calls 'numerical conjunctions' or 'addition' of individuals: '*A and B* is what is denoted by the concept of a class of which *A* and *B* are the only members.'¹⁷ Russell sways between denying that plural propositions can have genuine logical subjects and allowing that they do.¹⁸ He is also vague to the point of unintelligibility about the status of classes as many:

In a class as many, the component terms, though they have some kind of unity have less than is required for a whole. They have, in fact, just so much unity as is required to make them many, and not enough to prevent them from being many.¹⁹

Russell admits that he cannot find any individual like Frege's *Wertverlauf* (a word Russell felicitously translates as 'range') which is distinct from his own class as one. But whereas Frege's range is designed to obey the principle of extensionality, Russell's classes as one are mereological sums, and so do not.²⁰

Nevertheless, without a single object to *represent an extension*, Mathematics crumbles . . . But it is exceedingly difficult to discover any such object, and the contradiction proves conclusively that, even if there be such an object sometimes, there are propositional functions for which the extension is not one term.²¹

Russell's exasperation is clear. He is for the most part happy to regard the extension of a concept under which more than one thing falls as a class as many, but feels, in part under Frege's influence, the need for individuals to do the work of extensions. Why should mathematics crumble without these? Russell offers one brief example, and another reason is not hard to find. Firstly, consider a simple combinatorial problem: How many ways can m things be selected from n things, without regard to order, where $m < n$? The answer, $n!/m!(n - m)!$, is usually taken as the cardinality of the set of subsets of cardinality m of a set of cardinality n . This requires that we treat sets as members of other sets, i.e. use classes as one. But, on Russell's mereological view of classes as one, should any of the m things be a part of one of the others, the wrong answer would result. So we appear here to need something like Frege's range, which obeys the principle of extensionality while still being an individual. Secondly, Russell, like Frege, wants to give the logicist account of numbers as classes of equinumerous classes, but again if only classes as one can be members of other classes, the only number which could be thus defined is the number one, and that still remains a class as many. Russell again badly needs Frege's ranges: a number can then be taken as the range of the concept *equinumerous with M*, for suitable choice of concept (or range) M (I am ignoring Frege's difficulties about referring to concepts). But Russell's paradox has blocked for ever the unconditional guarantee of such handy individuals. Rather than admit the bankruptcy of logicism, Russell prefers to look to the complications of type theory, which he outlines in the second Appendix to *Principles*. From here on, the distinction between classes as one and as many ceases to play a role, and the whole idea of a class is eventually dropped in favour of a reduction to propositional functions.²²

§ 1.3 Bernays

Between the wars Zermelo's initial axiomatisation of set theory was modified and improved by various writers. Skolem made more precise

Zermelo's vague notion of a definite property, and Fraenkel proposed the Axiom Scheme of Replacement in place of Zermelo's Axiom Scheme of Separation, to allow unrestrictedly for transfinite ordinals. With Miriamoff's suggestion that all sets should be founded, so that for no set s_0 would there be an infinite descending sequence $\dots s_{k+1} \in s_k \in \dots \in s_2 \in s_1 \in s_0$, the shape of what is now always called ZF set theory was complete.²³ In 1925 von Neumann reinjected Cantorian ideas into set theory with a distinction between *sets* and *classes*.²⁴ This allowed axiom schemata to be replaced by axioms, and set theory was for the first time finitely axiomatized.

In a series of papers from 1937 to 1954, Paul Bernays developed von Neumann's treatment along somewhat similar lines. Bernay's treatment is usually taken as a mere variant of the approach of von Neumann, and the similar approach of Gödel 1940: the three are run together under the title NBG set theory. But there is a difference between the treatment of Bernays and those of von Neumann and Gödel which is quite crucial from our point of view. Whereas von Neumann and Gödel both regarded sets as classes, namely those classes which can be elements of other classes, even though Gödel, for example, used different faces for set and class variables, Bernays keeps sets and classes distinct from one another, allowing smaller and more tractable classes to *correspond* to sets. In his development of this theory he uses its finite axiomatization property to interpret it in a two-sorted first-order predicate calculus, with sets and classes comprising the different sorts, and two different primitive membership relations.²⁵ This is usually regarded as an unnecessary nuisance, since it complicates the symbolism and the treatment of mathematics, and the expedient of identifying sets with their corresponding classes is usually employed. But the thinking behind Bernays' treatment is clearly motivated by philosophical rather than mathematical considerations, as the following passage shows:

The two kinds of individual [sc. sets and classes], as well known, can in principle be reduced to only one kind, so that we come back to a one-sorted system . . . However it might be asked if we have here really to go as far in the formal analogy with the usual axiomatics. Let us regard the question with respect to the connection between set theory and extensional logic. As well known, it was the idea of Frege to identify sets with extensions (Wertverläufe) of predicates and to treat these extensions on the same level as individuals. That this idea cannot be maintained was shown by Russell's paradox.

Now one way to escape the difficulty is to distinguish different kinds of individuals and thus to abandon Frege's second assumption; that is the method of type

theory. But another way is to give up Frege's first assumption, that is to distinguish classes as extensions from sets as individuals.²⁶

Bernays' axiomatic theory of sets and classes consists in showing how to attain full freedom of set construction according to the intuitive principles laid down by Cantor, with sufficient power to derive classical mathematics, while avoiding the paradoxes. It thus constitutes a fulfilment of the idea, sketched, but never followed through, by Russell in Appendix A of *Principles*, of allowing unrestrictedly classes as extensions of propositional functions, while employing certain individuals as *Ersatz* extensions, Frege's ranges, in order to develop classical mathematics.²⁷

This is not to suggest that Bernays regarded classes as manifolds in our sense, that is, as 'many's' of individuals. Rather, he regarded them as individuals, though apparently as less substantial individuals than sets: useful fictions, perhaps.²⁸ However, he does speak of sets as *representing* classes. It would not therefore do excessive violence to at least the letter of his views if we were to regard classes, the extensions of predicates, as manifolds in our sense, and sets as individuals which are taken for mathematical purposes to represent the more tractable classes. Such an idea will be pursued further in § 6 below.

§ 2 Linguistic Phenomenology of Plural Reference

Plural reference was already introduced in the previous essay. Plural terms are expressions apt for referring to more than one thing at once. They contrast not with general, but with singular terms. A singular term is an expression apt for referring to, denoting or designating an individual. As the name suggests, it is (in Indo-European languages at least) usually inflected or otherwise modified for number, and when the subject of a clause, the main verb of the clause will usually agree with it in number. General terms, such as 'man', 'hooded crow', 'horse with a wooden leg' etc. are unfortunately so called, in that both general and singular terms might be assumed to be subsumed in a single category of terms. But I believe Frege was right in considering such general words and phrases (which I shall henceforth call *common noun phrases* (CNPs), where Frege called them 'concept words') as being inherently predicative rather than referential, although I do not consider CNPs to

be *simply* predicates, but rather to occupy a position intermediate in various respects between predicates and terms, constituting in fact a basic category of expression distinct from terms.²⁹

Singular terms should be contrasted rather with plural terms, which are also referential rather than predicative. Whenever we use a term, the syntax of English and many other languages compels us to treat the term as either singular or plural, and modify it accordingly. This can on occasion be a nuisance in ordinary discourse, and would be a considerable drawback in formulating an artificial language for logical purposes. The problem of how to deal formally with modification for grammatical number will be considered in the next section.

As outlined in the previous essay, plural terms fall into the same sub-categories as singular terms, namely proper names, descriptions, demonstrative phrases and pronouns, as well as having sub-categories not available, for obvious reasons, to singular terms, namely term lists. We have already seen how Bolzano, Russell and others drew attention to the possibility of forming term lists by using the word 'and' any number of times, flanked by that number plus one terms. The usual method of writing out a name for a finite set, as ' $\{a,b,c\}$ ' etc., constitutes, for those not under the impression that this expression denotes a new abstract unit, another feasible way of forming plural terms. Plural terms, like singular terms, may be different in sense and yet still designate the same things, while plural demonstratives, pronouns etc. are indexical in exactly the same way as their singular counterparts. Just as a singular term ('that man', 'the owner of 34 High Street'), may be used to refer to different individuals on different occasions of its use, so a plural term ('those men', 'John's children', etc.) may on different occasions of its use refer to different manifolds of things.

A plural term like 'the people in this room' is to be sharply distinguished from the (plural) CNP 'people in this room'. Whether singular or plural, CNPs are not terms. This difference is both syntactic and semantic. Semantically, CNPs do not of themselves make definite reference to things. Apparent exceptions, like 'People in this room have been smoking', can be set aside. In this case, although the CNP occurs alone as subject of the sentence, it is not a referential use, but quantificatory. The sentence means something like '*Some* people in this room have been smoking'. It is doubtful whether there is an exact logic for the quantificatory uses of CNPs in subject position. Sometimes, as in the above case, the meaning is existential, at others, as in 'Men are mortal', it

is universal, at others, as in 'People went home at midnight' it is probably majoritive, meaning something like 'Most people . . .', and in yet other cases ('Tigers have four legs', 'Gentlemen prefer blondes') the meaning is one of vague typicality, perhaps requiring some new kind of typicality-operator. Syntactically the difference varies according to language. In English, terms, unlike CNPs, may not be preceded by articles, demonstrative pronouns or quantifier phrases. In other languages the conventions differ: e. g. in Italian proper names require the definite article. In some languages, such as the Slavonic ones where articles are lacking, the difference is certainly less marked, and it might be preferable to regard the term/CNP distinction as somewhat parochial, especially in view of the long tradition of grouping proper and common nouns together in the one category of *name*.³⁰ Nevertheless, while the syntactic distinction may vary in strength according to language, the semantic distinction, between a nominal expression which is, and one which is not, marked for *definiteness*, whether this marking is morphological, syntactic, contextual or whatever, is one which cannot be ignored. As it happens, we shall not employ anything like common nouns in the formal treatment of § 4, but this is essentially a move away from ordinary language to the predicate/variable language of orthodox logic, where there is no CNP category.³¹

Mention must be made of collective nouns, like 'class', 'group', 'set', 'collection', 'aggregate', 'herd', 'flock', 'bunch' and the like. If *c* is a collective noun and *d* is some other CNP then '*c* of *d*s' is a CNP in the singular, yet we rightly regard such phrases as 'this flock of sheep' as referring to many individuals, though not one at a time. In the terminology of the previous essay, the expression may designate each of many sheep without *subdesignating* any of them, i. e. without containing a subterm designating any one. But, unlike a plural expression like 'these sheep' the expression 'this flock of sheep' is syntactically singular, and the question naturally arises whether we have here a singular term or an *ostensibly singular plural term*.³² Much of the appeal of the trinitarian concept of sets, whatever there is to be said against it, derives from the familiarity of cases where we use a grammatically singular expression to somehow characterise a plurality of individuals. The very words 'set', 'class' etc. are themselves collective nouns used for just this purpose. Do collective noun phrases refer to new, higher-order individuals, constituted by but distinct from their members, or do they simply refer to manifolds of individuals? I believe that, if we consider carefully, we

shall see that they do neither, although they share in part the behaviour of singular terms and in part the behaviour of plural terms referring to a manifold. To facilitate the discussion, I shall annex the word 'group' to describe what such terms refer to, or rather to describe, somewhat weakening my claim, what many or most of them seem to refer to. This answer is important, since on acceptance of it rests my suggestion that set theory (manifold theory) is a poor tool for ontological research (since most groups are not manifolds).³³

Two facts about groups have to be noticed: we shall then be clearer as to what a group is. Firstly, when we use a collective noun, we never, or hardly ever, use it without an accompanying CNP, linked to it (in English), by 'of'. We have classes of degree, sets of cutlery, clumps of trees, herds of cattle, collections of stamps and so on. In other words, groups are always groups *of* individuals, often of a specified sort. Secondly, to take up a point noticed by Stenius,³⁴ what makes certain individuals belong to a group is almost always more than their being several of the kind comprising the group. Not just any plurality of trees constitutes a clump, and not just any plurality of postage stamps constitutes a philatelic collection, and so on. The members of the group are linked, tied, connected or associated in some way. To borrow the terminology of Husserl from the first essay, between the members of the group there subsist various *foundation relations*. Such relations may take many forms. It may be that all the individuals in the group have a common relation to one thing, as for example when all the grapes in a bunch are connected, directly or indirectly, to one stem, or all the bees in a swarm are following the one Queen. It may be, alternatively, that the ties are simply relations holding between or among the members of the group, as for instance all the trees in a clump are relatively close to one another and further from other trees, or all the stars in a galaxy are relatively strongly attracted to one another gravitationally, as well as being closer to one another than to stars in other galaxies.

These facts distinguish groups in general from mere manifolds. For it is characteristic of a manifold that its members may be anything whatever. They need have no intrinsic ties or foundation relations: the only tie they need have is the purely extrinsic one of all being designated by one and the same term. Since we may form terms arbitrarily by listing, it is not surprising that the most bizarre bedfellows may be together in a manifold. Most of the manifolds we take any interest in are, mercifully, not of this kind. But the most important feature distinguishing most

groups from manifolds is this: the identity of a manifold is purely parasitic upon the several identities of its members: it obeys the principle of extensionality: manifolds are the same iff they have the same members. Groups on the other hand obey neither the 'if' nor the 'only if' part of this condition. A group may have different members at different times, and still be the same group. If a single tree is felled in a clump, the clump is diminished, but not destroyed. Likewise, if a new tree grows up in the clump, it is the same clump, but now augmented. Similar remarks may be made about other groups. Just as individuals, at least, those individuals which we call substances, may gain or lose *parts* to some extent without loss of identity, so groups may gain or lose *members* without loss of identity. I still attend concerts by the same orchestra I heard ten years ago, although the personnel has changed appreciably over that time. It is in this respect that groups are analogous to individuals, at least to individual substances, meriting the term 'higher-order objects' for groups. On the other hand, groups differ from individuals in being multiply constituted: a group may not *be* a manifold, but at any one time its members constitute a manifold. It is for this reason that the members of a group may be referred to using a plural term: we may refer to the trees in a clump as 'these trees', for example. It may be that the line between groups and individual substances is not a sharp one: a herd of cattle is certainly a group, and a multicelled organism like a man is certainly an individual, but certain colonies of insects resemble single organisms in various ways such as specialisation of role and balance of functions, while there is genuine dispute as to whether sponges are colonies of single-celled organisms or multicelled organisms of a different kind from most.³⁵

Because a group is not constituted solely by its members, but is the group it is in part because of the foundation relations among them, one and the same manifold of individuals may constitute, either successively or simultaneously, more than one group. To revert to the example of orchestras: in the days of the Empire, three of the orchestras of Vienna had the same personnel: when they played in the Court Chapel they were the Orchestra of the Court Chapel, when they played in the pit at the opera they were the Court Opera Orchestra, and when they played symphony concerts in the Musikverein they were the Vienna Philharmonic. Similarly two committees may have exactly the same members, yet not be one committee. In cases where two groups have the same members, we shall say they *coincide*. Because different groups have different persis-

tence conditions, two groups may first coincide and then not, or vice versa.

It would be as wrong to regard groups as mere successions of manifolds as it would be to regard individual substances as mere successions of 'genuine' individuals.³⁶ Just as we may regard individuals which can neither gain nor lose parts without ceasing to exist as a limiting case of individual substances, which can gain or lose parts, so we may regard manifolds as limiting cases of groups: those whose identity is exhausted by that of their members. In such circumstances the 'foundation relation' is the purely formal one of being just these several individuals and no others, although when we *refer* to a manifold using a plural term, this adds the weak extrinsic tie mentioned above.

Given that manifolds are groups obeying the principle of extensionality, manifold theory is powerless to describe the constitution of groups not obeying this principle, just as mereology is powerless to explain the nature of an individual which may gain or lose parts.³⁷ Nevertheless, it will not be wasted effort to develop the formal theory of manifolds, any more than it is a wasted effort to develop a mereology. Groups are, or are usually, 'many-fold', and a formal theory of pluralities will serve to show something of the logic of plural reference, as well as linking up more obviously with traditional set theory, where extensionality is always obeyed. To this end further aspects of the use of plural terms, those especially relevant to the basic notions of such a formal theory, should be mentioned.

Firstly, there is identity. We have spoken rather glibly of the identity of groups, but we need to be assured that there can be genuine identity predications involving plural terms. Sentences like the following:

The men in this room are John and Henry

resemble singular identity sentences in two important respects. Firstly, like singular identities, and unlike copulative sentences, the terms flanking the verb may be commuted without loss of sense, indeed without loss of truth (or falsity). Secondly, the logical properties of identity: reflexivity, symmetry, transitivity and intersubstitutibility in all extensional contexts *salva veritate* apply in the plural case also. There are apparent counterexamples to this last claim. Suppose John and Henry are the men in this room. Then while we may say

- (1) The men in this room are few
- (2) Max is not one of the men in this room,

the following sentences are less acceptable:

- (3) ?John and Henry are few.
- (4) ?Max is not one of John and Henry.

These facts do not however amount to a refutation of the proposition that intersubstitutability applies to plural terms. Sentence (1) is somewhat idiomatic as it stands: it would be far more acceptable to say the same thing by

- (5) There are few men in this room.

In this case, there is no plural to be substituted, and the problem vanishes. On the other hand, if by 'few' we mean something fairly definite, say, 'less than ten in number', then even if we accept (1) at its face value as containing a plural term, and tantamount to something like

- (6) The men in this room number less than ten (men),

then substitution gives

- (7) John and Henry number less than ten (men).

The readiness to drop the second occurrence of 'men' in (6) but not (7) may be explained by its having already occurred once in (6). (7) seems to me no less acceptable than (6). In case (2), again, if (2) is tantamount to something like

- (8) Max is not a man in this room

then the problem vanishes, whereas if we accept (2) at face value as containing a plural term, as I am more inclined to than with (1), then we may look on (4) as merely pragmatically or conversationally deviant, in that it is not usual to use different names for one person in close proximity, so that the need to make assertions like (4) does not often arise. Nevertheless, cases when assertions like (4) would be both apt and true are not hard to imagine: for instance

- (9) Tully is one of Vergil and Cicero, but not one of Plautus and Livy.

If sentences like 'The men in this room are John and Henry' are not plural identities, it is hard to see what they could be.

I thus take it as established that identity has a sense which is shared

between singular and plural identity propositions, involving the syntactic and logical properties mentioned above. Plural identities need not entail singular identities either: for instance

(10) John's parents are the two oldest inhabitants of the village

entails nothing about which parent is oldest and which is second oldest.³⁸

Next, there is membership and inclusion. We must distinguish between sentences like

(11) John is a man in this room

(12) These cows are brown

where the predicate does not involve a plural term, from those such as

(13) John is one of the men in this room

(14) These cows are among the cows owned by Brown

where the predicate does contain a plural term. The copulas 'is', 'are' (and their equivalents in other tenses) cannot be considered candidates for the vernacular equivalents of the ' \in ' and ' \subseteq ' of set theory, which are binary predicates, flanked by terms. For ' \in ' the nearest equivalent in English is 'is one of' or 'is (one) among', e. g. 'John is among the winners of Olympic Medals of 1964'. The nearest equivalent of ' \subseteq ' similarly appears to be 'are among' or 'are some of'.³⁹

Now the difference between 'is one of' and 'are some of', or between 'is among' and 'are among', appears to be no greater in principle than that between 'is' and 'are' or 'runs' and 'run': one of grammatical number. While this is only a linguistic point, and does not bear directly on set theory, it is worth recalling that the Peano-Frege distinction between membership and singular inclusion was not always regarded as commonplace. Some of the most notable logicians of the last century such as Schröder and Dedekind did not make the distinction, while in Leśniewski's Ontology the distinction between singular inclusion ' $a \in b$ ' and strong inclusion ' $a \sqsubset b$ ' is merely that the former is false if 'a' is not a singular name,⁴⁰ otherwise the two are equivalent. It is worth recalling also that the Peano-Frege argument rests on the assumption that sets can be *members* of other sets even when they contain more than one member, a view which Russell was, at first, not ready to accept at face value, and in which we agree with him.⁴¹ The case for there being a distinction

of *type* between individuals and pluralities thereof rested for Russell on there being certain predicates which applied to individuals which did not apply to pluralities, and vice versa.⁴² But 'apply to' is ambiguous. It can mean that the predicate may be predicated *truly* of the subject, or that it can be predicated *significantly* of the subject. Only the second yields evidence for a distinction of type. The first suggests, trivially, only that there are some predicates true of individuals and not true of pluralities, and, if incorporated into a logical system yields a type-free system like that of Leśniewski.

There are indeed predicates which are, at least, never true of individuals. Most obviously, there are the plural number-predicates, like 'are seven in number'.⁴³ Less obviously, there are predicates such as 'meet', 'disperse', 'surround', and those derived from relational predicates, like 'are shaking hands', 'are similar', 'are cousins'. (I have put these in the plural: it is of course trivially true that a predicate in the singular cannot *correctly* follow a plural subject and vice versa, but the underlying verbs 'be shaking hands' when used in the singular sometimes have the derelativised sense 'be shaking someone's hand', although this can hardly be said of all the predicates mentioned here.) Some of the predicates, like 'disperse' and 'surround', may be used in a grammatical singular number, but in such cases they apply not to individuals but to masses of stuff, as 'The fog is dispersing', 'Water surrounds the house'.

The existence of such predicates might be used to justify the introduction of type distinctions. But if one prefers to say that sentences like

?John surrounded the fort

?The cow dispersed to various parts of the field

are not nonsense but simply and necessarily *false*, as I confess I am inclined to do, although I have not usually succeeded in getting agreement on this, then the same examples may be used to stake a stronger claim for the legitimacy of plural reference. Whether plural reference is always eliminable in favour of singular, or singular reference together with quantification, is not in any case the main point. I certainly believe that even if plural reference is in principle eliminable, it would be at least highly inconvenient to actually eliminate it, and maybe practically impossible. However, I am not claiming its ineliminability or its practical indispensability, merely its existence and usefulness. It is not as if eliminating plural reference brings ontological economy. Manifolds do not exist over and above, or even alongside, individuals. A manifold is simply

one or many individuals. A manifold exists if and only if at least one individual exists.⁴⁴

Looking at the question of membership and inclusion from the point of view of plural reference, the semantic condition:

True iff everything designated by the subject term is also designated by the object term (where the subject term is the one before, and the object term the one after, the relational predicate)

applies equally to membership, inclusion, and indeed identity, which can be considered a limiting case of inclusion. Were we to replace 'everything' by 'anything' in the above condition, this would also let in the case where the subject term is empty.⁴⁵

§ 3 Problems of Formalisation

The phenomenon of agreement or concord in syntax arises whenever expressions in certain syntactic categories fall into subcategories in such a way that even when two expressions are of compatible categories, that is, categories such that when expressions from them are combined, the result is syntactically connected,⁴⁶ (as for instance adjective and common noun, term and verb), there are still restrictive rules, usually called *selection restrictions*, governing which combinations are to count as well-formed. When such rules are violated, we get the most obvious examples of bad grammar, as *'This books', *'They smokes' etc. Many languages utilise selection restrictions in connection with grammatical number, distinguishing singular from plural, among terms, verbs, nouns, adjectives etc., and sometimes a three-way distinction between singular, dual and plural (more than two).

It is interesting that in typed languages like that in *Principia Mathematica* the restrictions on forming formulas may be regarded as selection restrictions, with each category of expression: term, predicate etc. being divided into denumerably many different syntactic subcategories. The extreme inconvenience of such restrictions may be seen by the frequent resort made in describing typed languages to the device of typical ambiguity.

Despite our introduction of plural terms, it would be similarly inconvenient for us to have a formal language employing selection restrictions with respect to grammatical number. Suppose we had a language

with singular terms s, s', \dots and plural terms t, t', \dots and predicates P, P', \dots (one-place) and R, R', \dots (two place), and we require further that predicates always occur modified for number, so that if P, R , etc. are the unmodified predicates, then \dot{P}, \dot{R} , etc. are in the singular and \bar{P}, \bar{R} , etc. are in the plural. Suppose further that selection restrictions operate as follows: the number of a predicate is to be the same as that of its first argument. This procedure would resemble closely the practice in many natural languages. Now consider how we should state that a binary relation R is symmetric: in more orthodox formal languages it would go

$$(\forall xy)(xRy \supset yRx)$$

whereas in the suggested language it would go

$$(\forall s s' t t') ((s\dot{R}s' \supset s'R\dot{s}) \& (s\dot{R}t \equiv t\dot{R}s) \& (t\dot{R}t' \supset t'R\dot{t}))$$

where we have shortened it somewhat by using a biconditional in the middle conjunct. Similar encumbrances would accompany all other generalisations: consider how formidable the formula stating the transitivity of R would look, for instance. This problem has not arisen hitherto because formal languages have invariably employed only singular terms.

One possible weakening would be to make the modification of predicates optional rather than compulsory, allowing concord to be used for highlighting certain predications. This would however necessitate the postulation of equivalences like $(s\dot{R}t \equiv sRt)$, and would not result in the total disappearance of selection restrictions anyway, since $s\dot{R}t$ would still be ill-formed. Once modification of predicates becomes optional, it seems arbitrary to stop there; better to drop modification altogether. Predicates would then be *neutral* as between singular and plural, and there are no longer any selection restrictions. This does not stop us from continuing to divide the category of terms into singular and plural. We can reflect the difference between the singular 'is' of identity and the plural 'are' of identity by having two different identity predicates, '=' for singular identity and ' \approx ' for plural identity. Then rather than extract a *syntactic* penalty when a singular term flanks the plural predicate or vice versa, and declare the result ill-formed, we shall extract a *semantic* penalty, and declare the result *false*. This is preferable to declaring it meaningless, for we should then have to decide how to deal with well-formed formulae lacking truth-values in compounds, and this is a messy

affair⁴⁷ which we can quite easily avoid. However, in line with the view put forward in the previous section that there is a sense to the identity concept which is independent of the distinction between singular and plural, we shall employ an identity predicate ' \approx ' which is *neutral* as between singular and plural, in that it may be flanked by either singular or plural terms and still be true.⁴⁸

Suppose that s and s' are singular terms designating the same individual. What is the status of the ostensibly plural term ' s and s' '? It has the form of a list, yet, if it designates anything, it designates just one thing. Two policies are open here I think, only one of which will be pursued in the next section. We could take a list like this to be a plural term, as its syntactic form suggests, but because it is not *semantically* plural, regard it as empty, having no referent. On the other hand, we could regard it as having as referent the same individual as it subdesignates with both its subterms. This then poses the question whether a redundant list of this kind should be counted singular or plural: syntactically it looks plural, whereas semantically it looks singular. However, as in the case of predicates, there is no reason to regard the singular/plural distinction as exhaustive of all the kinds of terms there might be. It is highly expedient to employ *neutral terms*, which are neither singular nor plural syntactically, but can be either singular or plural semantically. In practice such neutral terms are far more useful than strictly plural ones, and in the formal language developed in § 4, neutral terms will be employed extensively. I do not know of any neutral terms in natural language, though there is no reason in principle why natural languages should not employ them. In the language of § 4, all term lists will be neutral: plural term lists could be used, but are not.

We must next decide policy on empty terms. There is a vast literature on the problem, since Russell first proposed the theory of descriptions. It would be impossible to review in detail which course is the best to adopt, although I believe that the course I shall adopt is both the best and the most natural. Russell's procedure for singular definite descriptions is to allow them as terms only where existence and uniqueness have been proved. Against this it has been objected that this complicates the notion of term in that there is then no general decision procedure as to which expressions are terms, and involves a further complication in case the uniqueness and existence formulas are not categorically, but only conditionally derivable.⁴⁹ Frege on the other hand proposed arbitrarily assigning a referent to every term which would otherwise be emp-

ty.⁵⁰ This procedure seems, indeed is, artificial, and a better and more natural alternative is available. While recognising, with Frege, the syntactic affinity between names and descriptions, that is, regarding them all as terms, we regard empty terms as being simply terms which do not designate anything. It has sometimes been said that an expression may denote without denoting anything. This seems absurd to me. Provided only that we are able to handle sentences containing empty terms in our logic, then there is no need to assign artificial referents to empty terms. It is the great merit of free logic that it allows empty terms to be handled for logical purposes without requiring the development of three-valued truth-tables for the connectives. Of the free logics available, the one which appears to me to have the best philosophical justification is one which allows some formulas containing empty terms to be true, others to be false, and yet others to have no fixed truth-value on an interpretation. This is the sort of free logic developed by Lambert and van Fraassen.⁵¹

The possibility will be admitted of all terms, whether singular, neutral or plural, being empty. This indeed could hardly be otherwise if the logic is to be free of existence assumptions in remaining valid for the empty domain, when all terms are perforce empty. There will in fact be a standard empty term ' \wedge ', and this will be neutral. Furthermore, the neutral identity predicate ' \simeq ' will be allowed to hold truly when flanked by empty terms. Indeed, to keep the system extensional, we shall require that if ' a ' and ' b ' are both empty, that ' $a \simeq b$ ' is true. Consideration of the intended semantics of ' \simeq ' shows why this is so: ' $a \simeq b$ ' is to count as true iff whatever is designated by ' a ' is designated by ' b ', and vice versa, and this is vacuously so when both terms are empty. The standard empty term ' \wedge ' therefore plays a role like that of ' \emptyset ' in standard set theory, except that \emptyset is usually taken to exist: ' $(\exists x)(x = \emptyset)$ ' is a theorem in standard set theory, while we shall have as a theorem ' $\sim E \wedge$ ', where ' E ' is the existence predicate. This appears to me to be a considerable intuitive advantage of the present theory of manifolds: set theorists were once wont to deny that there was an empty set, or apologize for it as a 'convenient fiction',⁵² though in latter days they have become more brazen about asserting its existence. Systems of pure set theory, without *Urelemente*, admit indeed nothing but \emptyset and the various sets compounded therefrom according to the axioms, which, from the point of view of intuitive considerations, is a total retreat from reality. We can, with a sensible logic allowing the manipulation of empty terms, gloss the notion of a "convenient fiction" not by reluctantly admitting the entity as having a shad-

owly sort of existence, but by allowing that a term may be highly useful and yet still be empty. To sum up: there is no empty manifold. But, skirting paradox, we might say, extensionality ensures that there are not two distinct empty manifolds.

When empty terms are admitted, but bivalence is retained, there are various ways in which quantifiers and variables may be employed. The first is to take variables as ranging over both actual and possible objects, with universal and particular quantifiers meaning roughly ‘for all (actual and) possible’, ‘for some (actual or) possible’, and an existence predicate separating the actual from the merely possible.⁵³ This line appears not only blatantly extravagant ontologically, but also rests on the dubious notion of a purely possible individual. A second possibility would be to follow Leśniewski and allow what has been called ‘unrestricted quantification’⁵⁴, so that for instance both ‘ $\forall xPx \supset Pa$ ’ and ‘ $Pa \supset \exists xPx$ ’ would be true even where ‘a’ is an empty term. I shall not follow this line here, since I believe it gives a non-standard meaning to the quantifiers.⁵⁵ However, I shall return in § 5 to a comparison with Leśniewski’s ontology, which could be readily interpreted as a calculus of neutral terms. The approach followed here maintains allegiance to the maxim that to be is to be the value of a bound variable, allowing all parameters to be empty, but not variables. This is the approach of free logic, as exemplified by Lambert and Van Fraassen.⁵⁶

Since we shall hold to Quine’s maxim on existence, and manifolds with more than one member may exist, it is only consistent to allow variables to range not only over individuals, but also over manifolds of individuals. We shall accordingly employ three kinds of variable, corresponding to the three kinds of term: singular, plural and neutral. We shall allow these all to be bound by quantifiers and the description operator, so the intended meaning of sentences and terms where variables other than singular ones are bound must be spelt out in the next section. It would be possible to dispense altogether with singular and plural terms and variables without loss of expressive power, but we have not done so in the present exposition, since the motivation for the introduction of neutral terms etc. was that there could be plural ones. Having established that plural terms do indeed play a role in natural languages, it would be somewhat ungrateful to banish them completely from our formal language, although the price to be paid for keeping in the three sub-categories of term and variable is, as we shall now see, a certain complication of the formalism.

§ 4 Axiomatisation of Manifold Theory

In this section we shall be concerned to present axioms for a theory of manifolds, remarking as we go on the intended interpretation of the axioms. No formal semantics will be set out, nor will any metamathematical results concerning the system be proved. Such tasks lie in the future. The first task is to make the basic ideas more familiar.

We shall speak about an object language without being too concerned as to what it actually looks like: all axioms and rules will be characterised metalinguistically, using schematic meta-axioms. Definitions will be regarded as semantically motivated metalinguistic abbreviations.⁵⁷ So, if a and b are terms such that ' $a := b$ ' is a definition, a and b are automatically intersubstitutable, and ' $a \simeq b$ ' is a metatheorem. Similarly, if A and B are formulas such that ' $A := B$ ' is a definition, A and B are automatically equivalent, and ' $A \equiv B$ ' is a metatheorem.

Primitive Symbols of the Metalanguage

The following constant symbols are used:

Connectives: 1-place: \sim ; 2-place: \supset .

Quantifier: \forall .

Determiner: ι .

Predicates: 2-place: \simeq ; \in .

Punctuators: ($;$).

The following metavariables range over expressions of the kind listed:

Terms: Singular: s, t, s', t', s'', \dots etc.

Plural: m, n, m', \dots

Neutral: q, r, q', \dots

All terms: a, b, c, a', \dots

Variables: Singular: w, x, w', \dots

Plural: h, k, h', \dots

Neutral: u, v, u', \dots

All variables: y, z, y', \dots

Predicates: 1-place: P, P', \dots

2-place: R, R', \dots

(predicates of greater adicity will not be considered.)

Well-Formed Formulas: A, B, C, A', \dots

Formation Rules

Those expressions which are terms and well-formed formulae (wffs) are specified by a double recursion.

A *term* is either a singular term, or plural term, or neutral term.

Singular Terms comprise singular parameters (if there are any in the object language), singular variables, and singular descriptions, and nothing else.

Plural Terms comprise plural parameters, if any, plural variables, plural descriptions, and nothing else.

Neutral Terms comprise neutral parameters, if any, neutral variables and neutral descriptions, and nothing else.

Descriptions have the following forms: singular: ιxA
plural: ιhA
neutral: ιuA

where A is any wff. Descriptions in general therefore have the form ιzA .

Terms may therefore be divided into singular, plural and neutral or into parameters, variables and descriptions.

Wffs comprise atomic and compound wffs, and nothing else.

Atomic wffs have the forms: Pa ; $a\bar{K}b$, where a and b are any terms.

Compound wffs have the forms: $\sim(A)$; $(A \supset B)$; $\forall zA$, where A and B are any wffs, atomic or compound.

The usual definition of free and bound occurrences of variables within terms and formulae will be understood. An *open formula* is one containing at least one free variable occurrence. A *closed formula* is a formula in which all occurrences of variables are bound. Assuming that the variables are given some linear alphabetic ordering, then if A is any wff, the *universal alphabetic closure* of A is that wff obtained from A by binding all the free variables remaining within it with universal quantifiers, working outwards in alphabetic order. If A is closed, then it is its own closure. In the following, the expression ' $\vdash A$ ' will mean 'the universal alphabetic closure of A is a theorem'.⁵⁸

' $A(b/a)$ ' will designate that formula obtained from A by substituting occurrences of b for all occurrences of a , while ' $A(b//a)$ ' will range over all formulae obtainable from A by substituting occurrences of b for occurrences of a in all, some or none of the places where a occurs. In each of these definitions it is assumed that if A contains a well-formed

part of the form $\forall aB$ in which the term b occurs free, this part is rewritten with a variable not otherwise occurring in A . We shall also dispense with parentheses wherever possible, following the conventions of Church.⁵⁹ Thus

$$A \supset B \supset C := ((A \supset B) \supset C)$$

$$A \supset . B \supset C := (A \supset (B \supset C))$$

and we shall continue this practice when other connectives are introduced.

The constants '&', 'v', '= ' and '∃' are defined in the usual way in terms of '⊃', '~' and '∀'.

Meta-axioms for Predicate Logic

- a1 If A is a tautology of propositional calculus, $\vdash A$.
- a2 $\vdash \forall x(A \supset B) \supset . \forall zA \supset \forall zB$
- a3 $\vdash A \supset \forall zA$, where z is any variable not free in A .
- a4 $\vdash \forall zA \supset A(y/z)$, where z is free in A , and y is of the same subcategory as z .
- a5 If A is a theorem and $A \supset B$ is a theorem then B is a theorem. (Modus ponens)

These axioms are of a form which is familiar in free logics. They differ from axioms for predicate calculus with existence assumptions by not having such theorems as ' $\forall xPx \supset Ps$ ' or ' $Ps \supset \exists xPx$ ', since the *dictum de omni* axiom a4 is restricted to the case where a variable is replaced by another variable.

The difference between these axioms and those for normal free logic lies of course in the fact that we have three kinds of variables. It should be made clear how these work. If D is any non-empty domain of interpretation, then an assignment of values to variables in D assigns individuals to singular variables, manifolds with at least two members to plural variables, and manifolds with at least one member (i. e. manifolds in general) to neutral variables. Of course, if D is a singleton, no values can be assigned to plural variables, and only individuals to neutral variables. In similar fashion, if there are any parameters in the object language, an interpretation over D assigns individuals or nothing to singular parameters, pluralities (by which I mean manifolds with at least two members)

or nothing to plural parameters, and manifolds or nothing to neutral parameters. The difference between parameters and variables thus consists in the possibility of parameters being empty even on non-empty domains.

So ' $\forall xPx$ ' means that the predicate P applies to all individuals, ' $\forall hPh$ ' means that P applies to all pluralities, and ' $\forall uPu$ ' means that P applies to all manifolds. From this it will be seen that care should be taken not to mix variables of different categories carelessly. This is catered for by the restrictions in a4. It also seems evident that both $\forall uA(u/a) \supset \forall xA(x/a)$ and $\forall uA(u/a) \supset \forall hA(h/a)$ should be metatheorems, since whatever is true of all manifolds should also be true of all individuals and of all pluralities. Such a metatheorem, $\forall uA(u/a) \supset \forall zA(z/a)$ is indeed forthcoming, but in order to prove it further axioms are needed which will serve to link the roles of the various subcategories of variables.

Meta-axioms for Identity

$$a6 \vdash a \approx a$$

$$a7 \vdash a \approx b \supset . A \supset A(b / a)$$

The predicate ' \approx ' is the neutral identity predicate, holding between terms a and b just when they designate the same manifold. The familiar properties of symmetry and transitivity are readily derivable. On the other hand, the extensional property, that $a \approx b$ when both a and b are empty, is not derivable from a1–a7, and has to be ensured by further axioms. It must be noticed that $a \approx a$ holds for *all* terms: in this lies its usefulness. However, with identity and quantification on hand, we could readily define an existence predicate and various other identity predicates.

In free logic, existence is usually defined in terms of identity and quantification, and we could proceed thus:

$$Ea := \exists u(u \approx a)$$

with singular and plural existence defined as follows:

$$E!a := \exists x(x \approx a)$$

$$E!!a := \exists h(h \approx a)$$

Notice that here the distinction between the three subcategories of variable allows us to define three closely related predicates. It will turn out that these are not the only ways in which existence could be defined, but they are intuitively appealing to some, in that they represent the maxim: to be is to be identical with something.⁶⁰ It should be here noted that there are indeed systems of free logic in which the existence predicate is present but the identity predicate is lacking. In such systems, not only is existence not defined in the usual way; it can be shown to be undefinable (I owe notice of this to Karel Lambert).

A neutral identity predicate which does not hold between empty terms may be defined thus:

$$a \cong b := Ea \& (a \simeq b)$$

and singular and plural identities as follows:

$$a = b := E!a \& (a \simeq b)$$

$$a \approx b := E!!a \& (a \simeq b)$$

while yet further predicates would cater for the cases where we allow singular-or-empty, and plural-or-empty terms. The predicate ‘ \simeq ’ is however taken as basic here because of the familiar properties represented by a6–a7, preserving the analogy with singular identity in our chosen system of free logic.

Inclusion

As indicated in § 3, membership is to be regarded as singular inclusion. To reflect this, we choose as primitive the predicate ‘ \in ’ of non-empty neutral inclusion. Its intended interpretation is as follows: $a \in b$ is true just in case (i) a is non-empty and (ii) every individual designated by a is also designated by b .⁶¹ This is captured by the first axiom a8 below. In addition we now introduce the principle of extensionality in a9: manifolds are the same if they have the same members (the converse follows from a7 and the quantification axioms). For technical reasons, I prefer to define the existence predicate E not in terms of identity, as in the previous subsection, but in terms of inclusion as follows:

$$Ea := \exists x(x \in a)$$

and $E!$ and $E!!$ similarly (but these predicates will not be used.) This could be expressed as: to be is to comprise at least one individual. It applies to individuals as well as to pluralities. That it amounts to the same thing as the previously suggested definition can be seen only if we grant that variables range over things that exist, singular variables ranging in ones, plural variables in twos or more, neutral variables in ones or more. This is the import of a10, which comes in three instalments for the three subcategories of variable. To formulate the condition for singular variables, we need to be able to say when exactly one individual satisfies a given condition. In fact we shall give a more comprehensive definition, which enables us also to say what it means for exactly one individual, exactly one plurality, or exactly one manifold, to satisfy a condition.

First, we define 'at least one' trivially as follows:

$$\exists_1 zA := \exists zA$$

and now we define 'at most one':

$$\exists^1 zA := \forall z \forall y (A \& A(y/z) \supset y \simeq z)$$

where it is a condition that y and z belong to the same subcategory. This sort of definition, without the complication about subcategories, is in any case already familiar from ordinary first-order predicate logic with identity. We now simply define 'exactly one' as usual as 'at least and at most one'.

$$\exists! zA := \exists_1 zA \& \exists^1 zA$$

The sense of $\exists^1 xA$ will be familiar already, but what of $\exists^1 hA$? This says: there exists exactly one plurality such that A , whereas $\exists^1 uA$ means: there exists exactly one manifold (whether singular or plural) such that A . Suppose for tax purposes an apartment block is divided into *households*, some of which are individuals, others *families*. Then $\exists! xA$, $\exists! hA$ and $\exists! uA$ respectively correspond to saying something like: there is exactly one individual/family/household in the block such that . . .

Meta-axioms for Inclusion

- a8 $\vdash a \in b \equiv Ea \& \forall x(x \in a \supset x \in b)$
 a9 $\vdash \forall x(x \in a \equiv x \in b) \supset a \simeq b$
 a10a $\vdash \exists! x(x \in w)$
 b $\vdash \exists_2 x(x \in h)$
 c $\vdash \exists x(x \in u)$

To understand a10b the numerical quantifier \exists_2 must be defined. This is done in the obvious way:

$$\exists_2 z A := \exists z \exists y (A \& A(y/z) \& \sim (y \simeq z))$$

where y and z must be of the same subcategory.

We can define different inclusion predicates in terms of the notions introduced up to now. Of these, the most interesting are singular inclusion, or membership, and inclusion which holds even when the subject terms is empty:

$$a \in b := E!a \& a \in b$$

$$a \subset b := \sim Ea \vee a \in b$$

Some ready metatheorems following from these axioms and definitions tell us e. g. that existence and self-inclusion come to the same thing: $Ea \equiv a \in a$, that everything is emptily or genuinely self-included: $a \subset a$, and that, when singular terms are in question as subjects, inclusion and membership amount to the same thing: $s \in a \equiv s \in a$. A metatheorem which will be of interest in the next section is the following

$$\vdash a \in b \equiv \exists u(u \in a) \& \forall u(u \in a \supset u \in b)$$

$$\& \forall uv (u \in a \& v \in a \supset u \in v)$$

where it will be noted that instead of using a singular variable and neutral inclusion predicate to express existence, we may equivalently use a neutral variable and the singular inclusion predicate. This suggests that we could manage with slimmer resources: neutral terms alone. That this is so is shown by these metatheorems, which express the 'ubiquity' of neutral terms:

$$\begin{aligned} &\vdash \exists u(u \simeq z) \\ &\vdash Ea \equiv \exists u(u \simeq a). \end{aligned}$$

Descriptions

When descriptions are introduced into a system with plural and neutral terms we must consider how their sense is to be specified. With singular descriptions we already know how to gloss 'the x such that', which (when completed, e. g. by ' x is in this room') is the nearest equivalent in a language without common nouns to a natural language description like 'the man in this room'. If we look at plural descriptions in natural language, such as 'the men in this room', then it is clear that something is comprised in the manifold of such men as a member if and only if it is a man in this room. However some predicates, unlike 'man in this room', apply to pluralities. Consider 'meet' for instance. A sentence of the form ' a met' can only be true if ' a ' is a plural term. Corresponding to this verb we get as plural description something like 'those who met'. But clearly an individual can truly be said to be among those who met: the manifold designated by 'those who met' is a plural manifold, but like all manifolds is comprised of individuals, even if the predicate used does not itself apply to the individuals individually, so to speak. Consider a complicated plural description like 'those who met either in the dining room or in the lounge'. Clearly an individual belongs to the manifold so designated iff he is one of those who met in either (or both) of those places: we could specify which manifold is designated here by giving a list of *individual names*. Suppose, for example, that John, Fred and Jim met in the dining room, while Mike, Sam and Fred met (later) in the lounge. Then those who met in either the dining room or the lounge are John, Fred, Jim, Mike and Sam. (This is a good example of a plural identity sentence at work.)

Neutral descriptions may be understood then as covering both the individuals such that . . . and the pluralities such that . . . The list of English monarchs comprises not only those who ruled alone, but also William and Mary, the joint monarchs.

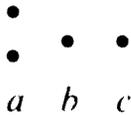
A pocket-size example will show the different kinds of description at work. Consider various collections of dots drawn on the page, and envisage them as being in a procession proceeding from left to right across the page. Let the one-place predicate 'R' be interpreted as 'forms a rank

in the procession.’ Any one or more dots in line abreast form a rank. Names will be assigned to the ranks: a token of each name appears below the rank in question. Where a rank consists of only one dot, the name is singular, and where it consists of more than one dot the name is plural. Then in the first procession



$\iota xRx \simeq b$, while $\iota hRh \simeq a$, and $\iota uRu \simeq a$ and b . We shall in fact write lists with terms between braces: a precise definition will follow. Here $\iota uRu \simeq \{a,b\}$. The term ιxRx is not empty, because there is a unique singleton rank, namely b . The term ιhRh is likewise not empty, because there is at least one rank with more than one member. The term ιuRu embraces all those things which are in any rank.

In the second procession



the term ιxRx is empty, because there is no unique singleton rank, while $\iota hRh \simeq a$ and $\iota uRu \simeq \{a,b,c\}$. It must not be thought here that because the term ‘ $\{a,b,c\}$ ’ contains three atomic subterms that the manifold thereby designated contains three individuals: here it contains four. It is also useful to have an expression marking out the manifold consisting of *all* individuals falling under R . We shall use the familiar notation $\{x|A\}$, for this purpose also, to stress further the analogy with normal set theory. The definition is this:

$$\{x|A(x/a)\} := \iota u(\exists x(x \in u) \& A(u/a))$$

and in the second procession $\{x|Rx\} \simeq \{b,c\}$.

In the third procession



both ιxRx and ιhRh are empty, since there are no plural ranks and there is no unique singleton rank. Here $\iota uRu \simeq \{x|Rx\} \simeq \{a,b\}$.

In the fourth procession

$$\begin{array}{cc} \bullet & \bullet \\ \bullet & \bullet \\ a & b \end{array}$$

both ιxRx and $\{x|Rx\}$ are empty, while $\iota hRh \simeq \iota uRu \simeq \{a,b\}$. If there could be such a thing as a null procession, all the descriptions would then be empty.

Descriptions, especially neutral descriptions, add greatly to the expressive power of the language. They enable us to define a great many constants in a way which makes the resulting theory begin to resemble more familiar set theory.

Firstly we define the universal manifold:

$$\forall := \iota u(u \simeq u).$$

and then, by analogy, we can define

$$\wedge := \iota u \sim (u \simeq u)$$

It will transpire that for every empty term a , $a \simeq \wedge$ is true. This is the extensionality principle mentioned earlier. Another metatheorem will be $Ea \equiv a \in \forall$: to be is to be comprised among the things there are. Only in the trivial interpretation over the empty domain is $\wedge \simeq \forall$. Like the night in which all cows are black, in the empty domain all terms, including the most comprehensive one \forall , are empty. The terms \wedge and \forall play a role similar to that of 0 and 1 in a Boolean algebra. The difference is that in Boolean algebras the zero element exists. However there are interesting analogies with Boolean algebras, which will be touched on briefly in the next section.

It is now possible to go ahead and define the usual Boolean operators of union, intersection and complement.

$$\begin{aligned} a \cup b &:= \iota u(u \in a \vee u \in b) \\ a \cap b &:= \iota u(u \in a \& u \in b) \\ a - b &:= \iota u(u \in a \& \sim \exists v(v \in u \& v \in b)) \\ -a &:= \forall - a \end{aligned}$$

These definitions could alternatively have been given using the notation $\{x\}A$ just introduced. In this guise they look more familiar, especially if we use the singular inclusion predicate. As it is, the following are forthcoming as metatheorems:

$$\begin{aligned} \vdash a \cup b &\simeq \{x\}x \in a \vee x \in b \\ \vdash a \cap b &\simeq \{x\}x \in a \& x \in b \\ \vdash a - b &\simeq \{x\}x \in a \& x \notin b \end{aligned}$$

From these definitions we may form arbitrary finite unions and intersections, and, because of extensionality, the usual Boolean identities and equivalences hold. The use of lists in normal discourse corresponds to expressions like $\{a, \dots, c\}$ in ordinary set theory, and we shall have an equivalent. We define term lists inductively as follows: if a is any term, then a is a term list, and if d is any term list, then $a.d$ is a term list. We get terms from term lists by surrounding by braces, and the resulting terms are defined as follows:

$$\begin{aligned} \{a\} &:= a \\ \{a.d\} &:= \{a\} \cup \{d\} \end{aligned}$$

This may seem like cheating, but it isn't. Given the motivation of the previous section, lists designate the individuals designated by each term in the list, whether it be singular or plural. We shall follow the convention that all term lists are neutral terms. Finite lists turn out to be indistinguishable from finite unions. This is in contrast to orthodoxy in set theory, but is motivated by the phenomenology of plural reference. It means that there are in the present theory no manifolds of manifolds distinct from manifolds of individuals. This point was defended as intuitively justified in the previous essay. In practice what it means is that manifolds do not stack up in an infinite hierarchy of types or ranks, but remain single-storied. This ought to appeal to the lovers of desert landscapes. So any expression formed out of terms by nesting lists to any finite depth may be replaced by a one-dimensional list, erasing all the braces except the outermost. Other cherished distinctions from orthodox set theory are casualties also. Firstly there is the distinction between a and $\{a\}$, as the last definition shows. In general it is only true that $a \in \{a\}$ when $\bar{\exists}!a$. a is a singleton. Where a is a singleton, not all set theorists

distinguish the element from the singleton set. As we mentioned before, Dedekind did not, and Cantor was not firm either way, while, in recent times, Quine has regarded the distinction as dispensable.⁶² The view that $a \in \{a\}$ only when a is a singleton embodies what I believe is right about Russell's distinction between classes as one and classes as many: that the only classes as one that exist are singleton classes!

One of the most powerful devices for generating sets in Zermelo's theory was the power set axiom. However, if we look at what must be our equivalent, the power set of a is $\mathcal{u}(u \in a)$. This turns out to be nothing but a , again, as our informal motivation would suggest. It is a metatheorem that $\vdash a \simeq \mathcal{u}(u \in a)$. For example if a is the pair $\{s, t\}$, then the power set of a is $\{\{s\}, \{t\}, \{s, t\}\}$. (There is no null manifold: even if we defined the power set in terms of ' \subset ' rather than ' \in ' the result would be the same, in any case.) Now, recalling that where braces are nested, we may remove all but the outermost, this manifold is revealed as $\{s, t, s, t\}$, which is simply an unnecessarily long way of designating $\{s, t\}$.

It might be thought that we are now crippled in terms of expressive power. How, for instance, can Russell's combinatorial problems be stated, and what is the status of the assertion that if a manifold has g members, then it has $2^g - 1$ submanifolds (minus 1 because there is no null manifold)? Firstly, we can talk about all the manifolds that satisfy a certain condition, rather than all the manifolds belonging to a (higher-order) manifold. There is nothing to stop us from making assertions about all pairs, for instance. But if we try to assemble the manifolds together into a single manifold in order to be able to 'handle' them (surely a manifestation of prejudice in favour of the singular), we shall find that we lose the original manifolds, getting landed simply with their union manifold. The manifold of all pairs is (assuming at least two things exist) simply \vee . The use of conditions instead of higher-order manifolds does bring a loss of expressive power however if one is not prepared to quantify over conditions. It may be said, however, that any ontological commitments incurred in quantifying over predicates is not lost when one trades predicates for sets: it simply reappears in a different form.⁶³ In any case, the paradoxes show that not every condition can earmark a distinctive individual as the corresponding set: this was where Bernays entered the fray in 1937.

We shall not develop number predicates or numerical quantifiers in detail, although it is clear from the definitions of $\exists_1^!$, \exists_2 etc. and the discussion of the previous paper how finite number predicates and numeri-

cal quantifiers can be defined. But it is important to realize that we may define similar looking but different numerical quantifiers by using different subcategories of variable. For instance, $\exists_2 xRx$ and $\exists_2 hRh$ do not mean the same thing, and neither means the same as $\exists_2 uRu$. Consider the processions examples again. $\exists_2 xRx$ means that two individuals are ranks: this is true in the second and third cases, false in the first and fourth. $\exists_2 hRh$ means that exactly two pluralities are ranks: this is only true in the fourth case. $\exists_2 uRu$ means that exactly two manifolds are ranks. This is only false in the second case. So we are quite able to say that

$$\exists_2^{\mathfrak{I}} x(x \in a) = \exists_{\frac{2\mathfrak{I}}{2\mathfrak{I}-1}}^{\mathfrak{I}} u(u \in a)$$

and indeed, given recursive definitions of the numerical predicates, the result could be proved as a metatheorem by mathematical induction.

Combinatorial problems such as would have warmed the cockles of Russell's heart could drop out of the system as metatheorems. For instance, a football manager with a squad of thirteen players has to pick an eleven to take the field. He can select any one of 78 different possible teams: but it would be surely only a matter of patience to prove the following as a metatheorem of the calculus of manifolds:

$$\exists_{\frac{78}{11}}^{\mathfrak{I}} u(u \in a \ \& \ \exists_{11}^{\mathfrak{I}} x(x \in u) \ \& \ \exists_{13}^{\mathfrak{I}} x(x \in a))$$

(I have not the patience.)

Having different styles of variable and accordingly different senses for numerical quantifiers also enables us to put a firmer gloss on the contention, made in the previous essay, that number predicates, when applied to individuals, have senses analogous to the sense they have when applied to pluralities or manifolds in general. The analogy comes out in the common form of the definitions of numerical quantifiers despite the use of different subcategories of variable. In this way the informal motivations of the previous essay link up with the formal treatment of this one. This distinction enables us easily to do the work which Stenius suggests requires a procedure he calls "second-order counting".⁶⁴ Indeed, it is more flexible, since it allows us to count arbitrary finite numbers of manifolds, not just those which are submanifolds of a given manifold.⁶⁵

But suppose, to revert to our example, that the football manager, not content with knowing how many teams he can pick, wishes to know how many ways he can slot his selected players into the eleven available positions, and arrives at the (I hope) correct answer of 39,916,800. How can this be expressed in terms of the 11 players, maybe the 2047 submanifolds thereof, without sets of sets? Surely it is here that we need sets of sets, or, as Stenius uses, arbitrary representative individuals to go proxy for sets. I am not convinced. Certainly simply considering the 11 players and submanifolds of them will never advance us to the relatively astronomical figure of 11!; but I do not think we are in this case counting men, or groups of men, at all. We are computing possible *ways* of slotting eleven men into eleven positions. This is the same as the number of different ways we may pair any two disjoint collections of eleven, or, speaking mathematically, the number of different bijections between disjoint sets of eleven. I would suggest that expressing combinatorial problems in terms of sets of sets, or sets of sets of sets, is merely a convenient device, and does not represent the ontology of combinatorial problems at all.

Meta-axioms for Descriptions

- a11a $\vdash E_1x A \equiv \exists_1 x A$
- b $\vdash E_1h A \equiv \exists h A$
- c $\vdash E_1u A \equiv \exists u A$
- a12 $\vdash E_1z A \supset \forall y (A(y/z) \supset y \in {}_1z A)$, where z is either neutral or of the same subcategory as y .
- a13 $\vdash s \in {}_1z A \supset \exists u (s \in u \ \& \ A(u/z))$
- a14 $\vdash \sim E_1z A \supset {}_1z A \simeq {}_1u (u \neq u)$
- a15 $\vdash a \simeq {}_1z (z \simeq a)$, where z is either neutral or of the same subcategory as a .

The three instalments of a11 present the conditions on the existence of manifolds designated by descriptions. a12 and a13 tell us about the membership of manifolds designated by such descriptions when they exist, while a14 “identifies” all empty descriptions in the way suggested by § 3. This treatment is most suited for mathematical applications, though the possibility of varying the axioms for other applications, e. g. in considering the logic of fiction, is not to be ruled out without further

consideration. The final axiom a15 states an identity not otherwise derivable. This in fact makes a6 derivable as a metatheorem.⁶⁶

The axiom a11c is quite a powerful one, and simulates union axioms in orthodox set theory after this fashion: If for instance Pu states a condition in one free variable on manifolds, then so long as at least one manifold satisfies the condition, the union of all such manifolds exists. Conversely, if such a union exists then at least one manifold satisfies the condition. That $\cup Pu$ is in effect a union can be seen by considering its membership conditions, using a12–a13. By a12, if $\cup Pu$ exists, then any manifold satisfying the condition is included in it, and by a13, any individual which is a member of $\cup Pu$ is a member of some manifold satisfying the condition. Notice that the individual need not itself satisfy the condition: this should be clear from the examples given before. In general, we cannot infer either that the union $\cup Pu$ itself satisfies the condition: if P is the predicate 'is a pair' then in any world containing three or more individuals, the manifold of pairs is not a pair.

This sort of consideration may put one in mind of Russell's paradox. It is worth seeing how it fails to arise in the present theory. All singleton manifolds are self-membered, and all pluralities are not. The manifold of non-self-membered manifolds is $\cup(u \notin u)$. In a domain with less than two members, this does not exist. In one with two or more members, it exists, and is identical with \forall . Now in such domains certainly \forall exists and $\forall \notin \forall$, but this does not entitle us to infer that $\forall \in \forall$: merely, and harmlessly, that $\forall \in \forall$ only when the domain has only one member, and then $\cup(u \notin u) \approx \Lambda$. The paradox simply does not arise, for precisely the reason originally suggested by Russell: there is a gulf between one and many.

In general $P(\cup Pz)$ only in the case where $\cup Pz$ is a singular description which is not vacuous, although cases arise with other subcategories: for instance, in domains with at least two members, the manifold of manifolds with more than one member is itself a manifold with more than one member, viz. \forall . We can indeed prove as a metatheorem the following general principle of comprehension:

$$\vdash s \in \{x|A(x/a)\} \equiv A(s/a)$$

though again the manifold $\{x|A(x/a)\}$ only satisfies the condition $A(\xi)$ when exactly one individual satisfies it, and this individual *is* the manifold.

Axiom of Choice

While the foregoing meta-axioms delineate a system with deceptive power, the following principle appears to be independent of them, and yet intuitively satisfactory, especially in the form given.

$$\text{a16 } \vdash \exists u A(u/a) \& \forall u \forall v (A(u/a) \& A(v/a) \& u \neq v \supset u \cap v \simeq \wedge) \\ \supset \exists u \forall v (A(v/a) \supset \exists ! x (x \in (u \cap v)))$$

What the axiom amounts to is this: if $A(\xi)$ is any condition in one variable satisfied by at least one manifold, and such that any two distinct manifolds satisfying it are pairwise disjoint, then there exists a manifold intersecting each manifold satisfying the condition in a single element. This is sometimes known as the weak or disjoint choice principle. It is hard to see how it could be questioned. In this form, the axiom is not really about choice or selection in any real sense: it is about the existence of certain manifolds.

In his 1908 paper on set theory,⁶⁷ Zermelo used the principle in just this disjoint form, although the pairwise disjoint sets were not those satisfying a condition, but those belonging to a set of sets. Notice that in our case we do not need to state that the sets be non-empty: this is taken care of by the variables, which range only over manifolds that have members. Zermelo's original 1904 proof of well-ordering⁶⁸ uses not this disjoint principle but the principle which he called in 1908 the General Choice Principle: that any set of non-empty sets possesses a choice function. In his 1908 paper on well-ordering he again uses the General Principle as premiss, but, as if by way of placation, assures that the General Principle is but a consequence of the Disjoint Principle,⁶⁹ while in his paper on the foundations of set theory the General Principle is derived as a consequence of his axioms. Now Zermelo gives the appearance of regarding the Disjoint Principle as more likely to secure acceptance from the sceptical, while proving that it is just as strong as the General Principle. But what he in fact shows is that the General Principle follows from the Disjoint Principle together with the other axioms of Zermelo's set theory. These include assumptions about set existence, especially the power set and infinity axioms, which are much stronger than we have employed. By these means, Zermelo is enabled to trade in arbitrary sets of sets for equinumerous but pairwise disjoint sets of sets, using pairs consisting of one element and one set. Such means are not

here available, nor indeed can the strong General Principle be formulated as one of set existence, which makes it appear rather different in kind from the Disjoint Principle. It has indeed been suggested that Zermelo's axiomatisation was motivated less by a desire to avoid paradoxes as to gain acceptance of the well-ordering theorem, in which the axiom of choice serves of course as premiss.⁷⁰

The interesting question left unanswered by this is whether, in the presence of weaker though still intuitively justifiable assumptions as to set existence, the General Choice Principle does not turn out to be stronger than Disjoint Choice.⁷¹

§ 5 Some Comparisons

The following remarks assess in broad outlines the affinities of the system presented in the previous section. In many respects the ideas, despite some obvious departures from current practice, represent a return to an older tradition, not fully distinguished in its time from general logic, namely that tradition running from Leibniz through Boole, Peirce and Schröder to Husserl, Lowenheim and Leśniewski, a tradition to be distinguished sharply from that running from Frege, Peano and Russell through to modern predicate logic on the one hand and from Cantor and Frege through Zermelo to modern axiomatic set theory on the other. Despite Russell's initial clarity about classes, he soon forsook that path in favour of a reduction of classes to propositional functions.

In many ways the present system is similar to Schröder's application of his calculus of identity and subsumption to domains taken in extension. Schröder developed a type theory of sorts.⁷² Church has suggested that this was essentially a substitute for a difference between set membership and set inclusion.⁷³ But Schröder introduces the type-like distinction rather to avoid paradoxes. These arise, in my opinion, through a lack of adequate understanding of the difference between a predicate's being applicable to a thing and a thing's being included in a domain,⁷⁴ together with an inability to handle empty terms. Schröder uses the symbol ' \in ' for subsumption: there are in his earlier, type-free system in addition the two domains 0 and 1 such that $0 \in a \in 1$ for every domain a . But Schröder does not distinguish between every element of one domain being an element of another, i. e. subsumption, and a subject's be-

ing characterised by a predicate. Thus he regards ' $0 \in a$ ' simply as signifying that '0 is subject to every predicate a '. Hence, if a predicate determines a domain a , then since $0 \in a$, that predicate applies to 0. So, in considering the predicate 'is equal to 1', Schröder regards the class of classes (domain of domains) satisfying this predicate as comprising just 1 and 0. But since 0 is subject to this predicate it follows that $0 = 1$, and all distinctions collapse: the night in which all cows are black.⁷⁵ Hence, Schröder concludes that classes of classes should be distinguished according to level from classes of individuals. That is indeed one way out, although it did not appeal to Frege. Frege suggested that Schröder's *Gebietenkalkül* was really only a theory of part and whole, and that in such a case there could be no null entity 0. I agree with the view that if this is how we interpret Schröder's system, as a mereology, then an empty individual indeed is out of place. But our system, like Schröder's, is intended not as a theory of part and whole but as a theory of extensions of terms. In such an extensional approach to classes, there should likewise be no null class, as Russell saw.⁷⁶ But we can retain the usefulness of Schröder's 0 without regarding it as an entity, by the scrupulous use of empty terms. Schröder's paradox does not arise in our system, even though we do not distinguish classes of classes from classes of individuals, because while $\Lambda \subset a$ for every a it does not follow that Λ exists, nor, if a is $\cup P u$ or $\{x|P x\}$, that $P \wedge$. *Pace* Frege,⁷⁷ the extension of a concept *does* consist of the things falling under it in the same way as a wood (as manifold, not group), consists of trees. Having on hand the concept of a manifold means that we can treat the extension of a concept as what it is: one or more individuals. An empty concept then is not a concept with an empty extension, if by this we mean that there is something, its extension, which happens to comprise no individuals. Rather, it is a concept without an extension.⁷⁸

The system of § 4 is a first-order system: we do not quantify over predicates. The differences, and complications, all arise from the introduction of plural terms and variables, with quantification over all manifolds, plural as well as singular. If the expressions involving terms other than singular are not employed, the remaining fragment is simply equivalent to a normal free logic, with ' \in ' equivalent to ' $=$ ', where ' $=$ ' differs from ' \simeq ' in not holding between terms which are empty.⁷⁹ On the other hand, the system cannot be proved consistent simply by interpreting all terms as singular (or empty), because it would be inconsistent when so interpreted: the axiom a10b would be interpreted as

$$\vdash \exists xw(x = h \& w = h \& x \neq w)$$

which is inconsistent.

The existing system of logic which our system most nearly resembles is Leśniewski's Ontology, sometimes called the calculus of names. In Leśniewski, names, like our terms, can designate one or more than one or they can fail to designate at all. On the other hand it is clear that Leśniewski's "names" comprise both what I should call terms, and common nouns. I had previously thought that the only possible interpretation of Ontology which made sense in terms of the sort of expressions to be found in natural languages was as a calculus of common nouns.⁸⁰ But it now seems to me that it can equally well be interpreted as a calculus of terms, whether these be singular, plural or empty. Leśniewski's calculus could be regarded as one involving solely neutral variables, with somewhat different principles and axioms governing quantification. In some Leśniewskian systems singular names are informally marked by use of capital letters, but this does not affect their substitutivity, which is why variables are all de facto neutral.

If we had adopted quantifiers without existential import, say \prod and Σ , such that $\Sigma uA := \sim \prod u \sim A$, subject to the axiom $\prod uA \supset A(r/u)$, where r is any term, empty or not, and axioms analogous to a2–a3, then we should have a ready way to interpret Leśniewskian expressions as follows:⁸¹

<i>Usual Leśniewskian Form</i>	<i>Interpretation</i>
$[a] \cdot A$	$\prod uA$
\in	\in
$\text{ex}(a)$	Er , or $\exists u(u \in r)$
$\text{sol}(a)$	$\exists^1 u(u \in r)$
$\text{ob}(a)$	$\exists^1_1 u(u \in r)$
\sqsubset (Strong inclusion)	\notin
\subset (Weak inclusion)	\subset
$=$ (Singular Identity)	$=$
\circ (Weak identity)	\simeq

I have preferred to develop the calculus of manifolds in such a way that it is recognisably an extension of the usual predicate calculus involving only singular and empty terms. The introduction of identity as a primi-

tive by a6—a7 seems especially preferable, since identity has stronger claims to be a logical relation than inclusion or membership, in terms of which it is usually defined in Lesniewskian systems. However, despite its unusual treatment of quantifiers, Ontology can be said to embody a theory of manifolds, although these cannot be construed as sets in the usual sense.⁸² Ontology could claim to embody a skeletal theory of extensions of expressions, whether these be construed as common nouns or as terms, exhibiting the algebraic similarities between a calculus of nouns and a calculus of terms. In this it could be said also to belong in the Boole-Schroder tradition. I should be unwilling however to give up the view that there is a syntactic difference between terms and common nouns, despite their many semantic similarities. Such an identification erases many distinctions to be found in the syntax of natural languages, even though these distinctions may not be strictly necessary for logical purposes. An enlargement of the present theory could introduce common nouns and quantifiers and descriptors adjoining them.⁸³

I have several times mentioned the quasi-Boolean properties of the calculus of manifolds. It is instructive to see how we can interpret the axioms in certain Boolean algebras. This has the advantage of enabling us to trade in some of the more unusual features, such as empty and plural terms, with quantifiers binding variables other than singular, for an interpretation in which all terms are singular and quantifiers are as in a normal first-order theory, without even empty terms to worry about. The system is also thereby shown to be consistent relative to the algebras in which it can be interpreted. As the simplest of these are finite, this is a heartening claim. Let us consider the particular case first, and then comment briefly on more general interpretations.

Consider any subset of the positive integers consisting of all the divisors of a number which is square-free, in the sense that it has no divisors of the form p^2 , for $p > 1$. The smallest such set is $\{1\}$, but there is no largest, so we can have models of any finite cardinality 2^n , where $n \geq 0$. Let M denote any such set of divisors, with subscripts e. g. M_{30} , to denote particular cases. We may interpret predicate parameters as predicates defined over M , though we shall not in general be interested in arbitrary predicates. We interpret term parameters as follows:

Singular parameters are assigned either 1 or a prime number.

Plural parameters are assigned either 1 or a composite number.

Neutral parameters are assigned any number.

Variables are interpreted to range over M .

Quantifiers are interpreted as follows. If A is any condition in one free variable, suppose A' is the associated condition defined over M .

A formula $\forall xA$ is true on the interpretation iff A' is satisfied by all prime numbers in M .

A formula $\forall hA$ is true on the interpretation iff A' is satisfied by all composite numbers in M .

A formula $\forall uA$ is true on the interpretation iff A' is satisfied by all numbers greater than 1 in M .

It is to be noticed that all universal quantifications are vacuously interpreted as true on interpretation over the domain M_1 .

Descriptions are assigned values in M as follows:

If a single prime number in M satisfies A' , then ιxA is assigned that number, otherwise it is assigned the number 1.

If at least one composite number satisfies A' , then ιhA is assigned the lowest common multiple of all those composite numbers satisfying A' (which is in M , by choice of the sort of set M is), otherwise it is assigned 1.

If at least one number greater than 1 satisfies A' , then ιuA is assigned the l.c.m. of all those numbers that do, whether prime or composite, otherwise it is assigned 1.

Notice that, as with parameters, 1 is playing the role of a null manifold, prime numbers are playing the role of individuals, and composite numbers the role of pluralities.

We interpret primitive formulas involving ' \simeq ' and ' \in ' as follows:

A formula $a \simeq b$ is true on the interpretation iff a and b are assigned the same number by the interpretation (so we are interpreting ' \simeq ' as '=').

A formula $a \in b$ is true on the interpretation just in case a and b are assigned numbers a' and b' such that (i) $a' \neq 1$ (ii) a' divides b' .

From these it follows that a term $\{x|A\}$ is assigned the product of all the prime numbers satisfying A' (which, by construction, is in M), or else 1.

It may then be checked that on any such interpretation all the axioms a1–a16 come out as true, indeed logically true. a1–a6 are quite straightforward, being valid according to the usual principles of quantification and identity in any first-order theory. a8–a16 get interpreted as follows:

a8: if a' divides b' (where we shall assume that when we say one number 'divides' another, that it is also $\neq 1$), then $a' \neq 1$ and every prime factor of a' is a prime factor of b' .

a9: if a' and b' have the same prime factors, they are equal.

a10: prime numbers have exactly one prime factor, composite numbers have at least two, and numbers greater than 1 have at least one prime factor.

a11: these are conditions for the number assigned to a description to be $\neq 1$: that they are met can be seen by checking the conditions for assigning numbers to descriptions given above.

a12: if a number corresponding to a description is greater than 1, then every prime factor of any number meeting the associated condition A' divides this number. This is so by construction of the number as prime or l.c.m.

a13: if a prime number divides the number assigned to a description, then it divides some number satisfying the associated condition.

a14: if the number assigned to a description has no prime factors, then it is equal to 1.

a15: every number is equal to the product of all the numbers equal to itself, or, if prime, then equal to itself, or, if 1, then equal to 1.

a16: if some number greater than 1 satisfies a given condition A' , and all the numbers that satisfy A' are pairwise relatively prime, then there exists a number in M such that its common factor with every number satisfying A' is a prime number. That this is so is easily seen. Since the numbers satisfying A' are relatively prime in pairs, if we select one prime factor from each, say the smallest, then no prime is selected twice, and the product of all these primes is in M and satisfies the condition by construction.

The sets M_n form Boolean algebras under division as the partial ordering. This suggests that we could model the calculus of manifolds generally in any Boolean algebra. However, the algebra must satisfy certain conditions for the interpretation analogous to that given above for the finite algebras M_n to go through. This interpretation is a particularly straightforward and appealing one. Let B be any Boolean algebra, with distinguished elements 1 and 0, under the partial ordering \leq . Let us suppose further

(1) that B is *atomic*, i. e. for all elements $b \in B$, there is an element $a \leq b$ such that for all $c \in B$, $c \leq a$ implies $c = 0$ or $c = a$.

(2) that B is *complete*, i. e. for any non-empty subset A of B , a supremum $\sup A$ exists relative to \leq , that is, an element $s \in B$ such that (i) for all $a \in A$, $a \leq s$, and (ii) for all $s' \in B$ such that $a \leq s'$ for all $a \in A$, $s \leq s'$.

(3) that B is *distributive*, i. e. for every subset A of B which is not emp-

ty, and for every element $b \in B$, $b \cap \sup A = \sup\{b \cap a \mid \text{such that } a \in A\}$.

We can now sketch how interpretation in any such algebra B will go: since the details are similar to the finite cases M_m , we can be brief. Predicate parameters are assigned predicates defined over B , term parameters are assigned elements of B : singulars to atoms or 0, plurals to non-atoms or 0, neutrals to anything. Universal quantifications are true in these cases: singular variable bound: true iff every atom satisfies the associated condition, plural: true iff every non-atom $\neq 0$ satisfies it, neutral: true iff every element $\neq 0$ satisfies it. Descriptions are assigned elements of B as follows: if A'' is the set of elements of B satisfying the associated condition, then singulars are assigned 0 unless A'' is a singleton whose element is an atom, when this is assigned to the description. Plurals are assigned the supremum of the set of all non-atoms satisfying the condition, or else 0, and neutrals are assigned $\sup A''$ if $A'' \neq \emptyset$, or else 0. The completeness property assures that such a supremum exists where the set is not empty. The distributive property assures that suprema behave nicely in formulas. It corresponds to the following metatheorem of the calculus:

$$\vdash b \cap \sup A(u/a) \simeq \sup\{v \mid \exists u(A(v/a) \& v \simeq b \cap u)\}$$

The axioms for manifolds can then be verified to be valid for all such Boolean algebras. The Axiom of Choice is interesting, because while its proof was trivial in the finite case, to prove the validity of its interpretation in the general case, where B may be infinite, requires – unsurprisingly – the disjoint choice principle. For the interpretation comes to this: if $A \subseteq B$ is a subset not containing 0, such that for any distinct elements $a, b \in A$, $a \cap b = 0$, then there is an element $c \in B$ such that for all $a \in A$, $a \cap c$ is an atom. To see how it is proved, consider any such set A whose elements are pairwise relatively atomic. For each element $a \in A$, let $A(a)$ be the set of atoms $\leq a$. Since $0 \in A$, $A(a)$ is non-empty in each case, and, since if $a \neq b$ are both in A , $a \cap b = 0$, so $A(a) \cap A(b) = \emptyset$. Applying the disjoint choice principle to the $A(a)$, we select an atom from each. Let the resulting set of atoms be C . By completeness, $\sup C$ exists, and has the property that $a \cap \sup C$ is the selected atom in $A(a)$ for all $a \in A$, proving the result.

It is known that all Boolean algebras may be represented by an isomorphic algebra of subsets of some set, but in addition, if the Boolean

algebra is atomic, complete and distributive, in the senses given above, it is isomorphic to the algebra of all subsets of the set of atoms.⁸⁴ With this we come full circle.

I have also recently discovered that it is possible to interpret manifold calculus in ordinary whole-part theory. We simply interpret all terms as singular, and the relation ' \simeq ' as ordinary singular identity in a free logic, and the relation ' \in ' as the ordinary part-whole relation, so interpreted that only existents can be parts. The resulting calculus of individuals differs from that of Leonard-Goodman only in that it allows empty terms: a perfectly laudable difference, and that it is (according to axiom a10) *atomistic*, which is not necessarily so laudable. We can then interpret singular terms as designating *atoms*, plural terms as designating *non-atoms*, and neuter terms as designating all individuals, atomic or not. The only difficulty concerns the description operator, which does not readily generalise to the normal description operator. In fact, for plurals and neuters, the description operator represents the Leonard-Goodman *sum* or *fusion* operator. This difficulty can be removed by defining a new operator: let us confine ourselves solely to neutral terms here:

$$JuA := \iota u(A \ \& \ \forall v(A(v/u) \supset v \simeq u))$$

It is then the operator J which generalises under the mereological interpretation to the normal description operator. We can give axioms rather for J than ι , which are symbolically exactly analogous to those for van Fraassen and Lambert's system FD₂, and then define ι as follows:

$$\iota uA := Ju\forall x(x \in u \equiv \cdot \exists v(A(v/u) \ \& \ x \in v))$$

where we assume we have already defined ' \in ' through ' \in '. This now preserves perfectly the parallel with the fusion operator of the normal calculus of individuals. As to be hoped and expected, under the present interpretation, ' \wedge ' remains an empty term, unlike the case when we interpreted the calculus in Boolean algebras. This agrees naturally with the intuition that there are no null heaps, as Frege pointed out in his Schröder review, and the difference is perfectly congruous with Tarski's demonstration that mereology is Boolean algebra save for a Boolean zero. Of course this heartening symbolic parallel between the axiom systems in no way reduces manifolds to heaped individuals: far from it. In

an enriched language having both plural terms and a part-whole predicate, there would be things we should wish to say that we could not say if that were so, e. g. that no plurality is an individual, and that no mereological sum is a plurality. All the reasons I adduce in "Number and Manifolds" for rejecting the group theory of number here rise up again to refute the identification of manifolds with heaped individuals. In particular, the unheapability of such items as impossible possibilities, and the generally wider applicability of the notion of manifold than that of mereological sum, applications of which are predominantly confined to the physical sphere, speak loudly against such an identification. So the subsumption relation and the whole-part relation, whatever their algebraic similarities, must always be distinguished. A square built up out of four other squares has each of the four component squares as parts: it is their sum. But it is not identical with the *squares*, for there are four of them, and only one of it. Nor is a part of one of the squares (a proper part) one of the four squares, while it is part of the one square. So the relations 'is one of' and 'is part of' are quite different. Whoever appreciates this will have no problems about the one and the many. The main *axiomatic* difference between manifold theory and whole-part theory consists in the self-evidence of the fact that all manifolds consist of individuals, and the lack of self-evidence of the proposition that all individuals consist of atoms, i. e. Axiom a10. It is worth recalling in this connection the independence of the atomic hypothesis from general mereology in Leśniewski, while the requirement that manifolds always reach back to individuals recalls the necessity felt for Miriamoff's grounding axiom in ordinary set theory.

§ 6 Sets as Representatives of Classes⁸⁵

Stenius⁸⁶ suggests that the most plausible way to regard sets-as-things, as he calls them, or classes as one, is to regard them as individuals arbitrarily assigned to serve as representatives of, go proxy for, classes. He develops the idea that the relation of representation can be seen as a genuine relation between individuals in the domain, with the membership relation \in being considered as the converse of representation. In this way the formal results of the theory of sets may be preserved, without engendering the problems of the trinitarian conception of sets. The idea is

appealing: if all the mathematician wants is *some* object to do the job of sets, why not let him have an individual as proxy-object, subject simply to certain conventions on how to assign such proxies.

The idea is not new, however. Frege's *Wertverläufe* are precisely individuals which do service for functions, and have the added advantage of being saturated entities.⁸⁷ Frege's realism induced him to worry about what such *Wertverläufe* were: he was unable to take the conventionalist step of letting them be arbitrarily assigned subject to conventions. That some restrictions were necessary Russell found to Frege's cost. In a late paper of 1940, Löwenheim⁸⁸ suggested the 'Schröderisation' of mathematics by using individuals to represent classes, subject to restrictions analogous to those of axiomatic set theory to avoid paradoxes. Bernays reviewed the article quite favourably, which is not too surprising, since, as we have seen, his classes can be regarded as representatives of *predicates*, and some of these classes may themselves be represented by sets. The axioms of set theory would then take the form of conditions on how individuals may represent classes.⁸⁹

It is interesting to see how such representation may be combined with a formal theory of manifolds as already presented. As will become clear, there are various possible ways in which representatives might be assigned. Looked at in this light, the different axiomatic set theories could be looked on not as different speculations as to what there is, but as alternative conventions, choice among which would be a matter of expediency rather than metaphysical anguish.

We shall not treat representation in detail, but sample a few of the leading ideas which would need to be developed in order to further the concept of sets as representatives.

The first point to note is that, assuming that the domain of individuals contains some fixed number α of individuals, by Cantor's diagonal argument, we should never have sufficient individuals at our disposal to represent, all distinctly, all the manifolds of individuals there are except in the trivial case when $\alpha = 1$, when it is true that $\alpha = 2^\alpha - 1$. So either every manifold gets an individual, but sometimes distinct manifolds get the same individual, as representative, or else not all manifolds are represented. This applies most obviously to finite domains: in a domain of 2 individuals there are 3 distinct manifolds, for instance.

Let us then introduce a new primitive relation ' \triangleleft ', where $a \triangleleft b$ is to be understood as meaning that *a* represents *b*. Now if any manifolds could represent others, we should trivially be able to use each manifold

as its own representative. But more interesting is the case where only individuals are representatives.

How is representation to be arranged? One obvious suggestion is that no manifold should have more than one representative:

$$r1. \forall xw\forall u(x \triangleleft u \ \& \ w \triangleleft u \supset x = w)$$

while a second is that no individual should represent two distinct manifolds:

$$r2. \forall x\forall uv(x \triangleleft u \ \& \ x \triangleleft v \supset u \simeq v)$$

These are in no sense metaphysical truths: they are stipulations. It would not be *false* for either of these not to hold, any more than it is false that there are two Senators to every State of the Union, or that the Queen is Head of more than one State. But we cannot combine r2 with universal representation, r3:

$$r3. \forall u\exists x(x \triangleleft u)$$

(except in the case of the one-member domain). For consider the manifold r defined as follows: $r := \{x | \exists u(x \triangleleft u \ \& \ x \notin u)\}$. Then on any domain with more than one member, r must exist, for suppose every representative were included in the manifold it represents. Then, since every manifold is represented, by r3, all three submanifolds of $\{s, t\}$ must have representatives in $\{s, t\}$, which they can only do if one of the representatives represents more than one manifold, contrary to r2. So r has at least one member. Suppose $s \triangleleft r$. By the theorem of comprehension, $s \in r = \exists u(s \triangleleft u \ \& \ s \notin u)$. If $s \notin r$ then s satisfies the condition because r exists, so $s \in r$. But then s must satisfy the condition of being a representative of some manifold it is not a member of. By r2, this must be r , so $s \notin r$, a contradiction. This is an exact analogue of the Cantor-Russell diagonal argument, and makes the point made above without recourse to the cardinality of the domain except that it must be greater than 1.

So some restrictions on representation are necessary. It is usual to have representatives only for the smaller, more tractable classes. This is the way of ZF and NBG set theory. Or we could restrict representation of classes which are the extensions of conditions of the form $\{x | A\}$ to

cases where the syntactic form of the condition is of a certain simple kind. This is the way of Quine.⁹⁰ We might have an individual which does not represent any manifold. If we have *only* one such, then it could be regarded as an analogue of the empty set. This can be expressed thus:

$$r4. \exists x (\sim \exists u(x \triangleleft u) \& \forall w (\sim \exists u(w \triangleleft u) \supset x = w))$$

In such circumstances, every individual other than this one, which we shall call \emptyset , is a representative. This provides an analogy with so-called pure set theory, where there are no individuals which are not sets. It would not be too inappropriate to regard $\emptyset \triangleleft \wedge$ as true in such circumstances. Pure set theory seems an extraordinary artifice in normal set theory, but its analogue in representative theory is no more than a recipe for not wasting individuals by having them not represent.

We may now see what an analogue of a set of sets is. It is simply a representative of a manifold of representatives. A theory of types among representatives would be a recipe for partitioning representatives and other individuals so that there would be manifolds u_0, u_1, u_2 , etc. with u_0 comprising individuals not representing anything (*Urelemente*), u_1 comprising representatives of manifolds included in u_0 , with perhaps an extra non-representative to serve as an empty representative, u_2 comprising representatives of submanifolds of u_1 , and so on. If we wished to continue indefinitely we should need to be assured of an infinite supply of individuals. Such a proposal is quite restrictive: it does not allow mixing of types, and every representative of a singleton is distinct from, and one type higher than, the individual it represents. On a countable domain with finitely many *Urelemente* every representative allowed by the theory could be forthcoming: n for *Urelemente*, the next 2^n for first-order representatives, the next 2^{2^n} for second-order representatives, and so on. But notice that not every manifold of individuals in the domain gets represented: there are not enough individuals to go round. Even u_1 will have gaps in it if the domain and the *Urelemente* are both of the same transfinite cardinality.

Systems of set theory designed to serve as foundations for mathematics all have axioms of infinity. It is important to notice that no such axiom is included in our calculus of manifolds. If we require that \triangleleft be irreflexive:

$$r5. \forall x(x \triangleleft a \supset x \neq a)$$

and add the further recursive requirement

$$r6. \exists u \forall x (x \in u \supset \exists w (w \triangleleft x \ \& \ w \in u))$$

then this can only be satisfied on an infinite domain. In particular, if representation is single-valued, satisfying r1, then it may be considered a partial function, and for any manifold u which is represented, we may denote its unique representative by $[u]$. Then r6 may be expressed as

$$r6a. \exists u \forall x (x \in u \supset [x] \in u)$$

If u is any given manifold, let the manifold generated from u by taking representatives of its members, representatives of these representatives and so on, be designated $Z(u)$. Then if there is a null representative \emptyset , the manifold $Z(\emptyset)$ is the manifold $\{\emptyset, [\emptyset], [[\emptyset]], \dots\}$, which is of course Zermelo's model for the natural numbers, or rather, an analogue of it.

If u is represented, let us represent this fact by the predicate R :

$$Ru := \exists x (x \triangleleft u)$$

One sensible stipulation regarding representation is that it be closed under the taking of submanifolds:⁹¹

$$r7. \forall u (Ru \supset \forall v (v \in u \supset Rv))$$

Another is that whenever a number of manifolds are represented, so is their union:

$$r8. R(\cup (Ru \ \& \ A(u/a)))$$

where A is some condition on manifolds. In particular, selecting the condition $a \simeq a$, r8 yields the result that the union of represented manifolds is represented, $R(\cup Ru)$. This is a sort of closure condition. We can get another sort in the following way. Let S be the predicate 'is a representative':

$$Sx := \exists u (x \triangleleft u)$$

then we could require that all manifolds of representatives be represented:

$$r9. \forall u(u \in \{x|Sx\} \supset Ru)$$

We may set up relations among representatives analogous to those holding among sets in ordinary set theory. For instance let η, κ be relations defined as follows:

$$s \eta t := \exists u(t \triangleleft u \ \& \ s \in u)$$

$$s \kappa t := \forall x(x \eta s \supset x \eta t)$$

Then η and κ are analogues of the membership and subset relations respectively. However, $s \kappa t$ & $t \kappa s$ only entail $s = t$ if r1 and r2 are satisfied. We can formulate as a stipulation an analogue of the power set axiom as follows:

$$r10. \forall x(Sx \supset R(\{w|w \kappa x\}))$$

while an analogue of the axiom of regularity is

$$r11. \forall x(Sx \supset \exists w(w \eta x \ \& \ \sim \exists x'(x' \eta w \ \& \ x' \eta x))$$

Given an infinite domain, single-valued representation and a null representative \emptyset , with this axiom we know that providing representatives are forthcoming at every stage, a manifold N such that (i) $\emptyset \in N$ (ii) $\forall x(x \in N \supset \{x, \{x\}\} \in N)$ and no other members besides those required by (i) and (ii), would furnish an analogue of von Neumann's version of the natural numbers. It would be the manifold $\{\emptyset, \{\emptyset\}, \{\{\emptyset, \{\emptyset\}\}, \dots\}$, and the relation $s \eta t$ among its members would be the natural ordering $<$.

Enough has perhaps by now been said to suggest that mixing manifolds with representatives offers a reasonable promise for keeping distinct Russell's and Cantor's two concepts of class, while not incurring the burdens of a Platonic ontology.⁹²

§ 7 Concluding Remark

"Sets", says Quine, "are classes... 'set' is simply a synonym of 'class' that happens to have more currency than 'class' in mathematical con-

texts".⁹³ Waiving the temptation to ask why Quine of all people should speak of synonyms, we might ask what underlies the claim. It is, I think, that there is identity, or at least continuity, between the mathematical concept of set and the familiar intuitive notion of a class. Modern set theory attempts to bite off as much of Cantor's Paradise as possible without biting off contradictions. It is worth asking whether in the process it has not forgotten what a class really is.⁹⁴

Notes

- ¹ Black, 1971, Stenius, 1974. It was from Black's paper that I obtained the view that plural terms and sets are counterparts, although, as I later discovered, Russell had arrived at the same idea much earlier. The extent of my agreement and disagreement with Black and Stenius (who are by no means in complete accord) will become clear through this paper. While I find that on the whole, their destructive comments are more successful than their constructive proposals, it still seems to me that they have been somewhat unfair to the earlier tradition of set theory, strands of which, as I show, come close to solving the difficulties. It is perhaps a reflection on the ahistorical way in which set theory is read and taught today that such strands should have been so completely overlooked.
- ² Cf. the remarks on this in the previous essay.
- ³ Abandoning the 'only if' part leads to Lesniewski's theory of 'collective classes', i. e. mereology. This kind of class is precisely Russell's class as one, for which see below. Lesniewski distinguishes collective from distributive classes. The latter do obey the extensionality principle. In Lesniewski however this is not a special set theory, but just the logic of names. It is interesting that Lesniewski was led to collective classes by consideration of Russell's paradox, and took a class as being most naturally conceived as the mereological sum. In view of the problems of the trinitarians, this is a natural attitude for anyone with nominalist inclinations. However, the calculus of manifolds, which I contend captures the notion of *class* rather than that of *whole*, bears affinities with Lesniewski's calculus of names, or 'ontology'. It also contains nothing a nominalist could find offensive.
- ⁴ Hence Leonard and Goodman's version of mereology, which they call the "calculus of individuals", might be thought well-titled. I am not convinced however, that masses of stuff (including limitlessly dispersed masses), which are amenable to mereological treatment, indeed cry out for it, are most aptly called 'individuals', since this term seems to apply most naturally to things falling under count concepts, whereas stuff falls under mass concepts. If there were a special grammatical form for mass nouns, distinguishing them from singular count nouns, then we should I think be far less inclined to heap masses and individuals together. However, this is a point with far-reaching consequences and ramifications, and cannot be pursued here. It should be emphasised that 'manifold' is to be understood in this paper as comprehending both individuals and pluralities. There is no difference between an individual and a single-membered manifold: the member *is* the manifold.
- ⁵ The predicate in manifold theory most closely analogous to '<' is not ' \in ' but ' \in '. The manifold-theoretic notion of an individual is analogous to the mereological notion of

an atom. But manifold theory and mereology part company over this notion, for, if there are to be manifolds, there must be individuals (which might be called relative atoms) to comprise them, whereas the existence of composite entities does not, *pace* Leibniz, Wittgenstein etc., entail that there must be absolute atoms, i. e. entities without proper parts.

⁶ The relation '∈' is one example of a predicate which is, in the terminology of the previous essay, not perfectly distributive. More precisely, the predicate ' $a \in \xi$ ' does not distribute over manifolds, because from ' $a \in b$ ' and ' $c \in b$ ' it does not follow that ' $a \in c$ '. It is also clear that the relation '∈' is an ideal or *formal* relation, in the sense that 'exists' is a formal property, i. e. corresponds to no material property in the thing(s) concerned. In Kantian terms, '∈' is "no real predicate", arises simply from *as* being among the things designated by '*b*', for instance.

⁷ With Zermelo's axiomatisation, set theory became just another mathematical theory, albeit a very basic one. But the logicist intuition that in some sense 'class' is a fundamental *logical* notion, not a general mathematical one, deserves a better run for its money, provided, naturally, that the intuition can be separated from the familiar paradoxes.

⁸ Letter to Dedekind, Cantor, 1899. So when Mostowski, 1966, p. 141 speaks of Cantor distinguishing between consistent and inconsistent *sets*, this is seeing Cantor through the eyes of von Neumann and Gödel. In fairness to Mostowski, Cantor occasionally talks of consistent pluralities *being* (rather than forming) sets, but it is also clear from the context that this is loose talk.

⁹ The contrast with one such 'working mathematician', Felix Hausdorff, could not be greater. In his justly celebrated book, Hausdorff, 1914, he passes the paradoxes by with a cursory wave. In a recent article, Moore, 1978, G. H. Moore has shown convincingly how Zermelo's attitude was also that of a working mathematician, and that he was spurred to axiomatise set theory not to lay the ghost of the paradoxes but to provide a convincing proof of the well-ordering theorem using as weak a choice principle as possible, to gain the assent of the community of mathematicians, who had remained unconvinced by his earlier proof. For more on the weak principle, see § 4 below.

¹⁰ In his letter to Dedekind, Cantor suggested the following principles:

(1) Two equinumerous pluralities are either both inconsistent or both consistent (Cantor in fact says, 'are both "sets"', which is an example of the sort of remark mentioned at n. 8 above).

(2) Wherever we have a set of sets, the elements of these sets again form a set (*not* loose talk). (Union principle.)

(3) Every sub-plurality of a set is a set.

The first property was made in von Neumann, 1925–6, a characteristic of the difference between sets and ultimate classes: an ultimate class (to use Quine's felicitous term) is one which is equinumerous with the class of all sets, which cannot be a set, by Cantor's diagonal argument (as Cantor recognized).

¹¹ Russell, 1903, § 74, p. 76.

¹² *Ibid.*, § 104.

¹³ *Ibid.*, § 74. Russell here suggests that the class as one may be identified with the whole composed of the terms of the class, cf. § 139. This has the effect of allowing that more than one class as many may correspond to the same class as one. It also runs into difficulties about heaping together pluralities whose members come from widely different ontological domains.

¹⁴ Russell changed his mind, between writing about classes in the body of the book, probably in 1900–1, and writing the Appendix on Frege, late in 1902, about the strength of the Peano-Frege argument. My sympathies are, as I hope is clear, with his first thoughts.

¹⁵ *Ibid.*, § 104.

¹⁶ *Ibid.*, § 74.

- ¹⁷ Ibid., § 71. Note the widespread use of the concept of 'Und-Verbindung' by psychologists of the period, e. g. in Husserl, 1891a, p. 75f, or in the essay by Reinach below, § 15.
- ¹⁸ Ibid., §§ 70, 74, 490. Russell however does I think distinctly favour the idea of there being propositions with more than one subject. It may be that there is interference between the linguistic idea of a single subject-expression, and the semantic idea of a proposition 'being *about* one or many things. Even a relational predication is *about* more than one thing, but unless the relation is expressed conjunctively (cf. the previous essay) these things are not all designated by one and the same subject-expression.
- ¹⁹ Ibid., § 70.
- ²⁰ Ibid., § 486.
- ²¹ Ibid., § 489.
- ²² Ibid., Appendix B. What is ironical about this is that the theory of types in the body of the book is motivated solely as a distinction between ones and many's, and rests on there being certain things which can be said of ones which cannot be said of many's and vice versa (§ 104). But it is of the essence of many's that they cannot be *members* of any class (ibid.), whereas all classes in the theory of types may be members of classes of the next higher type. So the theory of types enters at the expense of the one/many distinction, though it enters on the back of that distinction. There is therefore no justification for an *infinite* type hierarchy (§ 490), or even classes of classes.
- ²³ Cf. Miriamoff, 1917, Fraenkel, 1922.
- ²⁴ Von Neumann, 1925–6. The treatment is conducted entirely in terms of functions, but later commentators almost invariably present it more conventionally.
- ²⁵ In Bernays, 1937–54, these are symbolised '∈' for set membership, and 'η' for class membership. In § 6 we use the same pair of symbols in what is effectively the opposite way round.
- ²⁶ Bernays and Fraenkel, 1958, 41–2.
- ²⁷ Russell, 1903, § 489. The idea of representatives is further examined in § 6 below. Bernays also speaks of a set as *representing* a class in his 1937–54. A set *a* represents class *A* when $\forall x(x \in a = x \eta A)$. It is a consequence of his axioms that every set represents a unique class, but of course not every class is represented by a set.
- ²⁸ This can be seen in part by the circumstance that Bernays does not quantify over classes, preferring always class parameters (free variables). Levy, 1973, p. 196 describes the move as one of replacing the metamathematical notion of a condition by the mathematical one of a class, while in the preface to his 1976, Müller reports that, unlike von Neumann, Bernays did not regard classes as real mathematical objects (p. vii). Levy describes this reluctance as 'not taking classes seriously', 1976, p. 205. That others have 'taken classes seriously', to the extent not only of quantifying over them and defining them impredicatively, but even considering their being elements of 'hyperclasses' – none of this can be laid at the feet of Bernays, who is always on stronger ground philosophically than those writers who block membership *solely* to prevent paradoxes from arising.
- ²⁹ Cf. my 1978. Other writers to "take common nouns seriously" include Lewis, 1970. A predicate is, after all, a sentence save some names (terms): if common nouns were predicates, then '*John man' should be a sentence, and if they were proper names, '*Tree is rotten' would be an acceptable sentence of English. The situation may not be so clear with other languages, but in English there is a clear syntactic difference between proper and common noun categories. Cf. the fuller remarks in the text below.
- ³⁰ It is interesting in this connection that Leśniewski's Ontology is often (and in my view preferably) called a calculus of names. Cf. § 5 below.
- ³¹ Cf. my 1978. Although predicate logic was developed primarily to answer the sentence-forming requirements of mathematicians, it is noticeable that (mathematical texts no more avoid common nouns than other natural-language works. But since the official

formal syntax of modern mathematics does not use common nouns, their role is in part assumed by set-theoretic expressions. After so many years of familiarity with formal languages there is no reason why a fully adequate formalisation of noun-using mathematical language cannot be devised. No attempt has been made in this paper to do so, for this would involve greater complexity and unfamiliarity. Also, the arguments for accepting manifolds are independent of the use of common noun expressions.

³² Black, 1971, p. 104 in the reprint.

³³ 'Group' is to be taken here neither in the sense of McTaggart, 1921/7 nor that of Sprigge, 1970, nor, of course, in the mathematical sense.

³⁴ Stenius, 1974.

³⁵ Biologists evade the problem neatly by distinguishing between *Protozoa*, single-celled animals, *Metazoa*, multi-celled animals with two layers of cells, and sponges, which are set on one side as *Parazoa*.

³⁶ Such a view has to treat the identity of groups or individuals in flux as somehow second-rate. An obvious alternative, but one to be examined gingerly, is the view that there is also, or only, sortal-relative identity. Cf. Wiggins, 1967 or Griffin, 1977. In view of the distinctions between individuals, groups, wholes and classes made here I am hopeful that no such drastic expedient will be necessary.

³⁷ The view that the only genuine objects are those which can neither gain nor lose parts has a long history: it can be found in Leibniz and Hume, and has been defended most vigorously under the title of 'mereological essentialism' by Chisholm, e. g. 1976. For a rebuttal of this view, see Wiggins, 1979.

³⁸ It does of course entail a disjunction. Whether or not plural reference is eliminable, it is certainly useful. In any case, theoretical eliminability of certain kinds of expression, whether names, or variables, appears to me to carry ontological consequences only if it is supposed that ontology can be in some way "read off" linguistic facts.

³⁹ Cf. Russell, op. cit., §§ 68, 79, on 'is/are among'.

⁴⁰ Cf. § 5 below.

⁴¹ Russell, *ibid.*, § 74. But cf. his back-peddalling at § 491.

⁴² Cf. the remarks at n. 22 above. It is arguable that what Russell understood under the term 'theory of types' underwent changes, apart from the obvious one of the introduction of ramification, between 1903 and 1908. In that time, Russell was not always enamoured of the type-theoretic way out, advocating, not always at different times, at least three alternatives: the 'limitation of size' theory, anticipating ZF and NBG axiomatics, the 'zigzag' theory, anticipating Quine's NF, 1937, and most radically of all, the 'no class' theory, which took class expressions as incomplete symbols (Russell, 1973). Nothing illustrates more vividly the fecundity of Russell's intellect during this period than the apparent ease with which he could throw off radically new ideas.

⁴³ Cf. Russell, op. cit., § 71.

⁴⁴ As noted in n. 4, the concept of manifold includes also individuals; the word *plurality* is used for manifolds with more than one member. No single term bridges the gap between individuals and pluralities very well so the term 'manifold' is as good as any. But, so long as it is understood that the term is not understood as in the recent philosophical logical tradition, the term 'class' may be substituted for 'manifold' by anyone who finds the latter term barbaric.

⁴⁵ On the difference between 'anything' and 'everything' cf. my 1978.

⁴⁶ On syntactic connection cf. Husserl, LU IV, Ajdukiewicz, 1935, and other texts on categorical grammar, such as Lewis, 1970 or Cresswell, 1973.

⁴⁷ Just how messy can be seen by consulting Routley and Goddard, 1973.

⁴⁸ There are in fact two possible neutral identity predicates, one carrying, the other not carrying, existential import. Cf. the definition of ' \cong ' in § 4 below.

⁴⁹ Cf. Bernays and Fraenkel 1958, p. 49. But Bernays' solution is as artificial as Frege's.

- ⁵⁰ Frege, 1893, § 11.
- ⁵¹ That giving up bivalence may not be irredeemably problematical may be seen by consulting e. g. Humberstone and Bell, 1977. But complications of the sort their proposals involve ought to be resisted unless they are forced upon us.
- ⁵² See e. g. the introduction of the null set in Hausdorff, 1914.
- ⁵³ See Routley, 1966.
- ⁵⁴ See e. g. Henry, 1972, Part II.
- ⁵⁵ On the interpretation of the quantifiers in Leśniewski see e. g. Kūng, 1977. Orenstein, 1978 has disputed Kūng's contention that Leśniewski's quantifiers are not substitutional (Appendix B), but it turns out that 'substitutional' has more than one possible meaning. At any rate, the quantifiers are certainly not objectual in Quine's sense.
- ⁵⁶ See Van Fraassen, 1966, Van Fraassen and Lambert, 1967.
- ⁵⁷ As e. g. in Thomason, 1970, Ch. V, § 5.
- ⁵⁸ See Quine, 1940, §§ 14, 16.
- ⁵⁹ See Church, 1956.
- ⁶⁰ For a convincing defence of this, cf. Hintikka, 1959.
- ⁶¹ Schröder uses the symbol ' \Subset ' in his 1890–1905, and the symbol remained in use for some time afterwards, e. g. with Löwenheim and Zermelo, but then dropped out in favour of ' \subset ' or, more usually today, ' \subseteq '. Schröder designed it as a combination of a sign for identity and one for proper inclusion. We do not use it in that sense, since for us ' $a \Subset b$ ' is only true when ' a ' is not empty. It can be seen more as a generalization of the sign ' \in ' for membership or singular inclusion to all cases of non-empty inclusion, proper or improper.
- ⁶² Quine 1963, § 4.
- ⁶³ It will be noticed that the axiom of choice, a16 below, is in fact an axiom *schema*, since it uses predicate parameters. In this, the theory resembles ZF.
- ⁶⁴ Cf. Stenius, 1974. It seems to me that Stenius is here rather bent on preserving as much of orthodox set theory from the flames as possible. Black, too, seems to be too ready to allow orthodox set theory as a legitimate development of the naive theory suggested by plural reference, rather than as embodying distortions leading away from the original intuitions.
- ⁶⁵ Stenius, in his endeavour to pick up Cantor's result that to any set there are 2^n subsets if the set has n members, overlooks the other possible subsets of the power set, although more general "second-order" counting procedures could be added to his to allow for these.
- ⁶⁶ It is still preferable to treat identity separately first. A similar-looking metatheorem is $\vdash a \approx 1(u \Subset a)$, which identifies every set with its power set.
- ⁶⁷ Zermelo, 1908.
- ⁶⁸ Zermelo, 1904.
- ⁶⁹ Zermelo, 1908.
- ⁷⁰ See Moore, 1978.
- ⁷¹ Hints that the disjoint choice principle, which Russell called the multiplicative axiom, might, in an environment of axioms for set theory weaker than, say, ZF, be strictly weaker than the full axiom of choice, arise out of various oddities in set theory. For example the proposition that every Boolean algebra has a maximal ideal, which is equivalent to Stone's representation theorem for Boolean algebras, has to date only been proved using the axiom of choice. But it is known (Halpern and Lévy, 1971) that the prime ideal theorem does not entail the principle of choice. It is notable that in our interpretation of what 'class' means, the general principle can only be stated using the concept of a function, while the weaker principle uses only the more general notion of a predicate or condition.
- ⁷² Schröder, 1890–1905. Cf. Church, 1976.

⁷³ Church, *ibid.*

⁷⁴ There is indeed a considerable difference between a thing's falling under a concept and a thing's being included in a class. Frege was quite right to insist that the latter must be separated from the subordination of one concept to another, but there is nothing wrong in treating membership as singular inclusion.

⁷⁵ As Schroder says (p. 245), 'hier wäre dann alles "wurst".'

⁷⁶ Russell, 1903, § 73.

⁷⁷ Cf. the final section of Frege's review of Schroder, 1890–1905, Frege, 1895.

⁷⁸ Interesting discussion comes from a perhaps unexpected quarter in G. E. Moore's *Commonplace Book* (Moore, 1962), pp. 13–4, where Moore discusses *class* and *extension*. He denies that with the ordinary meaning of 'class', classes could have less than two members, but that *if we take* ' $\forall x(\phi x = \psi x)$ ' to imply ' ϕ and ψ have the same extension', then we must allow extensions having one or no members, so if we further identify classes with extensions, we must allow this for classes too. Moore seems very ready to throw over Russell's theory of classes on the strength of this somewhat grammatical point, and flirts with taking classes as pluralities, but in the end the discussion is inconclusive.

⁷⁹ In fact, because of the treatment of empty descriptions, it is the system FD_2 of Van Fraassen and Lambert, 1967.

⁸⁰ See Simons, 1978.

⁸¹ For an exposition of Leśniewski's Ontology, including the notions here interpreted, see either Lejewski, 1958 or Henry, 1972.

⁸² Asenjo, 1977b, takes Leśniewski *not* to have a set theory, but our disagreement with this is only a matter of how 'set' is to be interpreted.

⁸³ As e. g. is outlined briefly in my, 1978. Cf. n. 31 above.

⁸⁴ See e. g. Stoll, 1974, p. 214. (In the 2nd edition Stoll drops as redundant the requirement of distributivity.)

⁸⁵ I am indebted for some of the stimulus to writing this section to Wolfgang Degen, who is developing in detail a family of formal systems embodying alternative strategies for representation. Where I have concentrated my attention on the representation of classes only, Degen's work provides for the representation of predicate-entities in general. The Schröderian tradition and the idea of representation put forward in Löwenheim's 1940 were brought to my attention by Barry Smith; cf. his 1978.

⁸⁶ Stenius, *op. cit.*

⁸⁷ Frege, 1893, § 3.

⁸⁸ Löwenheim, 1940.

⁸⁹ Cf. Bernays' review of Löwenheim, Bernays, 1940.

⁹⁰ Quine, 1937.

⁹¹ Cf. the similar principle of Cantor, 1899, n. 10 above.

⁹² Goodman, 1977, indeed *defines* Platonism as the acceptance of classes in one's ontology.

⁹³ Quine, 1963, pp. 1, 3.

⁹⁴ I should like to thank David Bell and Barry Smith, and an anonymous referee of the *Journal of Philosophical Logic*, for comments on an earlier effort which helped me to make many improvements embodied in this essay. In at least one respect, I am conscious that more needs to be said, for nothing in this essay deals with the problem posed by vague predicates. Zermelo, 1908, was criticised for employing the unclear notion of a definite property. It must be said that most of what I have said in this essay was said without thought for what difference it might make if some properties entering into the formalism are not, in a suitable sense, definite. Are there vague groups and manifolds, or is this simply an unwarranted transference of an idea from the linguistic to the ontological sphere? I am heartened by the fact that we talk about vague groups, or at least

talk vaguely about groups, all the time in ordinary discourse, e. g. 'the trees in Austria', 'the utensils in this room'. This question will need separate consideration, but it cannot be offloaded as 'not our problem', as effectively happened with Zermelo set theory, as modified by Fraenkel, 1922 or Skolem, 1929.

I should also like to thank Prof. Karel Lambert for stimulating discussion of my ideas at a later stage. I owe to him correction of certain factual errors regarding free logic.

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Eugenie

On the Concept Dependence and

Editorial Note by Peter M. Simon

The author of this article Eugenie
Leopold Blaustein, a member of t
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Suppose we consider a whole a compounded of the colour and shape (understood as individual accidents, not as universal properties) of a particular brick in a wall. This is dependent upon the extension of the brick, and thereby also on the brick itself, but it is not dependent upon the wall, as – according to *Theorem II* – it should be. a requires the existence of the brick, but not the superordinate wall, since a can perfectly well exist whether the wall does or not, can predate and survive the wall, and would usually do so, as long as the brick did not change in shape or colour.

The criticism and counterexample fail because Ginsberg fails to observe the distinction which Husserl makes between *foundation* and *relative dependence*: an individual a of the species α is *founded on* an individual b of the species β if α s have to be supplemented by β s in order to exist at all, and b here does the job for a ; an individual is *dependent relative* to another individual if it is founded on something ‘within the range of’ the latter, i.e. is founded on some proper or improper part of it. Thus every case of founding is a case of relative dependence, but not vice versa. The brick example is just such a case: the colour+shape a is founded on the extension of the brick, and the brick is a piece of the wall, so a is dependent relative to the wall, but a is *not* founded upon the wall.

The mistake is quite understandable however, since Husserl is not always consistent in his terms, and introduces them in what, by modern standards, is a sloppy and haphazard fashion. The same confusion invalidates her other counterexamples, but does nothing to detract from the fact that she is the only author known to me to have taken Husserl’s semi-formal work on wholes and parts seriously enough to venture into print about it. In any case, as I have to confess in my own paper on the 3rd Investigation elsewhere in this volume, it is by no means easy to interpret Husserl here.

The article presented here gives a survey of modern whole-part theory, especially in connection with the distinction between dependent and independent parts. Ginsberg begins with Stumpf’s definitions of dependent and independent contents, proceeds to Höfler, Twardowski and Husserl, and ends with some suggestions of her own. Most space is spent discussing Husserl, which is in conformity with the 1929 article. The original language of this paper is Polish. This presents problems of translation, since the then current language of dependent and independent parts was German, and we are working in English. One of

the greatest incidental difficulties of whole-part theory is that there is a plethora of different terms and, worse still, conflicting uses of the same terms. Husserl uses the terms *selbständig* and *unselbständig*, which are usually translated as ‘independent’ and ‘dependent’, although Findlay occasionally uses the word ‘self-sufficient’ for the former, and translates the latter as ‘non-independent’ in order to bring out its negativity. There is arguably a closer notional similarity between ‘self-sufficient’ and *selbständig* and between ‘independent’ and *unabhängig* (which, together with its positive, Husserl also occasionally uses) than the other way round. However ‘self-sufficient’ perhaps carries additional connotations in English, and is in any case less common than ‘independent’. Findlay regards ‘dependent’ as too relative in its connotations to translate *unselbständig*, but as Husserl himself is at pains to point out, all dependence is also relative dependence. Husserl also points out that the notion which is expressed more commonly in German by the negative word *unselbständig* is in fact the more basic, and indeed in this context uses *Abhängigkeit* to make his point (LU III, § 5). It seems to me that since he makes no attempt to distinguish the senses in any explicit way, he understands the two German words as synonymous, and simply uses the commoner one. For this reason it appears unnecessary to use Findlay’s ‘non-independent’, but to render the most common German word by the most common English word, ‘dependent’.

While this policy works well enough for Husserl, it works less well for the more finely discriminated senses to be found in Ingarden’s *Der Streit um die Existenz der Welt* where four different senses of (in)dependence are distinguished. We give these in their German versions, with suggested English equivalents:

- (1) Seinsautonomie/Seinsheteronomie – (existential) autonomy/heteronomy
- (2) Seinsursprünglichkeit/-abgeleitetheit – (existential) originality-
/derivation
- (3) Seinsselbständigkeit/-unselbständigkeit – (existential) self-sufficiency/non-self-sufficiency
- (4) Seinsunabhängigkeit/-abhängigkeit – (existential) independence-
/dependence

These suggested translations differ somewhat from those employed by Helen Michejda in her translation of part of the work from its original Polish into English (*Time and Modes of Being*, Springfield, Ill.: Charles

C. Thomas, 1964, translation of parts of vol. I of Ingarden, 1947/48). While Ingarden originally wrote *Der Streit* in Polish, the same distinctions were forged by him in German for his 1929 Husserl-Festschrift paper "Bemerkungen zum Problem Idealismus-Realismus", and it is to this paper that Ginsberg refers in her paper here. My suggestion for fixing terminology is then this: where Ingarden's distinctions are made, or some similar distinctions resting on the same terminology, the scheme above should be adhered to. Where, on the other hand, as in Husserlian writings, the pairs (3) and (4) are not kept apart, there is no need to keep their English translations apart. This indeed is Fidnlay's practice.

As to Ginsberg's Polish; in her note on Ingarden she translates his *Selbständigkeit* by *samoistność* and *Abhängigkeit* by *zależność*, with the negative prefix *nie-* for the German *un-*, whereas Ingarden uses *samodzielność* for *Selbständigkeit* and *samoistność* as a synonym for 'autonomy's. Now *zależność* is the standard Polish translation for the English word 'dependence', while the dictionary gives two closely related English equivalents for *samoistność*: (1) autonomous existence or spontaneity, (2) independence or self-containedness. Translated morpheme-by-morpheme the word comes out as 'ability to exist by itself', which might emerge as 'self-existability'. Unfortunately, while the *phrase* admirably captures just what Husserl in particular is after, the *word* is somewhat barbarous, and certainly jars upon frequent repetition. Since Ginsberg alludes to Ingarden's distinction without actually employing it, the more common English words will be used to translate the more common Polish (and German), after the fashion suggested as appropriate for Husserl, but not for Ingarden. In short, except in the comment on Ingarden in footnote 18, we translate *samoistność* as 'independence' and *niesamoistność* as 'dependence', and similarly for their cognates.

On the Concepts of Existential Dependence and Independence*

The concepts of existential dependence and independence which are the subject of our deliberations belong to that order of concepts in philosophy which hold out the hope that they can not only be intuitively grasped, but also precisely defined. In the philosophy of the last decades we find a whole series of efforts to define these concepts precisely, though various authors apply different names to them. The attempts made by Stumpf, Höfler, and Twardowski provide us with a series of ever more precise definitions of the concepts we are here examining, culminating in the investigations of Husserl, who devoted a separate work to these problems. Nevertheless, there are still certain difficulties and obscurities in their definitions. Hence, our critical analyses are supplemented by an attempt at a new definition, which is doubtless itself by no means finally definitive.

The most precise of all possible definitions of the concepts of existential dependence and independence can be used either for the purpose of creating a science that treats dependent and independent objects, or they can be applied in many different areas of philosophical research and in other fields. We find the beginnings of an eventual deductive system in Husserl, who formulated a series of assertions concerning dependent and independent parts and wholes.¹ We have examined these assertions elsewhere.² On the other hand, the fact that the concept of dependence can be applied in many areas, e.g. in the area of sense contents, psychic facts, meanings, activities and products, objects of higher order, etc., testifies to the fruitfulness of the distinctions which are the subject of this paper.

* English translation of "W sprawie pojęć samoistności i niesamoistności", *Księga pamiątkowa Polskiego Towarzystwa Filozoficznego we Lwowie 12.II.1904-12.II.1929*, Lwów, 1931, 143-68. (The author's name is there given in its Polish married form: Eugenia z Ginsbergow Blausteinowa.)

§ 1 Analysis of Stumpf's Definitions

Stumpf, in taking up the relationship of the representation of extension to that of colour qualities,³ divides all such simultaneously represented contents into two classes, according to their mutual attachments, namely (1) independent contents (*selbständige Inhalte*) and (2) partial contents (*Teilinhalte*), which correspond with what were later called dependent contents. We encounter independent parts whenever the elements of a complex of representations by their nature can be separately represented, and partial contents wherever this is not the case.⁴ As examples of contents independent of one another Stumpf lists (a) contents opposed to each other, (b) qualities of the various senses like colours and tones, (c) qualities of the same sense like the notes of one chord. Extension and colour are examples of partial contents. According to Stumpf, it is endemic to extension and colour not only that the one cannot exist in representation without the other, but also that a change in the first causes a change in the second.⁵ In point of fact both of these contents can change independently of one another: colour does not cease to be red or green when extension changes, but despite this fact colour does participate to some extent in changes of extension. From this mutual functional relationship between extension and colour it follows that both of them are by their very nature inseparable from each other and in a certain way comprise one whole content, of which they are only partial contents.

Let us consider the definition of partial contents and ask whether, indeed, when two objects are partial contents, they can never be separately represented, albeit that Stumpf stresses that such an impossibility must result from the nature of these contents. Thus were we to take 'representation' in the broadest meaning of the word, including all acts which make present an object, then partial contents would not exist at all, since each content could be e.g. signitively represented separately from the others. Thus, for example, we can signitively (i.e. with the help of a word) imagine redness without simultaneously imagining extension. On the other hand, if we take 'representation' in the sense of 'imagination', then the definition of partial contents – as those which cannot be separately imagined – will be correct only in a limited sense, namely with respect only to what is imaginable. This definition will be important in connection with dependent sense qualities, to which the original concept of dependence was first applied and with which Stumpf

was perhaps solely concerned. But since the concepts of independence and dependence are also applied beyond the area of sense contents, e.g. in the sphere of such things as relations, which cannot be imagined at all, or in the area of psychic facts, concerning which it is doubtful whether they can be imagined, this definition seems too narrow. In any case, one cannot define partial contents as contents which inherently cannot be represented separately from others.

Stumpf brings up still another point characteristic of at least certain partial contents; namely, he stresses the dependence of changes of colour on changes of extension. But we shall discuss this point later in connection with the discussion of Husserl's views.

§ 2 Analysis of Höfler's Definitions

We later encounter the concept of independence and dependence in Höfler⁶ (though not under the same name). In his analysis of representations Höfler distinguishes three types of complexity of representations of contents. We find the first type of complexity in representations which contain 'parts' of the sort that may be represented separately and concerning which it is evident that each part of a simultaneously represented whole can exist independently of the remaining parts. Thus we can conceive each page of a book separately without conceiving the others, and one page may exist although the others do not exist, or may change although the others do not change. A second type of complexity is found in represented contents whose individual features [*Merkmale*] can, indeed, be distinguished from each other, but which cannot be separately represented and concerning which it is obvious that the qualities [*Eigenschaften*] of the object represented by these features cannot exist separately from the others. Thus colour cannot exist without any sort of shape, even though one can distinguish colour from shape. We meet the third type of complexity in represented contents which include features, say A and B, which are such that A can be represented without B but not B without A, and further one cannot conceive of B existing without A. For example, we may think of a colour which is not white, but we cannot, on the other hand, think of a whiteness which is not a colour; the same holds true with respect to foursidedness and shape, etc.⁷

What we have stated above can either be regarded as two divisions of the same class of objects made on the basis of two different principles, namely (1) whether particular elements of a content can be represented without representing the others, (2) whether particular qualities of the object represented by the elements of a content can exist separately from the others – or as one division, the principle of which is two different features. Should we prefer to see two divisions here, then several doubts which we have already mentioned in § 1 arise with respect to the possibility or impossibility of separate representation. And concerning the principle of the second division, Höfler stresses that this dependence or independence must be such that it can be stated with evidence. With his division of a content based on the principle of the possibility or impossibility of separate existence and with his emphasis on the point of evidence Höfler anticipates the later definition of independence and dependence of Husserl.⁸

But the principle of division which we are discussing is not always consistently expressed. For when Höfler discusses the first and second type of complexity he speaks of the possibility or impossibility of the *existence* of some qualities without the others, whereas in discussing the third type of complexity he speaks of the possibility or impossibility of *imagining* certain qualities *as* existing without the others. These two things are not, of course, the same, and it is doubtful whether they even go together; therefore they cannot serve us as one of the principles of division.

But it is also doubtful whether the possibility or impossibility of representing A without B and the possibility or impossibility of A's existing without B always go hand-in-hand, that is to say it is uncertain whether the classes of contents of representations derived from the divisions based on these two criteria are identical. For there is firstly no necessary connection between the psychological possibility or impossibility of representing something separately and the ontological possibility or impossibility of something existing separately. Further, there are cases such that A cannot exist without B but where they can be separately represented in imagination. Signitive thinking, which we referred to in our criticism of Stumpf, can again serve us as an example. We shall discuss further the division of objects from the point of view of the possibility or impossibility of their separate existence when we deal with the views of Husserl, who discusses this division in detail.

If, on the other hand, we were to accept the view that Höfler's pur-

pose was to present a single division based on two different features, that is, that belonging to the first class would be those contents which can both exist and be represented separately, etc., then this division would be incomplete, for there are those contents which do not belong to any of the classes which he distinguishes, e.g. those which can be separately represented but which cannot exist separately.

The above analysis concerns the definitions of Höfler published in the first edition of his *Logic*. In the second edition of 1922 Höfler changed the sections in which the question which interests us was discussed. We have considered the views contained in the first edition in such detail because we wished to present the history of the problem which interests us. In Höfler's arguments from the first edition of his *Logic*, as we have seen, there are two meanings of dependence, one of which was later developed by Twardowski, the other by Husserl.

In the second edition Höfler takes up the question we are discussing in connection with the problem of psychic analysis. There he distinguishes two kinds of parts: separable [*trennbare*] and inseparable [*untrennbare*] parts, and along with these, two kinds of psychic analysis. Psychic analysis of the first kind consists in imagining separable parts as existing separately, and that of the second kind, in differentiating between inseparable qualities of an object. Inseparable qualities of an object may be either mutually or unilaterally detachable [*ablösbar*].⁹ The arguments of Höfler mentioned here do not move the question under discussion forward, since they do not define the concepts of separability [*oddzielności = Trennbarkeit*] and detachability [*odłączalności = Ablösbarkeit*] more precisely.

§ 3 Analysis of Twardowski's Definitions

Many assertions on existential independence and dependence (though again in a different terminology) are found in Twardowski.¹⁰ In his deliberations this author uses the earlier concept of metaphysical parts. By 'metaphysical parts' he means parts which can be differentiated in the whole to which they belong through abstraction, but which cannot be separated from the whole in reality. According to Twardowski the concept of metaphysical part is interchangeable with the concept of quality [*Eigenschaft*],¹¹ while the expression 'quality' is used here to mean also an aspect of a relationship obtaining between a whole and

certain of its parts (sc. the *metaphysical* parts) as against the relationship itself. So qualities in this sense are extension, colour, weight, etc.¹²

In addition to the concept of metaphysical part we also find in Twardowski a concept homologous to that of inseparable part. Basing himself on the first principle of division which we find in Höfler, i.e. the possibility or impossibility of conceiving something separately, Twardowski divides the material parts of a representation into:

1. parts which are mutually separable, i.e. parts among which each can be conceived without conceiving the others,
2. parts which are mutually inseparable, i.e. parts which can, indeed, be differentiated from others, but which cannot be conceived without others,
3. parts which are unilaterally separable, i.e. such, for example, as parts A and B, of which one can, indeed, conceive A without B, but not B without A.¹³

Mutually separable parts of the content of representation are, for example, the representation of individual pages and covers of a book, insofar as they are the represented parts of one book. For one can conceive the individual pages independently [*niezależnie*] of one another, i.e. without conceiving the other pages or the covers of the book. Similarly, conceiving the cover is distinct from conceiving the individual pages of the book. A typical example of mutually separable parts of the content of a single representation is that of colour and extension: one cannot conceive colour without conceiving extension and vice versa. Representations of a genus and of a species subordinate to this genus stand in the relation of unilateral separability. For it is impossible to conceive a species without conceiving an appropriate genus. So in the representation of redness there must be a representation of colour, whereas the representation of colour does not of necessity imply the representation of redness.

Twardowski considers also the division based on the possibility or impossibility of separate existence, but he rejects it, since it assumes the existence of the parts of the object, while Twardowski regards objects and their parts as objects or parts of the objects of representation, abstracting from the reality, possibility or impossibility of their existence.¹⁴

We shall attempt to relate the concept of metaphysical part to that of inseparable part. When we say that some part is metaphysical, we de-

fine its relationship to the *whole* of which it is a part, but we say nothing of its relationship to the other parts of the whole, whereas a part [if inseparable] is inseparable from the other *parts* of its whole. Moreover, inseparable parts are parts which cannot be conceived in insolation from other parts, while metaphysical parts are parts which cannot exist separately from the whole. The question is, however, whether a given part is metaphysical when it cannot, as a matter of fact, exist separately from the whole, or whether for a given part to be metaphysical it is necessary for it to be inseparable from its whole by its very nature. Husserl resolves this question in reference to dependent parts by accepting the second possibility.

In addition to the already mentioned negative point, still another positive point appears in the definition of metaphysical part, namely, that a metaphysical part can be distinguished within the whole by means of abstraction. But since each part can be distinguished in the whole with the help of abstraction this point appears to be unimportant.

However with respect to the division accepted by Twardowski based on the possibility or impossibility of conceiving something separately, one can observe the same thing as above, namely, that depending on the meaning of the word 'representation' it either does not designate all inseparable parts or it leads to the result that there are no inseparable parts at all.

§ 4 Husserl's Definitions

Husserl took up the problem of independence and dependence in detail [in his third Logical Investigation].¹⁵ He encountered the concepts of existential independence and dependence in connection with his deliberations on parts and wholes. For according to Husserl a part of an object is everything that is 'in' a given object, that which a given object really constitutes and of which it is constructed – and what the object is of itself, i.e. in abstraction from all the relationships in which it is enmeshed.¹⁶ But in common parlance the word 'part' is not used in such a broad sense. When we attempt – according to Husserl – to clarify the differences that exist between the common concept of part and the concept defined in his way, we encounter the fundamental difference between independent and dependent parts. When we refer to parts in common speech, existentially independent parts or pieces [*Stücke*] are

meant, whereas Husserl's concept of part also includes existentially dependent parts or moments. And since each part can become an object (or content)¹⁷ of the representation directed toward it and itself may be regarded as an object (or content), the differentiation of independent and dependent parts points to a similar differentiation of objects (or contents), where the term 'object' is taken here in the broadest sense. Husserl defines the concepts of existential independence and dependence as follows:

An object is independent which by its nature is not conditioned in its existence by the existence of other objects; it could *a priori*, i.e. by its nature, exist as it is even if nothing else but it existed, or if everything around it were to arbitrarily change. In the "nature" of the independent object, in its ideal essence, there is no dependence on other objects. In reality it can be such that other objects are given along with the existence of this object on the basis of certain empirical laws, but in its essence the object is independent [*niezależny*] of them.

On the other hand, a dependent object is one which in its essence is conditioned by the existence of other objects, which in its essence cannot exist if, simultaneously, other objects did not co-exist with it, supplementing it. Due to the co-existence with other objects essential to it a dependent object comprises one whole together with them. Hence, a dependent object can only exist as part of a whole.¹⁸

§ 5 Analysis of Husserl's Definition of Dependence in Application to Objects

Before we undertake analysis of the Husserlian concepts of dependence and independence let us call attention to the fact that it is doubtful whether the division of *parts* into independent and dependent is coterminous with the analogous division of *objects*, as Husserl implicitly supposes. It is very probable that not all objects are part of some whole, where here the term 'object' is understood in its most general sense, as Husserl intended,¹⁹ while the word 'part' designates actual rather than possible parts.

Let us now examine the Husserlian definitions of the concept of existential dependence, first with respect to the division of parts. This definition does not seem valid to us in all respects, because it is not imperative either (1) that the supplementary object should co-exist alongside the dependent object,²⁰ or (2) that the dependent object be part of a

whole, which – in Husserl's opinion – it is supposed to form together with the objects supplementing it.

Ad (1). It is universally accepted that a relationship is a dependent object with respect to its terms.²¹ Despite this the existence of a relationship does not perforce require the existence of its terms. If, for example, we consider the following relationship of consequence: 'If wood is metal, then wood will melt in fire' – the terms of this relationship are the states of affairs [*stany rzeczy*] indicated by the antecedent and the consequent. The consequential relationship obtaining between these terms exists or obtains despite the fact that they themselves do not exist. Similarly, if the relationship of contradiction exists between states of affairs, then of necessity it is always dependent with respect to the one existing and the one non-existing term – if the principle of the excluded middle and the principle of contradiction are correct. If dependent objects do not require the existence of supplementary objects, then neither do they require anything to exist concomitantly with them. As examples of such objects, in addition to those already mentioned, we can instance certain objects of derelativized concepts, such as cause *qua* cause, and effect *qua* effect. But these last objects remain in a relationship of *formal* dependence to each other and not – as previously – in a *material* one (see below § 10).²²

Ad (2). Also invalid is the assertion that the dependent object can only exist as part of a whole which – in Husserl's opinion – it is supposed to form together with the objects supplementing it.²³ So, for example, the feature of difference of A from B is dependent with respect to both B and A. But despite this the feature of difference inhering in object A does not form any whole with object B.²⁴ Similarly when relationships are dependent with respect to their terms, they do not form wholes with them. But there are also dependent objects which can exist only as parts of a whole. So, e.g. in order to exist, colour requires an inherent subject of which it could be a part. But the dependence of an object requiring a whole is only one kind of dependence of objects.

The above deliberations incline us to the following conclusion: the Husserlian definition of dependence, insofar as it concerns all dependent objects, is too narrow, for there are dependent objects which are not subsumed under this definition. We should then reflect whether Husserl's definition could be retained in a narrower sense, namely, in reference to parts, whether parts of independent wholes or parts of dependent wholes.

§ 6 Analysis of Husserl's Definitions of Dependence in Application to Parts

Insofar as a part of an independent whole is concerned, it can be dependent only with respect either to the same whole or with respect to another part of the same whole. For if a part of an independent whole could also be dependent with respect to some other object existing beyond this whole, then the whole also would be dependent with respect to it, despite the fact that it is an independent whole.²⁵ It is worth noting here that it is probable that if some part is dependent with respect to its whole then it is dependent with respect to a whole that is directly superordinate for both of these parts,²⁶ but it can be either independent or dependent with respect to any other superordinate whole.

The definition of dependence given by Husserl is also important in reference to parts of independent wholes which are dependent with respect to a given whole as well as in reference to parts of independent wholes that are dependent with respect to other parts of the same wholes. The incorrectness of the second of our criticisms above in reference to the part of an independent whole stems from the fact that complementing this part may be either a whole whose make-up contains this part or some other part of this whole. Neither can we find any example of a part of an independent whole which would not fulfil the conditions enumerated in the definition of dependence given by Husserl. Consequently, both of the criticisms put forth above by us of the Husserlian definition of dependence are unimportant with respect to the parts of independent wholes.

We must still determine whether Husserl's definition is significant for dependent parts of dependent wholes. The matter presents itself differently here to the extent that for dependent parts of independent wholes we had only two possibilities (discussed above), whereas for dependent parts of dependent wholes we have three. For a part of a dependent whole may be dependent (1) with respect to its whole (2) with respect to an object lying beyond this whole (3) with respect to some other part of this whole. In the first two cases the matter presents itself just as above for parts of independent wholes. Husserl's definition – it would seem – is also significant here. Only the third possibility presents a difficulty, but only in a special case, namely, when part *c* of a dependent whole *C* is dependent with respect to an object *P* which lies beyond the whole *C* and with which *C* does not form a whole, nor of which is it a

part. In this case we are dealing with the relationship of a dependent object c [the original has ' c_1 ', which seems to be a mistake] to an object P supplementing it; for the fact that c is part of C has no bearing on the matter. But for dependent objects we pointed out that some exist for which Husserl's definition is not valid. For example, if there were some dependent whole whose part was a relationship that had terms which were not parts of the whole, then this relationship would be dependent with respect to its terms, hence objects lying beyond this whole, and could exist even if its terms did not exist.²⁷

Summing up then, we can say that the definition of the concept of dependence given by Husserl is valid for dependent parts of dependent wholes in cases (1) and (2), but that in case (3) the definition is sometimes invalid.

§ 7 Analysis of Another Husserlian Definition of Dependence

In Husserl we find still another definition of the concept of dependence. With respect to certain contents, writes Husserl, we have evidence that a change in, or disappearance of, at least one of the contents accompanying them (but not contained in them) must change or abolish them also. With respect to others we lack such evidence. Contents of the first kind can only be conceived as parts of a certain whole embracing them, whereas the second would be possible even if nothing else besides them existed. The first are dependent contents, the second independent ones.²⁸

In connection with this definition Husserl mentions the analysis of the concept of dependent contents made by Stumpf, with which, after certain modifications, he agrees. We shall once more present it briefly here, but in the form in which it appears after Husserl's modifications. Thus, a content dependent with respect to another is, for example, colour in relation to extension. In a certain genuine sense we can say that changes of colour are independent [*niezależne*] of changes in extension, in that colour *in specie* can remain the same when extension arbitrarily varies. But the colour given in concrete imagination remains in a relationship of functional dependency [*zależności*] on that extension in that concrete imagination. Colour thus conceived participates in changes of extension. In saying this we express the fact that a colour becomes

smaller, diminishes, and finally completely disappears solely because of the changes in and disappearance of extension.

It is just as difficult to agree with this analysis as it is with the above definition of the concept of dependent parts derived from it. The functional relationship between colour and extension here is only superficial. One can ultimately say that the coloured surface changes along with the changes in its extension; but the assertion that colour changes along with changes in extension is completely incomprehensible. The words 'smaller' and 'larger' applied to a colour are completely without sense. Neither can one say that the quality of a tone changes along with its intensity. The quality of a tone cannot be weaker or stronger. Perhaps in this as well as in the previous case the point is whether along with the diminishing of tone or extension the quality of tone or colour is given to me less clearly, but the point is, in our opinion, unimportant for quality or colour, since it relates only to a certain relative feature. Furthermore, insofar as insignificant changes in the extension of large surfaces are concerned, one cannot speak of different levels of distinctness. We shall observe, however, that even if the above analyses were valid, the definition of dependence which we have examined here could not be maintained. For let us take note of the same example of colour and extension. Just as colour is dependent in relation to extension, so also extension, given visually, is dependent in relation to colour. Thus there should be a functional dependency between extension, given visually, and colour. With a change of colour from green to yellow, extension should also become yellow, but this can hardly be said.

In the definition of the concept of dependent contents still another point was raised. For not only does colour change with changes in extension, but colour also disappears with the disappearance of extension. We shall not dwell on this second point any longer, for it was discussed above. The question involved here is the same as the question whether the existence of a dependent object also requires the existence of an object supplementing it.

§ 8 Analysis of Husserlian Definitions of Independence

Turning to the analysis of the first Husserlian definition of independence, we can assert that, just as we rejected above the definition of dependence in reference to all objects because it was too narrow, so

now we cannot agree with the definition of independence because it is too broad, precisely in reference to those objects for which the definition of dependence is too narrow. For since the concepts of independent and dependent objects are contradictory, so if some dependent objects do not fit the definition of dependence – and such, as we have seen, do exist – they must fit the definition of independence. But the definition is valid in a certain narrower range, namely, in the same one in which the Husserlian definition of dependence is significant.

But regarding the second definition of independence, according to which those objects are independent for which we lack evidence that a change in or the disappearance of at least one of the objects given along with them must change or annul them, this is also too broad. We gave examples above of dependent objects which fitted this definition of independence.

§ 9 An Attempt to Define Dependence and Independence

The attempts to define independence and dependence which we have discussed were of two kinds: (1) psychological, (2) ontological. The former defined independence or dependence with respect to the possibility or impossibility of separate presentation, and the latter with respect either to the possibility or impossibility of separate existence or to dependence on changes. Psychological attempts, as we have seen, encounter the following difficulty: that if the word 'representation' also included concepts, then there would be no dependent objects at all, but if it were to designate only ideas, then the division of objects into independent and dependent would be a valid division only for imaginable objects. But as far as ontological attempts are concerned, the definition of independence derived from the division based on the possibility or impossibility of separate existence turned out to be too broad (for it also included certain dependent objects like relations, states of affairs, etc.), whereas the definition of dependence was too narrow. And finally, with respect to the division based on dependence on or independence of changes we attempted to show that it cannot be maintained.

Hence, we shall now attempt to present a definition of independence and dependence which would be free of the above defects. We shall not attempt to define these concepts psychologically, i.e. by means of their relationship to psychic life, but we shall attempt to confine our defini-

tions to the ontological sphere. Following the example of Stumpf, Höfler, and Husserl in our definitions we shall also use the expression that something is necessary 'from the nature of a given object', for we wish to stress that the relations we are discussing, e.g. the relations of dependence holding between colour and extension, between a relation and its terms not only occur in reality, e.g. the relation between cause and effect, but also some other new factor appears along with them which is very difficult to define, but which we cannot ignore without violating our intentions. Later on in our definitions we shall encounter the concepts of state of affairs and subject of the state of affairs which we shall not further analyze here.

In speaking of independence and dependence one should distinguish relative and absolute independence and dependence.²⁹ Let us begin with the first. We state:

Object P_1 is independent with respect to object P_2 if for the existence or occurrence of P_1 , P_1 by nature does not require the occurrence of any state of affairs in which P_2 is the subject, " (besides the state of affairs that P_2 does not have to be the subject of states of affairs required for P_1 to exist or occur as well as all equivalent states of affairs in which P_2 is the subject.)

But object Q_1 is dependent with respect to object Q_2 if for the existence or occurrence of Q_1 the occurrence of some state of affairs in which Q_2 is the subject is necessary.

On the basis of the above definitions one can easily formulate definitions of absolute independence and dependence, according to which:

Object P is independent if, by nature, its existence or occurrence does not require the occurrence of any state of affairs in which some other object not entering the make-up of object P is the subject.

Object O is dependent if, by its nature, its existence or occurrence does require the occurrence of some state of affairs in which some other object not entering the make-up of object O is the subject.³¹

With regard to which states of affairs are required for the existence or occurrence of a dependent object (both in the relative as well as the absolute sense) one can distinguish various kinds of dependence. 'Q is dependent' may mean either that (1) Q for its existence or occurrence requires by its nature the occurrence of such a state of affairs that some other object exist simultaneously along with it; (this is the kind of dependence meant in the statement 'colour is dependent with respect to

extension', and this is the kind of dependence Husserl has in mind), or that (2) Q for its existence or occurrence by its nature requires the occurrence of such a state of affairs that some object has such and not other qualities, or that it has such and not another nature (e.g. the relation of difference subsisting between a plant and an animal, which is dependent with respect to its terms, requires that the plant possess certain features which the animal does not have and vice versa).³² Certain kinds of independence are coupled with the appropriate kinds of dependence. It is probable that one kind of independence or dependence is always vested in objects that belong to one category. So, e.g. the first kind of dependence is always vested in dependent parts of independent wholes. This fact could have some importance in establishing a criterion of independence and dependence.

§ 10 Possible Classifications of Dependence and Independence

Let us now reflect on a certain distinction in the realm of the concepts of independence and dependence, which we already discussed previously (§ 5). We say, for example, that part c is dependent with respect to the whole. This statement is ambiguous. For it may mean either that part c as a part of precisely this whole is dependent with respect to it; in this sense every part is dependent with respect to its whole; or it may mean that part c is dependent with respect to the whole not as a part of this whole, but as an object with such and not other features or such and not another nature. In this sense parts can be either independent or dependent with respect to the whole. Dependence in the first sense we call *formal* dependence, but dependence in the second sense, *material* or *objective* dependence.³³ We can similarly distinguish formal and material independence. We can speak of formal and material dependence not only with reference to parts and wholes, but also beyond this. For example, all objects of derelativized concepts remain in a relation of mutual formal dependence. Persons who are parents are only dependent parents with respect to their own children and vice versa; a change, which is the effect of some cause, is consequently dependent with respect to this cause; a man who is a serf is dependent as a serf with respect to his master, etc.³⁴ The dependence in these examples is formal dependence. For some object to be formally dependent with respect to another it is not sufficient for it to possess certain features in relation to

it. The fact that A is e.g. similar to B does not yet imply that it is formally dependent with respect to B. In defining formal dependence we shall be concerned with enumerating the conditions which an object must meet in order to belong to a certain class, but in defining material dependence we were concerned with listing the conditions which must be met for an object to exist or occur. The objects which we have discussed above as formally dependent are materially independent. Formal dependence and independence we could attempt to define as follows:

Object Q_1 as belonging to class K is formally dependent with respect to object Q_2 if for Q_1 to belong to class K the occurrence of some state of affairs in which Q_2 is the subject is required.

But object P_1 belonging to class K is formally independent with respect to object P_2 if for P_1 to belong to class K the occurrence of a state of affairs in which P_2 is the subject is not required.

Another division of independence and dependence is the division into absolute and relative independence and dependence. The pertinent definitions were formulated above in § 9. These concepts can be mutually related in the following way: (a) Object A, dependent with respect to B, is also absolutely dependent. (b) Object A, independent with respect to B, can be either absolutely independent or absolutely dependent,³⁵ (c) Object A absolutely independent must also be independent with respect to any B whatever.

In speaking of the divisions of independence and dependence one should note that objects which stand in relations of independence and dependence to each other can be divided into pairs of objects (1) unilaterally independent or dependent, (2) mutually independent, and (3) mutually dependent.³⁶

Ad (1). Objects A, B are unilaterally independent if A is independent with respect to B while B is dependent with respect to A, or conversely.

Ad (2). Objects A, B are mutually independent if A is independent with respect to B and B is independent with respect to A.

Ad (3). Objects A, B are mutually dependent if A is dependent with respect to B and B is dependent with respect to A.

Substituting for the expression 'A is independent (or dependent) with respect to B' our definition of relative independence (or dependence) we say:

Ad (1). Objects A, B are unilaterally independent if for the existence

of e.g. A, by its nature there is not required the occurrence of any state of affairs in which B is the subject, but for the existence of B, by its nature there is required the occurrence of some state of affairs in which A is the subject. And so forth.

§ 11 Elements of the Relation of Dependence

In our deliberations up to now we have spoken of objects between which independence or dependence holds. Now we shall examine the terms of the relation of dependence.³⁷ We say, for example, that extension is dependent with respect to colour or the pitch of a tone with respect to its intensity. These statements are ambiguous. For instance let us note a specific concrete red colour of a given book-cover. In this colour is found a certain concrete factor independent with respect to this redness due to which this specific colour belongs to the category of colour.³⁸ This factor is colouredness (being-coloured). It is the same (or not the same) in redness, greenness, and in all other shades of colour. It must be distinguished from the object of the content of general colour, i.e. colour *in specie* which – as some assert – is only one and does not really exist. All of these shades of colour are dependent with respect to this factor of colouredness. Factors similar to colouredness are found in a specific extension, pitch of tone etc., but we do not have separate names for them. We shall call them extension and pitch in contrast to a specific extension, specific pitch, etc. So the statement that extension is dependent with respect to colour can mean (1) that extension is dependent with respect to colouredness, or (2) that a specific extension is dependent with respect to colouredness, or (3) that extension *in specie* is dependent with respect to colour *in specie*.³⁹ We shall not examine the third case more closely, for it would be difficult to say anything well-founded. As far as the first possibility is concerned it should be stated that it does in fact occur. Indeed, extension *is* dependent with respect to colouredness. It could not exist if colouredness did not exist also.⁴⁰ But the matter is rather different in the second case. A specific extension is not dependent with respect to a specific colour. The specific extension of a brown board could also exist if the brown colour of the board did not exist, e.g. if we were to paint the board green. The extension of the board would then remain precisely the same. Thus, if it is indeed true that the same specific extension can be joined with various colours,

then it does not require the same thing by nature of a specific colour, i.e. it is not dependent with respect to a certain colour. Thus the question is in which sense one should take the word 'colour' in order to be able to state that a specific extension is dependent with respect to colour. In our opinion a specific extension requires for its existence not the existence of a specific colour, but the existence of any concrete colour whatsoever, any element whatsoever from the spectrum of the concept of specific colour.⁴¹

The question of the dependence of a relation with respect to its terms presents itself otherwise than with colour and extension. A definite relation obtaining between A and B is dependent with respect to A and B and thus with respect to its definite concrete terms. Thus it does not require just any terms whatsoever, in the way a definite extension requires just any sort of colour. If the terms were not A and B but some other terms C and D, then we would not be dealing with the same relation but with some other one.⁴² In all of the other cases distinguished above the question for relations is the same as for extension, pitch of tone, etc.

§ 12 Conclusion

After determining the definitions of the concepts of independence and dependence it will be interesting to turn our attention to the application which these concepts find in the most diverse areas of inquiry. The most widespread is the application of the concept of dependence to the area of sense contents. So, for example, we encounter the relation of dependence between colour, extension and shape in the visual sphere, between the elements of a tone, that is, between its quality, pitch and intensity in the aural sphere, between roughness and a certain degree of hardness in the tactile sphere, etc. The concept of dependence has wide application in the psychic realm. A characteristic example of dependence which we find in this sphere is the dependence of certain psychic facts with respect to their psychological foundation. We also find the application of the concepts of independence and dependence in the area of meanings, e.g. in the case of distinguishing categorematic and syncategorematic words. Husserl devoted a separate work to these problems,⁴³ where he based his science of semantic categories on the distinction between dependent and independent meanings. Further

areas of the application of the concept of dependence are found in the mutual relation between acts and products,⁴⁴ features and the object in which a feature inheres, 'derelativized' objects, in the theory of objects of a higher order, etc.⁴⁵

The possibility of applying this concept in various fields of inquiry indicates that it is a concept that does not lack scientific importance and is therefore worthy of detailed analysis.

Notes

(For biographical and bibliographical information, we are indebted to the chief librarian of the University Library, Łódź, Dr Bolesław Świdorski.)

¹ E. Husserl, 1922, p. 262f.

² See E. Ginsberg, 1929. In this paper we tried to show that of the six theses put forward by Husserl and supposedly having the character of deductively proven assertions, four are either wrongly formulated or have invalid proofs.

³ C. Stumpf, 1873, § 5, p. 106ff.

⁴ The distinction between contents which can be separately represented and those which cannot be separately represented is already found in Berkeley's *Treatise*. Indeed the concept of existential independence in a similar sense is used by Descartes and Spinoza in their definitions of substance, and they in turn derived them from still earlier thinkers.

⁵ Whether Stumpf would also accept the contrary, that changes of colour result in changes of extension is not clear from the text. Evidence that Stumpf would also accept this relationship in the opposite direction seems to be implied in the following: "Hieraus nun [namely from the relationship of extension and colour characterized above in the text] folgt, daß beide ihrer Natur nach untrennbar sind ..." (*loc. cit.*, p. 113). If only colour were in its changes dependent on extension, one could not conclude that both are by nature inseparable from each other, but only that colour is inseparable from extension.

⁶ A. Höfler, 1890, § 15.

⁷ In characterizing contents which we have called dependent Höfler further alludes to the dependence of changes of one quality or feature on the changes of the other qualities or features, but he does not go into the matter in more detail. This point as well as the division of contents based on the possibility or impossibility of their separate representation is common to both Höfler and Stumpf. A new element in Höfler in comparison with Stumpf is the division based on the possibility or impossibility of separate existence.

⁸ While for Höfler the evidence of experiences confirms dependence and independence, the corresponding point in Stumpf and Husserl is the nature of the contents or objects, from which dependence and independence derive. Thus while Höfler looks at the matter more from a psychological angle, Stumpf and Husserl treat it more ontologically.

⁹ A. Höfler, 1922, p. 121f.

¹⁰ K. Twardowski, 1894, Ch. 9-11.

¹¹ *Loc. cit.*, p. 58. 'An Stelle des Wortes Eigenschaft könnten Ausdrücke treten wie metaphysischer Teil oder Beschaffenheit.'

- ¹² See *Loc. cit.*, p. 58. Twardowski points out that the word 'Eigenschaft', and words which denote qualities, like 'redness', 'foursidedness' etc., are ambiguous. For sometimes they denote metaphysical parts possessed by the object, while at others they denote the relation of possession obtaining between an object and any of its parts. Later Meinong called attention to a similar ambiguity (1910, p. 57). He writes that by 'blackness' [*Schwarze*] or 'difference' [*Verschiedenheit*] is understood either the feature 'black' [*das Schwarz*] or 'different' [*das Verschiedene*] or else its position in a certain object, in virtue of whose possession it is black or different, i.e. the objective: being a black (or different) object.
- ¹³ *Loc. cit.*, p. 65. As we can see, unilaterally inseparable parts are not considered in this division. The reason for this probably lies in the fact that the extension of the concept of unilaterally separable part is identical with the extension of the concept of unilaterally inseparable part. If parts A and B are unilaterally inseparable, then they are also unilaterally separable and vice versa. The terms 'unilateral' – and 'mutual separateness' as well as 'mutual inseparability' come from Brentano, 1889. See Twardowski, *loc. cit.*, p. 65.
- ¹⁴ *Loc. cit.*, p. 51.
- ¹⁵ Husserl 1922.
- ¹⁶ *Loc. cit.*, p. 228.
- ¹⁷ The word 'content' is here used by Husserl in a broad sense, embracing all individual objects and their parts (1922, p. 219). In this entire work Husserl uses the terms 'content' and 'object' interchangeably, whereas we retain the second term for this meaning. Concerning various meanings of the word 'content' in Husserl see J. Blaustein, 1928, p. 44ff.
- ¹⁸ *Loc. cit.*, p. 236. In addition to the above definitions of the concepts of existential dependence and independence we also encounter certain others in Husserl, which are generalizations of Stumpf's analyses and about which we shall speak later in § 7. Pfänder (in his 1921, p. 307) accepted Husserl's definitions of independence and dependence. For Husserl and some other authors the starting point for the differentiation of the concepts of independence and dependence is the relation of part to whole. This differentiation can also be approached not only from the side of formal-ontological problems, but also from the side of existential-ontological problems, e.g. through distinguishing various meanings of the word 'existence'. Of course, this kind of analysis can be carried out only by someone who adopts a position which recognizes various kinds of existence. R. Ingarden takes up the problem of independence and dependence from this point of view in his 1929. There he distinguishes four pairs of contradictions:
- (1) existential autonomy and heteronomy [*Seinsautonomie, -heteronomie*]
 - (2) existential originality and derivation [*Seinsursprünglichkeit, -abgeleitetheit*]
 - (3) existential self-sufficiency and non-self-sufficiency [*Seinsselbstständigkeit, -unselbstständigkeit*]
 - (4) existential independence and dependence [*Seinsunabhängigkeit, -abhängigkeit*]
- [See editorial note preceding this essay.]
- ¹⁹ This most general concept of object in modern philosophy is encountered in Twardowski (1894, §§ 5, 7), then in Meinong, 1904, for whom this concept is basic for his theory of objects.
- ²⁰ We call P a 'supplementary object' to object A if A is dependent with respect to P.
- ²¹ E.g. we read in Meinong (1899, GA II, p. 386) 'There are objects of which we can say that they have by nature an inner dependence [*Unselbstständigkeit*]. I do not mean that dependence of occurrence whereby for instance a colour cannot be presented without an extension ... one can still call this [dependence] external by comparison with what I should like to call unfinishedness [*Unfertigkeit*], which attends e.g. the object 'differ

ence', when one tries to isolate it from that which is different.' See also Erdmann (1892, p. 57): 'Processes and relations are, as against things with properties, only dependently [*unselbständig*] real' (in the second edition the corresponding paragraph does not reveal the author's position so clearly) and Pfänder (1921, p. 307): 'The objects meant by concepts can be independent [*selbständige*] things or ... dependent [*unselbständige*] relations.'

²² It might be argued that the above criticism does not concern Husserl, since Husserl in his definition used the concept of existence in such a broad sense that he would have regarded both the above states of affairs and both terms of the relationship of contradiction as existing objects. If this interpretation of Husserl is correct, which is doubtful, then the above criticism would have to be regarded not as immanent but as transcendent, i.e. as carried out from a position which does not attribute existence to such objects.

²³ To be sure, in the place cited above there was only mention that a dependent object is something which is one with the objects supplementing it, not that it forms a whole with them; but we may express it in this way, since in another place Husserl writes (*loc. cit.*, p. 240): 'Dependent objects are objects of a pure species with respect to which there is an essential law that if they exist at all, they do so only as parts of more comprehensive wholes of a certain accompanying species.'

²⁴ The above criticism, which uses an argument taken from the realm of relative characteristics, is also not an immanent criticism, since Husserl deliberately takes only absolute characteristics into account in his analyses.

²⁵ See Husserl's first Proposition, *loc. cit.* p. 261.

²⁶ See Proposition 6a of my 1929, p. 12.

²⁷ If one were to assume with Meinong that psychic facts are dependent with respect to their objects, then an example would be easy to formulate. For suppose we take e.g. perceptive imagination (c) which is part of perception, i.e. the dependent whole (C). This representation is dependent with respect to its intentional object lying beyond C. In this case both of our criticisms again become valid. First, perceptive imagination does not form any whole with its intentional object, and second, the existence of the intentional object is not required for its own existence (e.g. in the case of hallucination).

²⁸ *Loc. cit.*, p. 230.

²⁹ We find this distinction in Husserl, *loc. cit.* p. 257. Our definitions of absolute independence and dependence are closer to Husserl's definitions than our definitions of relative independence and dependence.

³⁰ Since a correct judgment or true statement corresponds to each occurring state of affairs, one might in defining independence and dependence speak of the existence of a true statement instead of the occurrence of a state of affairs, and instead of an object which was the subject of a state of affairs, one could speak of the concept or name of this object.

³¹ Following the position which accepts the most general concept of the object (Twardowski, Meinong) one could try to define independence and dependence in yet another way. According to this definition an object is independent if by nature its existence or occurrence does not require some X to be an object. An object is dependent if by nature its existence or occurrence does require some X to be an object. Relative independence or dependence could be defined in a similar way. But we believe that the definitions given in the text are more fruitful. – If non-existing objects are also independent and dependent, then we can define their independence and dependence as follows: P is independent if for P to be an object it is by nature unnecessary for a state of affairs to occur in which some other object not part of the make-up of P is the subject. We can define dependence similarly.

- ³² According to the definition of dependence which we gave above, every object dependent with respect to some object requires for its existence the occurrence of a particular state of affairs in which this object is the subject and, hence, is derivatively dependent with respect to this state of affairs, in both this sense and the former sense.
- ³³ Pfänder's distinction between factual independence and dependence, and the independence and dependence attributed to objects due to certain thought forms (*loc. cit.*, p. 307f.) should not be confused with our distinction between formal, and material or factual, independence and dependence.
- ³⁴ Husserl (*loc. cit.*, p. 253) also gives this type of example, but he does not further deal with them, since he devotes his remarks to material dependence.
- ³⁵ For object A, independent with respect to B, can be dependent with respect to C, and so absolutely dependent, or else independent with respect not only to B but also to every other object, hence absolutely independent.
- ³⁶ See Twardowski, *loc. cit.*, p. 65 and Husserl, *loc. cit.*, p. 264.
- ³⁷ One should point out that the words 'independence' and 'dependence' at one time serve to designate a relation, and at another, to designate the relative characteristics built into this relation. However, this ambiguity is not harmful, since nearly everything we have said here about independence and dependence is equally valid in respect of meaning for both relations and relative characteristics.
- ³⁸ But this factor is dependent with respect to any concrete colour whatsoever, i.e. must stem from redness or greenness, etc., hence it is absolutely dependent.
- ³⁹ Yet further combinations are possible. We shall mention some of them later, whereas other absurd ones do not interest us.
- ⁴⁰ Here we may be permitted to speak of dependence as a requirement of existence, since for dependent parts of independent wholes we have accepted Husserl's definition.
- ⁴¹ In addition to these possibilities – as we have mentioned – there can be still others, so e.g. a specific extension is dependent with respect to colouredness but already in a derivative sense. For a specific extension is dependent with respect to extension, while extension is dependent with respect to colouredness and, therefore, by transitivity, a specific extension is dependent with respect to colouredness. Extension is dependent with respect to any specific colour in a similarly derivative fashion. For extension requires colouredness, and this any particular colour, and so on.
- ⁴² We take a position here according to which the terms are the *principium individuationis* of relations.
- ⁴³ *Loc. cit.*, Treatise IV.
- ⁴⁴ See Twardowski 1911.
- ⁴⁵ Recently Roman Ingarden used this concept to explain the successive 'unfolding' of the parts of a literary work (1931, p. 321 ff.)

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Barry Smith

Introduction to Adolf Reinach

On the Theory of the Negative Judgment

Adolf Reinach was born in Mainz on the 23rd December 1883. Between 1901 and 1905 he studied philosophy, psychology and jurisprudence in Munich, and his dissertation on the psychological foundations of jurisprudence, *Über den Ursachenbegriff im geltenden Strafrecht*,¹ bears traces of the psychologistic approach of his teacher, Theodor Lipps. Lipps' psychologism came under heavy criticism in Husserl's *Logical Investigations*, published in 1900/01, a work which enjoyed an almost immediate success in Munich.² A circle of philosophers was established, to which among others Reinach, Pfänder, Daubert, Theodor Conrad and August Gallinger belonged, the members of which adopted the *Logical Investigations* as their philosophical canon, awarding special significance to Husserl's account of material *a priori* relations among essences or kinds.

Between 1905 and 1909 Reinach spent long periods studying under Husserl in Göttingen, preparing in 1909 a Habilitationsschrift (*Wesen und Systematic des Urteils*) on the theory of judgment,³ many of whose ideas are summarised in the essay which follows. Reinach remained in Göttingen as a Privatdozent until 1914 when he was called to the front. He fell in Flanders on the 16th November 1917.⁴ Here we shall discuss the most important works from Reinach's Göttingen period on the material *a priori*, on the theory of judgment, and on the *a priori* structures of social acts.

§ 1 Kant and Hume on the Material *A priori*

Reinach's paper, "Kants Auffassung des Humeschen Problems", of 1911 embodies an attempted clarification of the concept of the synthetic *a priori* which was acknowledged by Husserl as having played a sig-

nificant role in the development of his pure (*a priori*) phenomenology.⁵ The paper begins by challenging the adequacy of Kant's understanding of the Humean concept of *relations of ideas*.

'Relations of ideas, in contradistinction of matters of fact,

depend entirely on the ideas, which we compare together ... 'Tis from the idea of a triangle that we discover the relation of equality, which its three angles bear to two right ones; and this relation is invariable, as long as our idea remains the same (*Treatise*, Bk I, Pt. III, § I).

Kant wishes to identify propositions expressing relations of ideas with judgments whose truth is, in his terms, 'grounded in concepts', i.e. with analytic judgments. This identification finds initial support in the fact that, in the *Enquiry* at least (see Sect. IV, Part I), it is the truths of algebra, geometry and arithmetic which are put forward in illustration of the concept of relation of ideas. In the *Treatise* however, a number of other relations of ideas are mentioned,⁶ for which the assumption of analyticity is, as Reinach argues, wholly implausible.

Consider, for example, the proposition 'red and orange are similar'. It would clearly be nonsensical to suppose that the concept of resemblance is 'contained' in the concepts of red and orange. Yet

Hume ascribes precisely the same basic determinations to the relation of resemblance and the other remaining relations of ideas as he does to the mathematical relations of ideas.

Further, not only did he

never directly characterise mathematical propositions as analytic: such a view would, when considered against the background of his other views, lead to the greatest absurdities.⁷

How, then, are we to understand relations of ideas, if not as analytic connections amongst (Kantian) concepts? A preliminary answer to this question is provided by a comparison with Locke, who is much closer to Hume at this point than is Kant. Locke separates knowledge that is genuinely instructive from analytic or 'trifling' knowledge. He explicitly contrasts the proposition 'The external angle of all triangles is bigger than either of the opposite internal angles' with empty propositions such as 'what is a soul is a soul' (*Essay*, IV, 8).

To be sure, this mathematical proposition says something about a 'complex idea', *not* however something which is 'contained in it', but rather something which is 'a necessary consequence of its precise complex idea'.⁸

Where Locke's identical propositions 'only affirm the same term of themselves', instructive propositions 'find out intermediate *ideas*, and then lay them in such order one by another that the understanding may see the agreement or disagreement of those in question.'

We shall return shortly to the question of how relations of ideas are to be understood in the framework of Humean (or Lockean) philosophy as this is viewed by Reinach. First however it is necessary to consider briefly the problem of causality ('das Humesche Problem' referred to in the title of Reinach's paper). Consider the two judgments: 'event *a* follows upon event *b*' and '*a* is connected with *b* by a relation of causal necessity'. What should be noted is that the second judgment materially enriches the content of the first:

In both cases, a relation between *a* and *b* is being asserted – in the first case only a relation of temporal sequence, in the second case one of necessary connection. The second relation, in a certain manner, includes the first within itself, but it goes far beyond the first with respect to its content. Therefore, whether I make the first or the second judgment signifies a fundamental difference in material content.

It is quite otherwise however in the case of a judgment such as ' $2 \times 2 = 4$ '. Here

I assert a relation between 2×2 and 4; but if I judge that 2×2 is necessarily 4, evidently I do not assert any new relation between the arithmetical terms (Reinach, *op. cit.*, p. 181f.).

In order to draw attention to the fact that, in the latter case, it is the state of affairs as a whole that is characterised in a specific way, Reinach designates the necessity involved as *modal* necessity. Where mathematical propositions exhibit only this modal necessity, causal propositions lay claim also to a certain kind of *material* necessity, that is to a necessary connection among the parts of the corresponding states of affairs. Fire *produces* heat, i.e. is, in its very nature, causally or necessarily connected with heat.

In Kant's view – a view which accords well with his narrow interpretation of relations of ideas as analytic connections amongst concepts –

Hume's inquiries were directed exclusively towards modal necessity, the necessity which causal propositions share with mathematical and logical propositions. Reinach, on the other hand, wants to defend the view that Hume's attention was mainly - though sometimes confusedly - directed towards *material* necessity:

What Hume wants to inquire into is 'necessary connection', and this is regarded by him as a relational predicate about which the question arises, to begin with, whether it is determined by the nature of its terms exactly as similarity is determined by the essence of two colours. Such a consideration makes sense only in the case of what we have called material necessity ... Kant repeatedly emphasises, with full justification, that necessity can never be given through experience. As contrasted with this, Hume tries to find it in experience. The reason for this lies in the fact that one of them is thinking of modal necessity, the other of material necessity (op. cit., p. 184 f.)

Hume's conclusion, of course, was that he was unable to discern in experience any evidence of necessary (material) connection between events. The inadequacy of this conclusion should not however blind us - as it did Kant - to the fine structure of Hume's presuppositions:

According to Kant, Hume saw only two possibilities: either the *foundation* of the causal judgment in pure reason, or the *explanation* of it from experience, i.e. from the mechanism of association and the 'subjective necessity arising from it', which is falsely taken to be objective. That for Hume there is a third possibility - the immediate grounding of necessity through experience - is overlooked by Kant and, from Kant's standpoint, must be overlooked. (op. cit., p. 186).

Hume could contemplate the possibility of grounding material necessity through experience only because for him, as for Reinach and Husserl, this necessity has both an epistemological and an ontological side, only because, that is to say, relations of ideas are also relations of things.⁹ Indeed, the most important conclusion to be drawn from Reinach's essay is that the treatment of the synthetic *a priori* had been set on the wrong road by the one-sidedly epistemological approach initiated by Kant.¹⁰

Relations of ideas are, in Reinach's vocabulary, *essential structures*, "structures wherein a predicate is 'conditioned by', or is grounded in, the nature of the terms that are placed in relation to one another" (op. cit., p. 166).¹¹

It is grounded in the nature of the numbers 3 and 2 that the former is greater than the latter; but there are no material things whose nature it is to lie beside each other. With exactly identical properties, things may be either near each other or far removed from each other. In the one case, therefore, the predicate is grounded in the essence of the subjects; in the other case, not so (op. cit., p. 164).

We shall see in what follows that, even amongst the things, events, and processes of the material world, a wealth of essential *a priori* connections is capable of being disclosed.

§ 2 The Philosophy of *Sachverhalte*

The concept of *Sachverhalt* or state of affairs plays a central role in Reinach's philosophy to a degree equalled, perhaps, only in Wittgenstein's *Tractatus*.¹² In the 18th and 19th centuries, logical orthodoxy in both Germany and England had rested on a conception of the judgment as a compound of concepts or presentations. Judgments thus conceived have no direct ontological correlates of their own: they are true or false in virtue of the existence or non-existence of a corresponding combination amongst the ontological correlates of their constituent concepts. Against this background, the recognition of the heterogeneity of the judgment as compared to (atomic or molecular) concepts or presentations – and in particular the isolation of the moment of assertive force by Brentano and Frege¹³ – was a considerable step forward. Yet the equally important step of recognising also a heterogeneous category of judgment-*correlates*, a category of entities in the world which would make judgments or sentences true or false, was taken neither by Frege nor by Brentano.¹⁴ The delineation of this category was first carried through effectively by Husserl in the *Logical Investigations*,¹⁵ receiving its most refined form in the essay by Reinach which follows. The distinction between propositions or meaning-correlates of judgements, and states of affairs or object-correlates of judgments, has since been taken for granted not only by continental philosophers influenced by Husserl, but also in much analytic philosophical work on logical semantics and on the ontology of facts.

Within Austrian philosophy however, particularly in the work of Bolzano, and of philosophers in the Brentano school such as Meinong

and Marty, this distinction was not made.¹⁶ Bolzano's *Sätze an sich* and Meinong's *Objektive* exhibit traits characteristic of both meaning-entities and object-entities,¹⁷ and it is difficult to see how these two sets of traits can be reconciled. *Objektive*, for example, are compared to ideal meanings in possessing an eternal or timeless existence, yet they are also viewed as being capable of containing real material objects as constituents.¹⁸ The same ontological brinkmanship is manifest in work of Moore and Russell on the proposition,¹⁹ and it can be discerned also in the work of Chisholm (another philosopher heavily influenced by both Meinong and Brentano), particularly in his conception of a generalised category of states of affairs which would include as sub-categories both events and propositions.²⁰

For Husserl, as for Reinach, the meaning-object dichotomy is firstly a distinction between the sense and quality of an act on the one hand, and the object intended in the act on the other: thus every act of judgment, for example, exhibits both a meaning and (at least if the judgment is true) an associated state of affairs. But it is secondly a distinction between the two separate disciplines of formal logic and formal ontology.²¹ A further dichotomy arises when we consider states of affairs from the point of view of existential ontology and ask after the *mode of existence* of statal entities.²² Is the existence of states of affairs dependent upon that of more or less distantly associated mental or linguistic acts? Or do they enjoy an autonomous existence, independently of mind or language? In Husserlian terms, is the state of affairs *this rose is red* a moment residing exclusively in the ontological orbit of the rose, or is it rather a moment of a larger whole constituted by, *inter alia*, a corresponding act of judgment. A variant of the first position is defended by Reinach in the essay below. The second position has been defended by, for example, Meinong and Strawson (compare the latter's claim that 'If you prise the statements off the world you prise the facts off it too'²³).

The most extreme affirmative position concerning the autonomy of statal entities is one which asserts that there is such an entity corresponding to every possible judgment, to every possible wellformed sentence, whether true or false. This position is characteristic of ontological rationalism or platonism as evinced, in different ways, by Bolzano, Frege, Meinong and Chisholm.²⁴ A view of this kind is defensible, I believe, only where it relates to entities belonging to the sphere of meaning (to Frege's 'realm of sense') or, as in Meinong's case, to some hy-

brid sphere of quasi-meanings. Where states of affairs are conceived as object-entities, tied down to the real world of Frege's 'ordinary referents', then it becomes impossible to develop intuitions which could support such all-embracing platonism: what mind-independent external referent, what constituent part or contour of the world, could correspond, for example, to a false sentence, to a counterfactual conditional, or to a judgment concerning the indefinite future?

Clearly some restriction is needed upon sentences to which autonomous *Sachverhalte* may be expected to be correlated. The most obvious restriction consists in denying objectual correlates to judgments that are false. An alternative, however, is to distinguish amongst the totality of autonomously existing states of affairs, subsistent states of affairs corresponding to true judgments, and non-subsistent states of affairs corresponding to those that are false. This is the position adopted by Meinong, by Reinach, and by Wittgenstein in the *Tractatus*.²⁵

Philosophers who adopt this latter approach may be inclined also to conceive statal entities as possessing an eternal existence, as custodians of (eternal) truth and falsity in a world of transient objects.²⁶ Wittgenstein, as is well known, adopted the opposite view, regarding objects as what is unalterable and subsistent, their configuration in states of affairs as what is changing and unstable (*Tractatus*, 2.027), a position which echoes the ontological atomism of Herbart.

The most serious controversy in the formal ontology of states of affairs however has concerned the relative status of positive and negative states of affairs. For Wittgenstein, as, for example, for Pfänder (*Logik*, Section I), all states of affairs are positive: Reinach however was insistent that there are both positive and negative states of affairs and that, whilst these have distinct epistemological properties, they are, in regard to their mode of being, indistinguishable.

Reinach's views on negative states of affairs were criticised by Ingarden in his *Der Streit um die Existenz der Welt*. Ingarden argued that if states of affairs are to be conceived as object-entities, dovetailed with the individual objects, events, properties and relations in the real world, then it is clearly justifiable to say of a state of affairs such as *this rose is red* that it exists autonomously, since here all of the constituents of the state, i.e. the rose and its individual accident of redness, themselves exist autonomously. Consider however the negative state of affairs *this rose is not blue*. Here whilst the rose itself exists autonomously, the property involved is only thought or intended; it is carried into the si-

tuation from outside by our act of judgment. Therefore, argues Ingarden, the mode of existence of such a state of affairs must be distinct from that of the positive, autonomous state of affairs.²⁷

Hence we have distinguished, at this level of generality, three alternative positions regarding the autonomy of states of affairs: Ingarden's position, according to which only states of affairs corresponding to positive, true sentences exist autonomously;²⁸ Wittgenstein's position, according to which both subsistent and non-subsistent *Sachverhalte* exist, but all are positive; and Reinach's position which allows both positive and negative, subsistent and non-subsistent *Sachverhalte*.²⁹

In the present essay Reinach considers in detail only those properties of *Sachverhalte* that are of relevance to the theory of judgment, and specifically to the theory of the negative judgment.³⁰ The clarity of Reinach's own exposition makes superfluous the duplication of his arguments here. It is however worth pointing out that, in contrast to most modern philosophical logicians, he is concerned not merely with logical (deductive) and semantic properties and relations amongst judgments (or propositions) considered in abstraction from their contexts of use, but also with the judgment as a mental act, bound up with other mental acts of recognising, thinking, arguing and inferring.³¹

His account rests on a distinction between two types of mental formation: spontaneous, temporally punctual and typically linguistically articulated *acts*, on the one hand, and non-spontaneous, enduring *conditions* or *states* typically only loosely associated with language, on the other. To the first category belong (episodic) acts of assertion, denial, questioning, etc., acts of perceptual or cognitive apprehension and of evaluation of objects or states of affairs, acts of intending an object (e.g. of meaning so-and-so by the use of a given proper name), acts of promising, commanding, forgiving, requesting, etc. To the second category belong states of conviction or belief, of having something (some object or state of affairs) before or on one's mind, of enjoying some sensation, of feeling obliged or committed to someone, etc.³² Reinach's principal charge against previous accounts of the judgment was that the distinction between judgment as assertion and judgment as conviction had been ignored, or, more generally, that the relation between the two spheres had been thoroughly misunderstood, whether in accounts of the dependence of judgment as assertion upon an underlying conviction or belief, or of the dependence of, say, an act of promising upon an underlying intention or volition. (Reinach's application of these ideas to

the sphere of legal or quasi-legal formations such as acts of promising will be considered briefly in the section which follows.)

A note on influences: In considering the influence of 'On the Theory of the Negative Judgment' it would almost certainly be wrong to assume any awareness of Reinach's work on the part of the author of the *Tractatus*, despite the similarities between the respective *Sachverhalt*-ontologies of Wittgenstein on the one hand, and of Reinach and the other Munich phenomenologists on the other.³³ These similarities are almost certainly to be attributed to a shared influence upon both Reinach and Wittgenstein of the work of Meinong, and perhaps also of Stumpf and Husserl.³⁴ Where Reinach did exert a substantial influence was upon the members of the Munich-Göttingen circle of phenomenologists themselves, and in particular upon Ingarden, whose 1925 analysis of the category *problem*³⁵ owes much to the Reinachian approach to states of affairs. Reinach exerted an importance influence also upon Otto Selz, a Würzburg psychologist who applied Husserlian and Reinachian ideas in his work on the psychology of thinking.³⁶ Consideration of these and other influences will however have to be postponed for another place.

§ 3 The Theory of Social Acts

The fundamental principle of Reinach's philosophy may be expressed as follows: for every domain of objects, whether psychological or material, mathematical or grammatical, a determinately structured family of essences can be discovered, standing in *a priori* relations to each other, as a reflection of which corresponding *a priori* laws hold of the objects in question.

These laws are certain and unchangeable; they are prior to any human convention and would obtain even though never actually recognised by any thinking subject.³⁷ A system of material essences can be disclosed, for example, for the domain of human emotions, for human value-phenomena, including the phenomena of ethics,³⁸ and – as in Reinach's own essay below, – for the sphere of judgment and of cognitive phenomena in general.

Reinach's own most original contribution to philosophy lay in the application of this principle to legal and quasi-legal formations as these occur in human societies, and in particular to acts of promising, apolo-

gising, forgiving, requesting, commanding, and so on, and to the social phenomena associated with these. The peculiar characteristics of acts of this kind were re-discovered several decades after Reinach's work by Anglo-Saxon speech act theorists.³⁹

Reinach's work on social acts is presented in his "Die apriorischen Grundlagen des bürgerlichen Rechts" (The *A priori* Foundations of the Civil Law), first published in volume I of Husserl's *Jahrbuch* in 1913.

Consider, first of all, the social formations of claim and obligation. Many philosophers before Reinach's day had been tempted to regard these formations as reducible in some way to the mental experiences, or to the beliefs or dispositions of the subjects involved. All such theories however, as Reinach argues,⁴⁰ completely by-pass that which is essential to these formations. For whilst there is, certainly, such a thing as a belief in the existence of a claim or of an obligation, this is something which, as a matter of principle, can be identically constituted whether or not the purported claim or obligation really exists, and even independently of whether it belongs to the subject in question or to some alien subject. Further it is clear that claims and obligations can exist in the absence of any knowledge or beliefs of this kind, as they can exist also in the absence of any feelings of entitlement.⁴¹

Claims and obligations are like mental entities, however, in the sense that they always and of necessity require a bearer (typically an individual person): they can, like mental experiences, be regarded as individual moments of their bearers. In addition we can see that claims and obligations of necessity require a determinate content: every obligation has as its content some future conduct (*Verhalten*, that is action or forbearance) on the part of its bearer, and this content is shared by the claim or claims with which the obligation is interwoven. A claim or obligation also has a specific temporal structure: no claim or obligation comes into existence without some specific ground or reason for this existence. In the terminology of Husserl's third Logical Investigation, then, they are founded upon (require of necessity to exist in a more inclusive whole with) events of specific types, for example acts of promising.⁴² It is a synthetic *a priori* truth that an act of promising immediately and of necessity brings about a mutually correlated claim and obligation.

The commonly held view of the act of promising had been that it is simply the expression of an act of will or of an intention to act in the interests of the party before whom the declaration is made. The most ob-

vious inadequacy of this account is that it throws no light on the problem of how such an utterance should bring about a claim and obligation of the appropriate kind. It is after all clear, that the bare intention to do something has no quasi-legal consequences of the given sort, and it is difficult to see how any essential difference is made by the simple expression of such an intention.

In the wake of Austin and Searle the fundamental categorial distinction between promising and communicating one's intention to do something is readily accepted. Reinach sought, in his 1913, to provide a complete and systematic theory of all such phenomena. Both types of act, he points out, involve deliberate linguistic utterance. They fall within the (wider) class of what Reinach calls *spontaneous* acts, i.e. acts which consist in a subject's bringing something about within his own psychic sphere, as contrasted with passive experiences of, say, feeling a pain or hearing a noise.⁴³ Not all spontaneous acts are linguistic in character (consider, for example, a deliberate turning of the attention toward something). Further, there are certain types of spontaneous act which may involve an overt linguistic utterance but for which this moment of overtness is non-essential: for example, acts of judgment (one may pass judgment without giving any outward sign of having done so), or also acts of forgiving, praising, cursing, blaming, praying, and so on. In each case an overt linguistic utterance is a possible but not a necessary, constituent of the act in question.

One can already clearly see, however, that for certain other types of spontaneous act this dispensability of a linguistic utterance does not apply. The utterance of a command, for example, or of a promise or question, is clearly a necessary or essential component of the total act. Reinach accordingly divides spontaneous acts into two classes, which he calls *internal* and *external*, according to whether the act's being divulged, being brought to overt expression, is a dispensable or separable piece, or an indispensable, inseparable moment of the act-complex in question.⁴⁴

A further division, one which applies not merely to spontaneous acts but to mental phenomena generally, is that between *non-self-directable* acts, i.e. acts which demand of necessity an alien subject toward whom they are directed (whether internally or externally), and acts such as love and hate which may be directed toward one's self.⁴⁵ Again, an act of commanding clearly presupposes essentially the existence of one or more alien subjects to whom the command may be addressed; such an

act, if it is to exist at all, demands not merely that – like all mental phenomena - it have a bearer, but also that one or more additional subjects should exist toward whom it is directed. It is necessary, in other words, that the bearer of the act should exist as part of a more inclusive whole whose constituents are connected together in some specific way (in this case through the enduring relationship between commander and commanded which consists in the fact that the former has authority over the latter).

A further peculiarity is possessed by certain specific types of external, non-self-directable acts, that they are such that their constituent utterance must of necessity be *grasped* by the subject(s) toward whom the act is directed: the issuer of a command must not merely utter the command in public; he must direct this utterance to its addressees in such a way that it is received and understood by them in an appropriate way. This peculiarity of commands, that they stand in need of being directed to and perceived by their addressees, is absent, for example, from acts of forgiveness. Reinach introduces the term *social act* to designate those spontaneous acts which stand in need of being addressed to and of becoming correlately perceived by their addressees. A social act

is an action of the subject to which is essential not only its spontaneity and its intentionality, but also its being directed towards alien subjects and its standing in need of being perceived by those subjects. What has been said of commands holds also for requests, admonitions, questionings, informings, answerings, and many other types of act. They are all social acts which are, in their execution, cast toward an alien subject that they may take hold of or bring about effects inside him (*einem anderen zugeworfen um sich in seine Seele einzuhacken*).⁴⁶

It is, Reinach argues, essential to the social act that it be a single, integral whole articulated both internally and externally: that it should exhibit both internal (psychological) moments and external (physical) moments (in being, typically, an overt linguistic utterance). Inner experiences of many kinds – shame, or love, or anger – may equally be brought to overt expression; but here the outward-facing moment is to some degree an arbitrary and dispensable supplement in the total act. In the case of social acts proper, in contrast, it is not as though we have, in the act, two independent parts, more or less intimately associated with each other: the act, in being executed, constitutes itself as an inextricable whole within which the internal and external moments exhibit a

manifold of interdependencies and can be distinguished from each other only abstractly.

This peculiarity of social acts is clearly expressed by Reinach in his critique of Hume's account of the act of promising. Hume, as is well known, seeks to discover a type of mental act which might accompany the utterance of a promise and thereby lend it its peculiar status as a promise. But the attitude in which this search is carried out is mistaken from the start. Hume

wishes to discover an experience which becomes expressed in a promise, which therefore could be present *without* the presence of any accompanying expression. And of course he cannot succeed in exhibiting such an inner experience. He rejects quite properly the experiences of resolving, wishing, willing; but what he does not see at all, is that besides such inner experience there exist also activities of the mind which do not merely find in words their accidental, supplemental expression, but which come to execution in the act of speaking itself and of which it is characteristic that they announce themselves to another by means of this or some similar external appearance.⁴⁷

So, too, it is easy to see that there is no independent and self-contained mental experience which is somehow brought to expression in the issue of a command. There could not even in principle be such an experience. Yet it is nevertheless true that social acts of necessity presuppose or are founded upon appropriate conditions of mind on the part of their bearers.

The act of imparting presupposes a conviction of the content which is imparted. An act of questioning essentially excludes such a conviction, requiring instead a state of uncertainty in relation to its content. The psychological presupposition of a request is the wish that that which is requested should happen or, more specifically, that it should be brought about by he to whom the request has been addressed. A command has for its foundation not merely the desire, but the volition, that the addressee should carry out that which has been commanded.⁴⁸

All of this is of course to assume that one distinguishes a request as such from a sham (merely apparent) request, a question as such from a conversational ploy, etc. Indeed every social act is of necessity subject to a system of essential modifications of these and other specific types, subject to secondary *a priori* laws of their own. Thus when a speaker executes an act with the intention of presenting it as a social act of a given type but from which the necessary underlying foundation is missing,

then of course we do not have a mere string of words: lies and dissimulations, too, are social acts of determinate types.⁴⁹

We might summarise the above by means of the following examples:

The internal and external moments of the social act of imparting or informing: I can be convinced of (believe in) a particular state of affairs without ever giving expression to this conviction in an assertion, or only in an assertion which I keep to myself. For an imparting of the state of affairs to be effected it is essential not only that an assertion be made, but also that this assertion be specifically addressed to a second subject who is in a position to perceive and understand it.

Here, as in all subsequent cases, the social act in question can be considered as an abstract moment of a whole consisting of two (or more) subjects reticulated together in a specific way. The present example involves the simplest possible whole of this kind, consisting merely of two subjects connected together momentarily by a single, fleeting cartilaginous band, namely the particular utterance itself. In subsequent cases the encompassing wholes with which we have to deal will exhibit more complicated structures. Consider, for example, the encompassing whole which must exist if a social act of joining together in holy matrimony is to be effected.

The internal and external moments of the social acts of requesting and commanding: These two acts may share an identical content and exhibit identical physical appearance (differing, perhaps, only in their emphasis or tone of voice). They differ most importantly in the presence in or underlying the latter of a prior enduring relationship of subjugation, a legal formation of a quite specific kind, different from any merely psychological dispositions which may equally accompany an act of requesting.

An important difference between these acts and acts of imparting is that where, in the latter, the addressee's becoming aware of the content of the act is of itself sufficient to bring to fulfilment the project which is announced in the act (or, as we might also say: to saturate the whole which is initially projected by it), this leads only to a provisional staging-post in the case of acts of requesting or commanding. Here it is only the subsequent realisation of the request or command in appropriate

conduct of the addressee that can truly close the circle which has been opened with the execution of the initial act.

The internal and external moments of the social acts of questioning and answering: Questioning, too, is a type of social act which somehow demands or calls forth a specific response on the part of the addressee, namely, a further social act of answering. The latter does not demand in its turn any subsequent action on the part of he to whom it is addressed, but rather presupposes some prior action, and indeed presupposes always and essentially a social act of a determinate type.

Thus we can distinguish, following Reinach, social acts of, e.g. answering or ratifying, which presuppose other, prior social acts; and social acts of requesting, questioning, commanding, promising, etc., which aim towards subsequent social acts or toward activities of other kinds. And we can see that the relationships holding amongst social acts and among the act-complexes into which they enter as essential constituents may exhibit certain specific kinds of temporal determinations. It is not, however, as though a social act of, say, promising, can *necessitate* the future realisation of appropriate conduct: it can at most establish what has been called an *a priori* tendency,⁵⁰ a necessitation of a type which is conditional upon the absence of supervening considerations (e.g. of a moral nature, or relating to physical impracticability).⁵¹ As a result of the episodic tie between hearer and receiver which is the act of promising, these two individuals become affected, as regards their subsequent conduct in relation to each other (their *sich zueinander Verhalten*), in virtue of the fact that the relation between them is not a saturated relation (*befriedigte Beziehung*), resting complete in itself.⁵² It is such as to call forth a specific type of action on the part of the bearer of the obligation brought into being by the act, namely the realisation of the relevant content. An obligation therefore carries in itself the determination to bring itself to an end, and a tendency also towards the dropping apart, as it were, of the two individuals who have become related to each other through obligation.

§ 4 On the Nexus of Representation

It would be impossible for us to go into detail here concerning the whole range of issues discussed by Reinach in “The *A priori* Founda-

tions of the Civil Law". In particular we cannot consider Reinach's discussions of legal rights and of the origins of legal rights, claims and obligations, nor his discussion of collective and divided ownership, of contract theory, and of those particularly important kinds of speech acts which, when executed by jurists, result in the establishment of new law. We cannot consider either the various possible modifications of social acts – of which the modification of the act of imparting information in an act of lying is just one amongst a whole range of examples. We shall however discuss one particularly important species of such modification, since this may serve to clarify further aspects of Reinach's theory of social acts and of the relations of this theory to his general philosophical methodology.

It is a commonplace that acts of, say, thanking, blessing, commanding, promising, accepting (delivery of something), perhaps also acts of murder and the like, may, under appropriate conditions, be carried out by one individual *in the name of* another.⁵³ The determination of which acts are, of their nature, subject to this proxy-modification is one task of the *a priori* theory of social acts and of related formations.

The execution of an act in the name of another is of necessity quite a different thing from its being carried out 'in his spirit', that is, in such a way as to conform to his presumed or expressed intentions. I can act in the spirit of another without acting in his name, and I can act in his name without acting in his spirit. Indeed it is not any kind of knowledge of the intentions of the represented party which forms the inner state or mental condition on the basis of which the acts of the representative are executed, but rather the intention that the consequences which ensue from this execution should fall not to him, but to the person in whose name he acts. Thus, when I execute a promise on another person's behalf, no obligations are acquired by me thereby, but rather – in appropriate circumstances – by this other person. As Reinach points out, this possibility – that the rights and obligations of a person may be transformed or brought to an end, or that he may acquire new rights without being himself aware of it – seems at first quite extraordinary.⁵⁴ Clearly however I cannot execute, say, promises in the name of another purely at random; not even the will of this other to acquire specific obligations suffices to make my promises in his name effective. Some relevant conduct on his part is presupposed. One might initially suppose that the execution by him of a prior act of promising would be what was required. But the consequences of a promise are in no way identical with

those of a genuine relationship of representation. For if the principal promises his representative that he will do that which is promised in his name, then the obligation which he acquires is an obligation which arises not from any promise which is subsequently executed by proxy, but from his own promise. And moreover it is then not the relevant third party to whom he is obliged – as it would be in the case of the obligations flowing out of an effective relationship of representation, – but the representative himself.

And if instead he should promise this third party that he will indeed effect that which his representative might subsequently promise in his name, then whilst there arises here an obligation of the required content and in relation to the appropriate person, it is one which flows directly out of this initial promise: the subsequent act of the representative serves merely to make known, in regard to this particular obligation, what its particular content is. The obligation is not at all one which springs from the promising act which becomes executed on the principal's behalf, as it would be in the case of a genuine nexus of representation.

Nor, either, is the representation relation rooted in social acts of commanding. The representative is not, for example, a mere messenger of the represented party. Even should a representative be charged, in the fullest detail, with executing only a single act of promising in the name of someone else, he remains therein a representative; he retains at least the authority to execute just this act. The messenger, in contrast, is not the executor of any legal-social act at all: the social act of imparting information which he does perform stands, rather, in the service of the legal-social act which is executed through it by another.

How, then, does the nexus of representation become established? We have seen that persons are able, by executing acts of promising, to acquire obligations for themselves. How can they come to have the authority to generate obligations for others? Clearly this is something which can be granted only by the person who will in fact acquire the obligations in question. The act of granting such authority is not however an act of transfer (comparable, e.g. to a transfer of property), for the authority remains at the same time and will continue to remain in its original hands. It is, rather, as if this authority is something *sui generis* which can be re-generated, re-created anew in the person of another.

The act of granting such authority by the principal to his agent will of course typically be supplemented by further information-giving social

acts expressing the will of the principal as to how his representative should proceed on his behalf in given circumstances. But such acts are not essential constituents of the act of granting authority itself (they can be wholly absent without the latter thereby suffering the least effect). This essential separateness of granting authority and conveying information is further revealed in the fact that should the representative subsequently act in the name of but against the stated intentions of his principal, this can in no way be taken to have damaged his status as a representative, and he will suffer retributive consequences only to the extent that he had himself entered from the start into an *additional* obligation to abide by such intentions.

In Husserlian terms we can say that a representative *b*, in his capacity as a representative of *a*, and *a* himself, *qua* represented party, are mutually founding, interdependent moments. As a result of *a*'s having granted to *b* an appropriate authority the two individuals are mutually adjusted to each other and remain so adjusted as long as this authority does not expire or become rescinded in a new social act on the part of *a*. During this interval *a* and *b* need have no further dealings with each other: their mutual adjustment is as it were relatively isolated from the consequences of their respective behaviour.

In the course of his operations *b* will typically execute a variety of social acts in the name of his principal. Fixing on one such act, say a proxy act of promising in which *b* becomes momentarily tied to another party *c*, we can see that as a consequence of this momentary tie *a* automatically becomes enduringly reticulated with *c* – even though the two parties may in fact have had no direct dealings with each other, – since the obligations to *c* arising from the promise accrue to *a*. Hence here, as in other cases, there are two levels of interconnection between the various constituents of the founded wholes before us. *a* and *b*, first of all, must have become mutually adjusted to each other in an enduring nexus of representation. Only thereby can *b* (in his capacity, etc.) proceed to erect a temporally punctual link between himself and *c* which will have among its automatic consequences the immediate establishment of a second temporally extended mutual adjustment between *a* and *c*.

Proper names, too, according to a familiar theory, are said to go proxy for their objects (see e.g. *Tractatus*, 4.0312); and whilst Reinach himself did not discuss this matter, it may be of some interest to round off this introduction to his essay with a brief consideration of the proxy theory of names in the light of the Reinachian theory of social acts.⁵⁵ It

will help us to gain some initial insight into the name-object nexus if we pause to consider that species of social act of most immediate relevance to the theory of proper names, the act of legal baptism. The possibility of baptism demands, first of all, that a corresponding authority be vested in some individual in virtue of which appropriately constituted acts of this individual have as their *a priori* consequences certain legal effects. We might be tempted to suppose that his acts have the nature of commands, in establishing certain obligations on the part of the members of the relevant community to act in certain ways. This could imply at most however that the community is obliged to behave *as if* 'Gottlob', say, were Gottlob's name, which could suffice only to establish a certain regularity of association of 'Gottlob' with Gottlob: the fact that this is his *name* would remain thereby untouched.

Rather, we must recognise that the baptismal authority is empowered to generate new legal formations of a quite peculiar, hitherto unrecognised kind, namely *names*. It is by now well-understood that an act of promising has, as a matter of *a priori* necessity, the immediate effect of bringing into being the mutually correlated legal formations of claim and obligation. So, here, the act of baptism has as its immediate effect the bringing into being of the legal formation which is the baptismal name. The peculiarity of baptismal legal formations is that they are also units of language and belong, in this capacity, to the subject-matter of the various linguistic disciplines of phonology, etc.

But what is the nature of such formations *qua* objects of the *a priori* theory of law? Once again it is foundation relations which we have before us. 'Gottlob' *qua* name-of-this-particular-individual depends for its existence upon the existence of this individual; as a knighthood, for example, or an individual knowledge of Greek, is necessarily dependent upon the existence of its bearer. Gottlob's name is, then, a moment founded upon the whole which is (the baptised) Gottlob – and this will help us to explain how 'Gottlob' *qua* name of this particular individual can differ from 'Gottlob' *qua* name of that particular individual. What is before us here is not any arbitrary convention to treat one and the same linguistic unit as if it were the name of two separate individuals, but rather legal formations which are already *two different objects*, as my knighthood or headache is already, in virtue of being mine, a different object from your knighthood or headache.

We can now see the sense in the analogy between legal representation and the naming relationship: the proper name (*qua* name) can

serve as an adequate proxy for its bearer (qua named object) in virtue of being founded upon the latter in the Husserlian sense. Just as, no matter what geographical distance may separate a representative from his principal, the two remain inextricably bound together, so too a proper name, even when at work in contexts which are far flung in relation to its bearer, functions in its jurisdiction as that object's name. A range of further applications of the methodology of foundation relations, not to proper names, but to sentences and to sentence-using acts, will be found in the essay which follows.

Notes

¹ Leipzig: Barth, 1905.

² 'It is reported amongst the earliest of Husserl's students that in the Summer of 1902 – that is between one and two years after the appearance of the *Logical Investigations* – a student rode by bicycle from Braunschweig to Göttingen where, thick with dust, he entered the Husserl house. The two of them argued for hours about the *Logical Investigations* until Husserl rose excitedly, called his wife into the room, and said, "Here is someone who has read – and understood – my *Logical Investigations*!" It can be said that this conversation initiated that school of philosophy which has come to be known as the phenomenological movement. The student in question was Johannes Daubert' (F. G. Schmücker, *Die Phänomenologie als Methode der Wesenserkenntnis unter besonderer Berücksichtigung der Auffassung der München-Göttinger Phänomenologenschule*, Dissertation, Munich, 1956, p. 1).

³ Unfortunately the manuscript of this work, along with other substantial Nachlass material, seems to have been destroyed: see E. Avé-Lallemant, *Die Nachlässe der Münchener Phänomenologen in der Bayerischen Staatsbibliothek*, Wiesbaden: Harrassowitz, 1975, the apparatus of which includes a useful general account of the Munich-Göttingen school. Cf. also vol. I, ch. 4 of H. Spiegelberg, *The Phenomenological Movement. A Historical Introduction*, 2nd ed., Haag: Nijhoff, 1965 and Schmücker, op. cit.

⁴ During these final years Reinach began a work on the phenomenology of religion to be called *Das Absolute*, and the ethical and religious ideas he developed in this period – together with his conversion to Christianity – exerted a considerable influence on his contemporaries in Göttingen. Reinach's conversion was followed by that of his wife, his sister, his brother and sister-in-law and many of his friends and pupils, including Edith Stein (cf. Stein's autobiography, *Aus dem Leben einer jüdischen Familie*, Freiburg: Herder, 1965, esp. pp. 172–219). His sister Pauline joined the Benedictine nuns of Mont Vierge in Wépion, Belgium, and it is reported that before her arrival at Mont Vierge the Prioress addressed the chapter as follows: 'Tomorrow we shall have the great joy of welcoming to our community a new member, who is, even according to the flesh, a sister of Our Lord' (reported by J. M. Oesterreicher, *Walls are Crumbling*, New York: Devin-Adair, 1952, p. 118).

⁵ Reinach's paper was published in *Zeitschrift für Philosophie und philosophische Kritik*, 141, 176–209; cited here (with a number of small amendments) according to the

translation by J. N. Mohanty in *Southwestern Journal of Philosophy*, 7, 1976, 161–88. Husserl's acknowledgment appears in his obituary of Reinach in *Kantstudien*, 24, 1917, 147–49. Cf. also Husserl's remark, 'It was really Reinach who introduced me to my *Logical Investigations*, and in an excellent way,' quoted by Oesterreicher, op. cit., p. 87. Reinach was involved in the initial reworkings of the text of the *Logical Investigations* in 1911 for a planned second edition of the work (K. Schuhmann, *Husserl-Chronik*, Haag: Nijhoff, 1977, p. 155).

⁶ See Book I, Pt. III, Sect. I, and compare the passage from the appendix to Book III of the original edition appended as a footnote to Book I, Pt. I, Sect. VII by Green and Grose (p. 328): "Tis evident, that even different simple ideas may have a similarity or resemblance to each other; nor is it necessary, that the point or circumstance of resemblance should be distinct or separable from that in which they differ. *Blue* and *green* are different simple ideas, but are more resembling than *blue* and *scarlet* ... 'Tis the same case with particular sounds, and tastes and smells ...'

⁷ (Reinach, op. cit., p. 168f.) Kant's interpretation of Hume was nevertheless, at least in Reinach's day, almost universally shared. One exception is the dissertation, *D. Humes Lehre vom Wissen. Ein Beitrag zur Relationstheorie im Anschluß an Locke und Hume*, Leipzig: Engelmann, 1901, by Paul Linke, another philosopher who began his career as a student of jurisprudence and came to the study of philosophy and psychology as a result of the impact of Theodor Lipps.

It is noteworthy how many philosophers in the Austrian tradition reveal the influence of Locke and Hume as contrasted with the Kantian and Hegelian ideas dominant in the North of Germany. See e.g. Meinong's *Hume-Studien*, and G. Davie, "Edmund Husserl and 'the as yet, in its most important respect, unrecognised greatness of Hume'" in G. Morice, etc., *David Hume. Bicentenary Papers*, Edinburgh University Press, 1977, 69–76.

⁸ Reinach, op. cit., p. 179. The issue before us can be summarised as follows: are we constrained to understand Locke's term 'consequence' here as signifying analytic (logical) consequence, or can we appeal to some rudimentary notion of *ontological* consequence in the sense of § 6.4 of the essay by Smith and Mulligan above?

⁹ This two-sided view of ideas finds echoes in the work of Herbart, a German philosopher who stands close to Hume also in many other respects. See Husserl's discussion of Herbart's views in § 59 of the Prolegomena to the *Logical Investigations*.

¹⁰ A further, related criticism of Kant's epistemologism is presented by Reinach in his 'Die obersten Regeln der Vernunftschlüsse bei Kant', *Kantstudien*, 16, 1911, 214–33.

¹¹ Here both 'predicate' and 'term' have not only an epistemological or logico-grammatical but also an ontological significance. Compare n. 8 above.

¹² The dominant role played by this concept in Reinach's thought can almost certainly be ascribed, at least in part, to his legal background. As had been continually stressed by German philosophers of law from the 18th century onward, that which is of primary importance within the orbit of the law (or within any given legal trial) belongs not to the category *event* or *action*, but to the category *state of affairs*, i.e. is a matter of (actual and possible) relations of conduct or *Verhalten* amongst individual legal subjects. See the preamble to my "Law and Eschatology in Wittgenstein's Early Thought", *Inquiry*, 14, 1978, 425–41 and the references there given, as well as the discussion in § 1 of the essay by Smith and Mulligan above.

¹³ The molecular conception of the judgment survives even in Bolzano's theory of the *Satz an sich* as a complex of *Vorstellungen an sich*, and it can be discerned also in the multiple relations theory of judgment advanced by Russell. The account of assertive force put forward – in echo of certain ideas to be found already in Descartes – by Brentano (for references see the discussion in § 5 of Reinach's essay), is of course distinct from that of Frege. In particular, Brentano distinguishes both positive and nega-

tive moments of force (moments of assertion and denial), where Frege, more correctly, distinguishes only positive force: see M. A. E. Dummett, *Frege. Philosophy of Language*, London: Duckworth, 1973, ch. 10. In this respect Reinach, despite his roots in the Brentano-Husserl tradition, is closer to Frege than to Brentano: see §§ 13, 16 and 19 of the essay below. For a criticism by Reinach of Frege's theory of number see his 1914 Marburg lecture 'Über Phänomenologie', first published in Reinach's *Gesammelte Schriften*, Halle: Niemeyer, 1921, 379–405, Eng.trans. by Dallas Willard, *The Personalist*, 50, 1969, 194–221.

- ¹⁴ Brentano did advance a theory of *Sachverhalte* (called by him *Urteilsinhalte*) in his early writings, but he later came to conceive all talk of judgment-correlates as necessarily eliminable: see part III, 3 of Brentano's *Wahrheit und Evidenz*, Hamburg: Meiner, 1958, 'Gegen sogenannte Urteilsinhalte, Sätze an sich, Objektive, Sachverhalte'.
- ¹⁵ Aspects of the theory of the *Sachverhalt* were anticipated in certain scholastic writings (see I. Habbel, *Die Sachverhaltsproblematik in der Phänomenologie und bei Thomas von Aquin*, Regensburg: Habbel, 1960) and in the works of a number of 19th century German logicians, especially Beneke and his followers. Here it is perhaps the writings of Julius Bergmann which are of greatest importance. Almost uniquely amongst philosophers outside the Brentano tradition, Bergmann employs the word *Sachverhalt* as a technical term of his logic. See e.g. Bergmann, *Allgemeine Logik*, part I, *Reine Logik*, Berlin: Mittler, 1879, *passim*. The official Brentanist account of the origin of the term is given by Reinach in § 8 of his essay below. Cf. also my article, "Sachverhalt. I" in K. Gründer, ed., *Historisches Wörterbuch der Philosophie*, Basel: Schwabe, forthcoming.
- ¹⁶ As Reinach, as early as 1914 expressed it, "alle Österreicher verwechseln Satz und Sachverhalt beständig" (*Nachlass B II 5*, p. 375).
- ¹⁷ On the meaning/object dichotomy see my "Essay in Formal Ontology" *Grazer Philosophische Studien*, 6, 1978, 39–62 and "Frege, Husserl and the Ontology of Reference", *Journal of the British Society for Phenomenology*, 9, 1978, 111–25.
- ¹⁸ The *Objektive: that there are horses*, for example, contains horses among its constituents.
- ¹⁹ Cf. Dummett's criticism, *op. cit.*, p. 153: "Since Moore and Russell drew no distinction, for what they considered to be genuine proper names, between sense and reference, the meaning of a proper name, that is, the object for which it stood, was for them an actual constituent of the proposition. While the proposition was intended by them not to be a full-blooded denizen of the real world, so to speak, the fact that among its constituents were actual objects belonging to that world gave it a curious hybrid status." Certainly in the case of Russell one can detect Meinong's influence in the development of his theory of the proposition.
- ²⁰ See e.g. his *Person and Object*, London: Allen and Unwin, 1974, ch. 4.
- ²¹ Consider the discussion in the Prolegomena to the *Logical Investigations* in which Husserl considers the question of what makes the activity of scientists science. This "is certainly not the psychology of scientists' acts, nor any real context into which these acts of thought are fitted, but a certain objective or ideal interconnection which gives these acts a unitary relevance, and, in such unitary relevance, an ideal validity ... Two meanings can be attached to this objective interconnection which ideally pervades scientific thought, and thus also to science as such: it can be understood as an *interconnection of things* to which our thought experiences (actual or possible) are intentionally directed, or on the other hand, as an *interconnection of truths* in which this unity of things comes to count objectively as being what it is. These two things are given together *a priori*, and are mutually inseparable." (§ 62).
- ²² The opposition between formal and existential ontology is taken from Ingarden, *Der Streit um die Existenz der Welt*, Tübingen: Niemeyer, 1964/65, esp. vol. I and ch. 7 of vol. II/1. Ingarden also distinguishes a third discipline of material ontology.

- ²³ P. F. Strawson, "Truth", *Proceedings of the Aristotelian Society, Supplementary Volume*, 1950, as repr. in *Logico-Linguistic Papers*, London: Methuen, 1971, p. 197.
- ²⁴ Consider, for example, Frege's talk of an 'eternal realm of thoughts' (discussed in detail by C. Thiel in the final chapter of his *Sense and Reference in Frege's Logic*, Dordrecht: Reidel, 1968; cf. also my "Frege, Husserl and the Ontology of Reference").
- ²⁵ Meinong employs a slightly different terminology according to which all *Objektive* subsist (*bestehen*), but only some are factual (*tatsächlich*). For Wittgenstein's view see e.g. *Tractatus* 2.04: 'Das Bestehen und Nichtbestehen von Sachverhalten ist die Wirklichkeit'. A valuable account of this aspect of Wittgenstein's early thought is provided by R.-A. Dietrich, *Sprache und Wirklichkeit in Wittgensteins Tractatus*, Tübingen: Niemeyer, 1974.
- ²⁶ See Chisholm, op. cit. A related position was advanced by M. Honecker in his *Gegenstandslogik und Denklogik. Vorschlag zu einer Neugestaltung der Logik*, Berlin: Dümmler, 1921, pp. 110ff. Honecker puts forward a conception of states of affairs as the locus of existence of the past. According to this view not objects, but states of affairs, would form the subject-matter of the discipline of history.
- ²⁷ Ingarden's criticism of Reinach appears in § 53 of vol. II/1 of *Der Streit um die Existenz der Welt*; a translation by A. Szylewicz is to appear in B. Smith, ed., *For Roman Ingarden. Studies on the Borderlines of Philosophy, Linguistics and Literary Theory*, Amsterdam: Benjamins, forthcoming. For an account of the dispute between Reinach and Ingarden and of its parallels in analytic philosophical discussions of facts see my "Essay in Formal Ontology".
- ²⁸ Ingarden awarded a merely intentional existence to states of affairs correlated with sentences expressing negative judgments, and with certain other types of sentences (e.g. sentences expressing empirical possibilities, and fictional sentences); see *Der Streit um die Existenz der Welt*, ch. 9, and *Das literarische Kunstwerk*, Halle: Niemeyer, 1931, *passim*.
- ²⁹ The fourth combinatorially possible position, according to which both positive and negative states of affairs exist, but only those corresponding to true sentences, is defended by Honecker, op. cit.
- ³⁰ As Reinach's essay below makes clear – see especially the discussion of Windelband and Brentano, – a controversy had raged amongst logicians concerning the status of negative judgments which were, for a number of reasons, taken to be more problematic than their positive counterparts. Reinach's essay can be regarded as a contribution to this debate bringing to bear ideas derived from Husserl's *Logical Investigations*.
- ³¹ The judgment is, in the terminology of Husserlian part-whole theory, a whole exhibiting both a concrete phonetic moment and a concrete psychological moment. On the abstract character of the proposition as this is conceived within modern philosophical logic see Dallas Willard, "The Paradox of Logical Psychologism: Husserl's Way Out", *American Philosophical Quarterly*, 9, 1972, 94–99.
- ³² On this distinction see also F. Bassenge, "Hexis und Akt. Eine phänomenologische Skizze" *Philosophischer Anzeiger*, 4, 1930, 163–68. The opposition between act and condition is an outgrowth of the Aristotelian dichotomy of $\delta\upsilon\nu\alpha\mu\iota\zeta$ and $\epsilon\nu\epsilon\omicron\gamma\epsilon\iota\alpha$.
- ³³ Thus consider, for example, the following passage from Otto Selz, *Über die Gesetze des geordneten Denkverlaufs*, vol. I, Stuttgart: Spemann, 1913, a work influenced by both Husserl and Reinach: "It was Stumpf who introduced the expression 'Sachverhalt' as a technical term. Instead of this we use the expression 'Sachverhältnis', in order to bring to expression with the word 'Verhältnis' (relation) the peculiar nature of the Sachverhältnis as *ein sich zueinander in einer bestimmten Weise Verhalten von bestimmten Gegenständen*" (p. 131 n., my italics). Compare e.g. *Tractatus*, 2.03: Im Sachverhalt verhalten sich die Gegenstände in bestimmter Art und Weise zueinander.

- ³⁴ See my "Essay in Formal Ontology" and also "Wittgenstein and the Background of Austrian Philosophy", in *Wittgenstein and his Impact on Contemporary Thought*, Vienna: Holder-Pichler-Tempsky and Dordrecht: Reidel, 1978, 31–35.
- ³⁵ "Essentielle Fragen", *Jahrbuch für Philosophie und phänomenologische Forschung*, 7, 1925, 125–304.
- ³⁶ See especially *Über die Gesetze des geordneten Denkverlaufs* and the paper "Existenz als Gegenstandsbestimmtheit" which Selz contributed to the *Lipps-Festschrift* in which "On the Theory of the Negative Judgment" itself appeared (cf. *Münchener Philosophische Abhandlungen*, 259–93). Selz's later psychological writings (see the bibliography at the end of this volume) have been unfairly neglected by contemporary psychologists: he has made important contributions, in particular, to our understanding of phenomenal intensity, and of our experience of spatial and temporal wholes. Other philosophers influenced by Reinach included Kurt Stavenhagen, whose book *Absolute Stellungnahmen. Eine ontologische Untersuchung über das Wesen der Religion*, Erlangen: Philosophische Akademie, 1925, rests heavily on Reinach's classification of acts and in particular on his theory of *Stellungnahmen* or position-takings. Wilhelm Schapp's *Die neue Wissenschaft vom Recht. Eine phänomenologische Untersuchung*, 2 vols., Berlin: Verlag für Staatswissenschaft und Geschichte, 1930/32 also displays evidence of the influence of Reinach, as does Herbert Spiegelberg, *Gesetz und Sittengesetz. Strukturanalytische und historische Vorstudien zu einer gesetzesfreien Ethik*, Zurich und Leipzig: Niehaus, 1935.
- ³⁷ See in particular Reinach's Marburg lecture "Über Phänomenologie", Eng. trans. p. 213: 'It is states of affairs which are *a priori*, in that the predication in them – the being-B, let us say – is required by the nature of the A; that is, in that the predication is necessarily grounded in that nature. But states of affairs obtain indifferently of what consciousness apprehends them, and of whether they are apprehended by any consciousness at all. In and for itself, the *a priori* has not even the least thing to do with thinking and knowing' - nor, we may add, with speaking.
- ³⁸ Analytic philosophers have often put forward the realm of feelings as a paradigm of the subjective. As another Munich phenomenologist, Max Scheler, has shown however, feelings exhibit an objective order no less determinate than the logical order exhibited by a set of propositions. See for example Scheler's *Zur Phänomenologie und Theorie der Sympathiegefühle und von Liebe und Hass*, Halle: Niemeyer, 1913; *Wesen und Formen der Sympathie*, Bonn: Cohen, 1923, and "Der Formalismus in der Ethik und die materiale Wertethik", *Jahrbuch für Philosophie und phänomenologische Forschung*, 1, 1913, 405–565, 2, 1916, 21–478. Cf. also A. Kolnai, "Der Ekel", *ibid.*, 10, 1929, 515–69.
- ³⁹ See John F. Crosby, *Phenomenology and the Philosophy of Law: The Apriori Foundations of Civil Law*, mimeographed, University of Dallas, 1979. Crosby presents Reinach's *a priori* theory as a radical critique of the conventionalist presuppositions underlying modern analytic speech act theory. On Reinach's philosophy of law in general see J.-L. Gardies, "La philosophie du droit d'Adolf Reinach", *Archives de la Philosophie du Droit*, 14, 1965, 17–32. Perhaps the most important difference between Reinach's work on social acts and the work of the speech act theorists turns on the fact that Reinach, with his use of the methodology of part-whole relations, is able to produce a highly systematic and yet elegant theory of the whole range of social acts. The speech act theorists, in overconcentrating on linguistic differences between different types of acts, have been able to produce no complete theory of comparable elegance.
- ⁴⁰ "Die apriorischen Grundlagen des bürgerlichen Rechts", *Jahrbuch für Philosophie und phänomenologische Forschung*, 1, 1913, 685–847, ch. 1.
- ⁴¹ *Op. cit.*, p. 696.

⁴² Reinach draws a parallel here between the coming into existence of claims and obligations and the appearance of alterations in the natural world: both require a sufficient reason. There are however two important differences between the two: (i) The relation between cause and effect in the natural world does not exhibit the kind of necessary connection between ground and consequent into which an immediate insight is open to us: that fire generates smoke is a state of affairs not rooted in the essence of fire. Of an event sufficient to establish a claim, however, we can recognise immediately that whenever an event of exactly the same type recurs, a corresponding claim must also once more be established. That it results in the establishment of a claim is an essential, not an accidental moment of the event in question. (ii) The type of act in which an effect in the natural world comes to givenness does not require the execution of a correlated act in which the corresponding cause is grasped. The former does not stand in need of foundation by the latter. An effect in the natural world can, at least in principle, be brought to presentation independently of its cause. It is in contrast impossible to grasp the existence of a claim or of an obligation independently of a return to the corresponding ground or reason. Only through an act in which I reestablish the existence of, say, a promise, can I establish the existence of that which follows from it. Cf. Reinach, *op. cit.*, pp. 701 ff.

⁴³ Cf. §§ 2–3 of “On the Theory of the Negative Judgment”.

⁴⁴ “Die apriorischen Grundlagen”, p. 706.

⁴⁵ *Loc. cit.*

⁴⁶ *Op. cit.*, p. 707.

⁴⁷ *Op. cit.*, p. 728.

⁴⁸ *Op. cit.*, p. 710.

⁴⁹ *Op. cit.*, pp. 710f.

⁵⁰ See Crosby, *op. cit.*, p. 20 and pp. 69–79.

⁵¹ The concept of *a priori* tendencies is considered also, from different points of view, by August Gallinger in his *Das Problem der objektiven Möglichkeit*, Leipzig: Barth, 1912, esp. p. 78 on the concept of *sachliche Motivation*, and K. Duncker, “On Pleasure, Emotion and Striving”, *Philosophy and Phenomenological Research*, 1, 1941, 391–430.

⁵² Reinach, *op. cit.*, p. 723.

⁵³ Reinach’s theory of representation is presented in § 7 of “Die apriorische Grundlagen”, pp. 782–800.

⁵⁴ *Op. cit.*, p. 785.

⁵⁵ Some analogies between legal and linguistic representation have been considered already by H. Gomperz and K. Bühler; see the latter’s “Über den Begriff der sprachlichen Darstellung”, *Psychologische Forschung*, 3, 1921, 282–94.

On the Theory of the Negative Judgment*

I Investigations of Judgment in General¹

§ 1 The Ambiguity of the Term 'Judgment'

It is of the utmost importance to uncover an equivocation bearing on the term 'judgment', one which, as it seems to me, makes itself felt in the form of confusions in very many logical contexts. On the one hand one understands by 'judgment' what one tends otherwise to characterise as 'conviction', 'certainty', 'belief', also as 'consciousness of validity'. On the other hand one means by this term also 'affirmation' or 'assertion'. Now certainly conviction and assertion stand in close relation to each other, but they are by no means identical. And whilst there is no doubt that one may well use 'judgment' to designate both, for this very reason one must then emphasise much more carefully the fact that the two delineate – in quite different ways – two wholly heterogeneous logical spheres, and that they thereby divide the total field of the theory of judgment into two neighbouring but absolutely separate sub-fields. This must now be shown in more detail. We must separate the two just-mentioned concepts of judgment and at the same time distinguish them from other related formations with which they may be, and indeed have been in the past, confused.

* English translation of "Zur Theorie des negativen Urteils", *Munchener Philosophische Abhandlungen*, Festschrift for T. Lipps, A. Pfander, ed., Leipzig: Barth, 1911, 196–254; repr. in Reinach's *Gesammelte Schriften*, Halle: Niemeyer, 1921, 56–102; new edition, Munich: Philosophia, forthcoming. (Chapter and section headings have been supplied by the translator.)

We begin with a term which has cropped up frequently in writings on the theory of judgment since the influential investigations of Franz Brentano in this field. Brentano designated the positive judgment as an 'accepting' [*Anerkennen*] and opposed it to the negative judgment as a 'rejecting' [*Verwerfen*]. Certainly these terms cannot be understood unambiguously at this stage; and further those theorists who have employed them have by no means always avoided the dangerous ambiguities which they contain. One speaks of acceptance and rejection first of all in the sense of an *evaluative* turning towards or away from; thus a moral deed is accepted, an immoral deed rejected. Brentano (1889, p. 56) and Marty (1908, p. 233) quite rightly emphasised that this concept of a 'holding dear to oneself' or of a 'feeling of agreeableness' has no place in the theory of judgment. What should it mean to say that in the judgment ' $2 \times 2 = 4$ ' the identity of 2×2 and 4 is 'esteemed', or that in the judgment ' $2 \times 2 \neq 5$ ' the identity of 2×2 and 5 is in this sense 'disapproved of'? But the danger of a confusion of this kind is not great; much more must we be on our guard against another kind of confusion.

There is a notion of acceptance which has nothing in it of an actual esteeming, one which can more precisely be characterised as a consenting. I hear, say, the judgment ' a is P ' expressed; I understand it, reflect upon it, and then I utter a consenting 'Yes'. In this 'Yes' lies a consent, an acceptance; but even here the acceptance is not a judgment. For which judgment should it be? The judgment ' a is P '? Certainly not. For this judgment evidently relates to the being P of a , to this state of affairs, but the acceptance which we now have before us relates rather to the *judgment* ' a is P '. And that the state of affairs is not the same as the judgment which posits it requires no special emphasis. I can even bring in the original judgment alongside, and say: 'Yes; a is indeed P '. Here we have consenting acceptance and judgment next to each other, as evidently different. I first of all consent, with my 'Yes', to the original judgment, and then I judge for myself that a is P . Now one can designate this judgment too as an acceptance, that is, again, as the acceptance of the state of affairs which is the being P of a . And it is precisely here that there lies the danger of the confusion mentioned above. For consenting acceptance and judging acceptance are fundamentally different, both as acts and in regard to their objectual correlates. If one wanted to make use of equivocation it might be said that what consenting acceptance accepts is precisely a judging acceptance.² Many confusions

in the theory of judgment are thus to be explained as arising through the substitution of consenting acceptance for the genuine judgment. It is to a large extent the term 'acceptance' which leads us astray here,³ and the same considerations can be applied, of course, to the expressions 'rejection' and 'disapproval'.

§ 2 The Sphere of Conviction and the Sphere of Assertion

After excluding acceptance and rejection both in the sense of positive and negative esteem and in the sense of consent and refusal, we come to the question whether 'acceptance' possesses any unambiguous sense at all, at least within the sphere of judgment. We have already indicated that this is not the case. Let us take a concrete example. Imagine that there has arisen a question between myself and someone else concerning the colour of a particular object. I step up to the object and I see that it is red. The being red of the object is here given to me, and as it comes to be given to me there develops within me the relevant conviction or belief that the object is red. Here one can very properly talk of a judgment. Indeed we have here the point about which the concept of 'belief' in English philosophy is oriented.

Let us follow through this case a little further. I turn away from the object, step up to the other person and say: 'The object is red.' What is involved here? The conviction originally attained can endure, I can hold fast to it, even when the object no longer stands before me. With this conviction I turn to the other person and utter the just-mentioned words. But it is not at all as though there was nothing involved here except the conviction of the given state of affairs and the utterance of these particular words. In uttering these words I *mean* or intend something by them, something objectual which they designate, and I mean this in a positive, affirmative manner. Such positing or asserting displays the presence of an act of a quite peculiar type. This is shown by the fact that if I say: 'Is the object red?' I also have something objectual in view, indeed I have the same thing in view as with the sentence: 'The object is red.' Here however we have not, as before, an assertive but rather an interrogative directedness. If we pay careful attention to the two cases then that which is characteristic of the case of assertion becomes clear. And again one may move to the case in which someone else makes the assertion '*a is P*' and I repeat his sentence, understanding it, without

however sharing in its assertion. Once again it is exactly the same state of affairs which is meant in the two cases, but only in the first case is it posited *assertingly*.⁴ How the understanding repetition of an assertive sentence is to be characterised positively must be left open; but in any case we can rule out any talk of its being itself an assertion. Thus we see that there are quite peculiar acts of positing or asserting; acts which are present in every positive judgment which we make. We shall study this asserting as it appears in judgments which are *uttered*, but we must take care not to reduce it to something purely linguistic. Thus one can accept that we could nowhere point to an assertion not linguistically clothed. But this does not mean that the asserting and the linguistic clothing are one and the same. For both in the case of speaking proper and in the case of inner, silent speech we have acts of asserting.

The speaking *is* characterised quite differently in the two cases - but we shall of course guard against the temptation to characterise this difference by viewing inner speech as a mere *presentation* of speech, for the presentation of uttered speech and inner speaking are clearly two quite different things. But while the form of speech is thus altered in a quite specific fashion, the assertion to which it gives expression in the two cases remains one and the same. And in whatever way this alteration may be more precisely characterised, the specific moment of asserting is certainly not subject to it, and this is sufficient proof of how mistaken we would be if we were to identify asserting with speaking.

Now this assertion too, which is gradually beginning to come into prominence before us, can also be designated as a judgment in a second use of this term - perhaps an even more appropriate use than in our first use of 'judgment' as conviction or belief. Thus we have arrived at two concepts of judgment both of which are concealed within the ambiguous term 'acceptance': indeed besides acceptance as esteem and acceptance as consent we have two separate cases of *judging* acceptance. And whilst linguistic usage seems at bottom to allow us to designate only *assertion* and not conviction as an accepting, since assertion and conviction are continually being confused, the latter is to that extent also included under this term. Brentano's theory of judgment gives us an example of this. He speaks of the judgment as an acceptance and this initially points us - when we leave out of account those meanings which do not belong at all to the sphere of the theory of judgment - to the sphere of assertion. But Brentano speaks on the other hand of differences of degree of a judgment and, as is not difficult to see, this leads us

immediately into a quite different sphere. In his *Psychologie* Brentano even spoke of 'intensities' of judgments in analogy with the intensity of feelings (1874, p. 292).

This view he later somewhat modified. "It is false ...", he tells us (1889, p. 57) "that the so-called degree of conviction is a grade of intensity of the judgment which could be brought into analogy with the intensity of joy and pain." But Brentano wants to accept degrees of judgment just as much as before. And Windelband similarly speaks of a possibility of gradations in the 'feeling of conviction' or of 'certainty' (1884, p. 186). When applied to assertion such a claim yields no sense at all. Either something is asserted or it is not asserted; degrees of assertion simply do not exist. Certainly one can speak of a hesitant or reluctant assertion; but it is clear that such an assertion is not thereby somehow an inferior assertion, somehow less of an assertion. The situation is quite different in the case of conviction. Here there is indeed a good sense to talk of levels or degrees. Alongside conviction there lie conjecture and doubt and with each of these the 'degree of certainty' sinks lower and lower. Thus in this context Brentano cannot have in mind the judgment in the sense of an assertion, he must rather be thinking of judgment in the sense of conviction; and an expression to this effect forces itself upon Brentano in the passage indicated above. Here the dangerous ambiguity in the concept of acceptance shows itself extremely clearly, and hence we wish to avoid this terminology completely in what follows, where for cases of positive judgments we shall always use the term 'assertion'. Note, however, that we have managed to bring out a first fundamental distinction between conviction and assertion, a distinction which we wish to pursue somewhat further in what follows.

In psychological and logical reflections we often find the act of judgment placed alongside other more or less closely related acts of consciousness. Sometimes we find judgment placed in opposition to doubt and conjecture, at other times in opposition to questioning or wishing. If we look more closely then we see that the term 'judgment' is here figuring in the two senses at present under review. It is unacceptable to rank conjecture and doubt with assertion; they belong rather alongside conviction, as different grades of certainty. On the other hand the acts which find their expression in the words 'Is *a* *P*?' or 'If only *a* were *P*!' undoubtedly find their place not alongside conviction but rather alongside assertion.

Thus far we have only indirect indications of the differences between

our two types of judgment. Direct corroboration here, as in other cases, can be achieved only through immediate perception. But we can already see with indubitable clarity that *conviction* or belief on the one hand, that which develops in us in the presence of a particular object, always involves some aspects which we may designate if not as feelings, at least as states of consciousness, at all events some *condition* of consciousness; but that *assertion* on the other hand, which does not 'develop' within us but is rather 'made' by us, is totally different from any feeling, from every condition, and is much rather to be characterised as a *spontaneous* act.

Further both conviction and assertion realise themselves within time; one can determine the point of time in which they come into being. But whilst we can speak of convictions of any arbitrary temporal extent, assertion essentially excludes any talk of a temporal extendedness; it has no temporal *course*, but rather exists as though it were something *punctual*.

We are far from affirming an absolute unrelatedness between conviction and assertion; indeed it is precisely because there exist very close relations between the two that they have been continually confused. No assertion is possible which is not accompanied by an underlying conviction which is such that both the assertion and the belief relate to something strictly identical. In contrast it is not necessary that *every* conviction or belief founds an assertion, and it is even excluded that an assertion should underlie a conviction. One may wish to question our first proposition by pointing to the case of lying, which seems to be something which is essentially an assertion in the absence of conviction. Closer consideration shows however that one may not at all speak of lying as a case of genuine assertion. We have to deal rather with a quite peculiar modification of assertion, a quasi-assertion as it were, lacking proper vivacity, and something for which we may find an analogy in the quasi-questioning which is a frequent occurrence in conventional conversation. *Genuine* questioning as much rules out a prior belief in that which is being questioned as genuine assertion excludes the disbelief in that which is being asserted. A conventional 'question', one with respect to which we know perfectly well what is being asked about, is not a genuine question; and a lie, something which involves a disbelief in that which one purports to assert, is correspondingly not a genuine assertion. We cannot go further into this not in itself unimportant correspondence between the two cases. For us they have merely the function

of bringing clearly into light the separation of conviction and assertion. Essential connections of the kind observed are clearly possible, and understandable, only where we have to deal not with some *one* thing that is merely *expressed* in different ways, but rather with two quite distinct formations. We wish to pursue the difference between the two a little further.

§ 3 Acts of Presentation and Acts of Meaning

It is well known that Brentano separated presentation and judgment from each other with extreme strictness, but that he at the same time brought them into close relation by postulating that of necessity every judgment have a foundation in a presentation. Every acceptance and every rejection necessarily presuppose, he claimed, the presentation of that which is accepted or rejected. Thus the object which is judged is taken up twice into consciousness: once as something presented, once as something accepted or rejected. If now we, for our part, ask after the relation between presentation and judgment, then we must of course distinguish two separate sub-questions; and what holds of judgment in the sense of conviction or belief need not at all hold of judgment as assertion. One thing certainly holds equally of both cases: there is no possible conviction and no possible assertion which is not conviction or assertion *of* something; the relatedness to something objectual, with respect to which the conviction is held, and towards which the assertion is directed, is something which is essential to both cases. We could, in this context, speak of the intentional character of the two types of judgment, but we would then have to guard against drawing over-hasty conclusions from this 'intentionality'.

To say of an experience that it is intentional is to say that it possesses a 'directedness towards' something objectual, and this in turn presupposes that something is 'at hand' for consciousness. But this being at hand – in the widest possible sense – is not a being presented, or at least need not involve any being presented.⁵ Certainly it is not easy to delineate firmly the concept of presentation. Husserl has shown the many ambiguities by which it has come to be affected (1900/01, II, pp. 493 ff).

Leaving out of account here the popular meaning in terms of which one speaks of presentation⁶ as something *opposed* to perception, we

can speak of presentation as something which includes equally not only perception but also memory, phantasy, and other related acts. A close examination of the expression 'presentation' [*Vor-Stellung* literally: setting before] will help us to circumscribe this very wide class of acts. It reveals that what is to be counted as an object of presentation is anything which we have 'before' us, or which – since we wish to avoid any suggestion of a spatial conception – is 'present' to us, 'there' for us. The sheet of paper which I am now perceiving is present to me, as is Milan Cathedral which I am now bringing to mind, as is a past experience of grief which I am now remembering, and a landscape which I am now imagining. However fundamentally different all these acts may be, still everything which is grasped within them is 'there' for me, stands as it were in front of me, is 'set before me' in the pregnant sense indicated above.

This concept of presentation extends itself far beyond the sphere of sensible objects in which it has its root. Even the beauty of a work of art, as something of which I am aware, is present to me, as is, say, the number 2, something which I bring to mind in relation to two arbitrary individual objects. Thus we by no means fail to appreciate the richness of phenomena which are to be distinguished here. If we take sensible perception alone then it is immediately clear that that which is 'actually' perceived, that which stands in the foreground of our perceptions, is something quite different from the co-presented background, and that both are in their turn different from the small segment toward which my attention is, at any given moment, principally directed. In each of these cases however, we can speak of an existence⁷ of the object of the act, as we can also in the various other spheres of bringing to mind, remembering, imagining, being aware of, and (as e.g. in the case of numbers) thinking of. The objects of acts of each of these types are all there for me, and this is what allows us to consider all such acts, together with all other acts whose intentional correlates are present in the same sense, as belonging together within a single group. One might now wish to question whether all acts whatsoever which involve a relation to something objectual are not admitted by this account, and whether every intended object whatsoever is not thereby also something which is 'there' for me. This would be quite wrong however, as we shall now show by demarcating a class of intending acts whose objectual correlate is in no sense presented (set before the subject of the act), in the hope that we may thereby throw some further light upon the exposition thus far.

We shall turn our attention to linguistic expressions. Suppose I am counting off, say, the mountains of Germany, either by calling out their names to someone else or by reciting them to myself. In doing this I utter a large number of names, perhaps very quickly one after another, but obviously there is much more involved here than mere utterances; in uttering the words I mean something by them, i.e. precisely the mountains which they designate. Anyone wholly ignorant of language would be limited to the utterance of the words without understanding of them; that is without meaning by the words the objects correlated with them. In contrast, whoever utters the words understandingly thereby aims – with them or through them – through and onto something other, and it is this ‘something other’ which is all-important. The acts now under consideration have a spontaneous directedness to something objectual; but it is not difficult for an unprejudiced observer to perceive that there can be no talk of a ‘presentation’, of a ‘presence’ of these objects in the sense determined above. Certainly they *may* be present; I can call out the name of a mountain and at the same time perceive it or bring it to mind in my memory. It is then of course presented, but one sees immediately that this accompanying presentation is normally not to hand, or at least need not be to hand. And further one sees that even in the cases where the object signified by the name is presented we would still have to distinguish from this act of presentation the act of meaning which is tied up with the utterance of the words. For even here it is not as if there were nothing more involved than a presentation of the mountain and the bare utterance of a word. A careful consideration reveals much more that the following is the case: that presentation is an act of its own peculiar type, a bare receptive ‘having’ of the object which may be of a longer or a shorter duration. If now an utterance of the name of the object is adjoined to it then – should the name be uttered with *understanding* – there becomes bound up with that act of presentation another quite peculiar act, which we designate as an act of meaning or of being directed towards. This latter act, that is to say, appears *alongside* the presentation, distinguishing itself already from the act of presenting, on the one hand in being always linguistically clothed, and on the other hand in being such that a spontaneity of directedness and a temporal punctuality are essential to it. Presentation and meaning [here always in the sense of ‘*Meinen*’, i.e. meaning or intending something objectual *by* a given expression – Tr.] are certainly not without any relation to each other in our example. It is of course

precisely the same object which is at one and the same time presented and meant. But this identity of reference-point of the two acts cannot sanction the identification of the acts themselves, i.e. by allowing the dissolution of the punctual act of meaning within the stretched-out act of presentation, the former conceived as somehow insignificant in the presence of the latter. Much more is it the case that the two exist alongside each other, such that, according to circumstances, one would designate the whole situation either as one in which the object, at first merely presented, is then grasped in an act of meaning, or as one in which an at first merely meant object is then further brought to givenness in an act of presentation.

We do not fail to recognise the concomitant dangers of misunderstanding in designating the particular act which we are at the moment trying to throw into relief as an act of *meaning*. For 'to mean an object', 'to aim at an object', can signify also an involved 'turning towards' the object – or whatever other expression may be offered for an *interested concern*,⁸ – something which naturally falls outside our present field of investigations. For the kind of meaning or intending which involves an interested concern essentially presupposes the presence of the object which is 'meant' in this fashion, and here we are concerned with that type of meaning an object whose distinguishing peculiarity is precisely this: that it neither presents the object to us, nor in any way presupposes its being presented. And no other mode of expression is open to us, besides that of 'meaning' or 'intending' or 'being directed towards an object', for that type of act which is bound up with the understanding utterance of words and in which we are related to something objectual which is yet not brought to presentation. Hence there is nothing for it but to warn of the dangers of confusion due to ambiguity in this manner of expression, especially ambiguities involving the notion of an involved 'turning towards', a notion which must quite deliberately be kept to one side.

At the same time these considerations may serve to bring to light one principal difference between our concepts of meaning and presentation. Anything that is presented is such that we can turn toward it with a specific interest, raise it up out of its surroundings, concern ourselves with its specific traits. In the sphere of meaning in our sense however there is no possibility of such modifications. Consider for example the situation in which in the course of speaking we direct ourselves, in succession, towards a series of objects. In such a case there can be no talk of a turn-

ing towards the objects, a raising of them into prominence. For of course whilst it is possible to turn one's attention towards objects which are at first merely meant, this can never occur within the act of meaning itself; it requires its own new act, one which will bring these meant objects to presentation, and only what is thus brought to presentation can then be the subject of an attentive turning towards. We can only advert attentively to that which is thus presented.

We get even closer to the root of the fundamental opposition between presenting and meaning through the following reflections. The acts in which objects are brought to presentation are quite different according to the class of objects toward which they are directed. Colours are *seen*, sounds *heard*, things of the external world are *perceived by the senses*, numbers are *thought*, values are *felt*, etc. Thus even in the case of tones and colours it is an obvious requirement that we everywhere strictly distinguish the object of the act from the act itself, through which it is brought to presentation. Once given this distinction we recognise that there exists an abundance of the most interesting essential connections which correlate of necessity the various types of object with corresponding types of presenting acts. Colours can after all *only* be seen, numbers *only* thought. One sees immediately that a quite different situation obtains for the case of acts of meaning an object. We speak understandingly of colours, tones, values, numbers, physical things, for all of these objects are meant, but here there are no qualitative differences on the side of the acts of meaning which would correspond to the qualitative differences among the objects. Certainly the act of meaning a colour is different from the act of meaning a number, precisely in virtue of the fact that in the one case it is a colour, in the other a number, which is meant; but it is an act of meaning, still, which lies before us in the two cases: there is no fundamental difference between the acts which would parallel the difference between seeing and thinking which we meet in the case of presentations of colours and numbers.

§ 4 The Independence of Meaning and Intuition

At first one may want to identify this difference with that between acts which are intuitionally filled and acts which are intuitionally empty, an opposition much discussed in the most recent logic and psychology,

especially in connection with Husserl's *Logical Investigations*. It is acts which lack any intuition – one would then argue – which are distinguished by us here as acts of meaning. Such a view would however be wholly wrong; indeed we have to deal here with two completely separate pairs of opposites. For in fact we have both intuitional fulfilment *and* intuitional emptiness as much in cases of presentation as in cases of acts of meaning an object. A presentation which lacks intuition is by no means thereby turned into an act of meaning, and nor, conversely, is an act of meaning which is enlivened by intuition at all to be conceived as a presentation.

To make this clear we need only bring into view the various possible cases which may arise. If we restrict ourselves to the case of sensual presentation, then it is the perception of physical things which provide the best examples of presentations whose intuitional content may exhibit a greater or lesser degree of fullness, distinctness and clarity. As we draw nearer to a physical thing the intuitional content which represents it becomes ever richer and clearer, new aspects of the object offer themselves to us with an ever greater distinctness. From the very beginning the object stands before us; to the extent that it is brought to presentation our intuitions take on ever new forms; and this increase or decrease of intuition takes place along various different gradational axes, even though the characteristic of being presented as such does not itself admit of degrees. Here we see quite clearly how the concept of presentation, characterised by the presence of something objectual, has to be precisely differentiated from the concept of intuition, as something which may vary greatly in the continued presence of one and the same object. The independence of the two goes so far that something objectual may be presented without its being possible to establish the slightest trace of intuition directly representing that object. For consider again the case of sensual perception. A book lies before me; the whole book is presented to me, and yet only parts of it are intuitionally represented. The rear side of the book, for example, is in no way intuitionally given to me: I neither perceive it, nor, under normal circumstances, do I attempt to draw any intuitional representation of it from memory or phantasy. Perhaps one would wish to say in regard to this situation that only the intuitionally represented part of the book is in fact brought to presentation. But what is to be found before me is after all the *book*, the whole object and not an object-torso. If we discover of a presented object, say a vessel, that its rear side is missing, then we expe-

rience frustration. The intention, which had directed itself toward a whole object, is partially unfulfilled – and such a non-fulfillment, or better, such a frustration, is only possible if the original presentation were such that its intention had extended to the original object *qua* complete whole i.e. *including* a non-intuitively given rear side, such that with the rotation of the object there can arise a conflict between that which had at first been non-intuitively presented and that which is now intuitively given. Within each and every perception of a physical thing we find that there are components of the presentation which are non-intuitive in this way. Now certainly it would be possible, according to one linguistic usage mentioned above, to designate the corresponding objectual components as co-‘meant’. But we surely no longer need to emphasize the fact that we are not dealing here with an act of meaning in the sense before us in the present paper: for what is essential to the latter is, of course, the non-presentedness of the object which is meant. We could imagine a case in which acts of ‘meaning’ in both senses were simultaneously to hand. Thus we might observe a physical object whose rear side is co-‘meant’ in a non-intuitive presentation and at the same time utter understandingly the sentence: ‘The rear side of this object is ...’ Here there appears alongside the enduring non-intuitive presentation a quite different type of act of meaning or intending, one which is linguistically clothed, temporally punctual, self-contained. No one could deny the essential differences between these two acts; thus we see most distinctly that a non-intuitive presentation is in no way identical with the type of linguistically carried meaning-of-an-object which concerns us here.

It is not an easy task to uncover a *wholly* non-intentional intention within the sphere of presentation; in the sphere of meaning, in contrast, it is non-intentional acts which are the first to urge themselves upon us. In the flow of speech we talk at will of entities of arbitrary complexity. Meaning-act is followed by meaning-act in the most rapid succession; and we direct ourselves toward each and every one of the entities designated by our words, though we can normally, in an unprejudiced observation, perceive nothing of an intuitive character in this directedness towards (or meaning of) the entities involved.⁹ Certainly there are from time to time various types of intuitive image which rise to the surface in the course of speaking; vague, indeterminate impressions, either of the objects of which we are speaking, or else of other, associated objects, sometimes heeded, mostly however and in the normal case, escap-

ing our attention. They rise to the surface and in many cases outlive the act of meaning which they accompany, and then they once more disappear. They appear to have only an insignificant influence upon the steady succession of acts of meaning, like ripples upon the surface of a river. Now one may designate those acts of meaning which are accompanied in this fashion by 'illustrative' images as intuitive acts, but then one must not overlook the fact that we have to deal here with intuitiveness in a quite different sense from that which arises in cases of presentation.

What immediately forces itself upon us concerning the intuition which we encounter in amongst acts of meaning, is that this is intuition whose function is fundamentally different from that of the intuitiveness of perception, (and of presenting acts in general). In every presentation the intuitional content *represents* the presented object to me, it exhibits it to me. In that which is intuitionally given to me in a sensual perception the whole object stands before me, just as the remembered or phantasied object is grasped 'in' the intuitional content which is momentarily to hand in an act of memory or phantasy. Now whatever may be the results of a closer analysis of this very difficult matter, we can see already that in the sphere of acts of *meaning* we have to deal with a quite different situation. When intuitional schemata rise up and fall away here they lack any kind of representing function. They do not 'exhibit' or 'present' anything [*Sie stellen nichts 'dar' oder 'vor'*] – for of course in the sphere of meaning there is absolutely nothing to hand which is presented. Rather, they partake of an existence which floats quite free from that of the object which is meant. They belong to a quite different stratum to that of the intuitional content of presentation, not being truly immanent to [the sphere of] meaning. Where we can speak of an intuitiveness *of* presentation it is more apt, in a case of meaning, to speak not of *its* intuitiveness but rather of intuitive images [*Bildern*] which *accompany* it.

§ 5 Judgment and Presentation (Critique of Brentano)

Our analyses have sufficiently demonstrated the absolute difference between presentation and meaning. In particular they have made clear that the non-intuitive presentation of an object is in no way identical with an act of meaning that object and that the meaning which is ac-

accompanied by intuitive images is in no way identical with a presentation. The many recent discussions of the question whether there exist acts of consciousness which are absolutely free of intuition have overlooked the fact that we have to deal here with at least *two* questions: with the question concerning intuition-free presentations and that concerning intuition-free acts of meaning. It seems to us to be indubitable that there are intuition-free acts of meaning. It is, on the other hand, very questionable whether there exist absolutely intuition-free presentations. It is true that, as already mentioned, the rear side of every perceivable thing is non-intuitively presented; but then we do not have a self-contained, independent presentation: much more is it the case that the rear side is co-presented in the presentation of the object as a whole. Let it suffice here to point out that the variety of different opinions in regard to the above question may perhaps be laid at the door of an inadequate separation of the two spheres of presentation and meaning.

We now return to the question whether every judgment is necessarily founded in a presentation. For the case of assertion this question may immediately be given a negative answer. One need only observe how, in the course of speech, assertion may follow on assertion without that which is asserted ever once needing to be brought to presentation. Here one must not allow oneself to be led astray by the apparently self-evident thesis that I can only judge about that which I know, and which is, therefore, somehow present to me. For although it is certainly correct that I must be related in some determinate way to that about which I make assertions in order to be able to make those assertions, it is wrong to suppose that it is only presentation in our sense which can be considered as providing this relation. I am related to objects also in cases of acts of meaning, acts outside the sphere of presentation. Indeed an act of this kind constitutes the necessary foundation for every act of assertion. Further, in the act of assertion as such that which is asserted is not presented, not brought before the mind of the subject involved, even though in every case it would be possible for a presenting act to join up with or follow upon the act of assertion. This is not the place to draw the consequences for the theory of knowledge which follow from this fact. For us it is important only to remark that the meaning act can be qualified in a variety of ways. If I say, e.g.: 'Is *a* *P*?' and then: '*a* is *P*', then in both cases something is meant, indeed what is meant is identically the same state of affairs; but in the first case it is put into question,

in the second case assertingly posited. We could distinguish, within the total complex which we designate as the assertion of a state of affairs, the specific moment of assertion on the one hand from the constituent of meaning or intending on the other. The assertion is constituted from them both.¹⁰ The moment of assertion attains *through* the meaning-component its relation to the relevant state of affairs; it is necessarily *founded* in this component. It is, in contrast, excluded that a conviction should be founded through such an act of meaning or intending. I can, of course, be convinced of a given state of affairs and at the same time mean that state of affairs. As just indicated this is always the case when a state of affairs is asserted; but then it is the *assertion* which is founded in the act of meaning and not the underlying conviction.

The question now arises as to how conviction or belief acquires a relation to its objectual correlate. Let us recall the case which served as our point of departure: I stand before a flower and I see its being red; on the basis of this act of seeing there develops in me the conviction of the relevant state of affairs. It is clear that in this case it is a presentation, in the exact sense determined above, which lies at the basis of the conviction. And thus one might be tempted to say, with Brentano, that the judgment is founded on a presentation. But there are two reasons for caution at this point: first of all we are not dealing here with judgment in general but only with judgment in the sense of conviction; and secondly, whilst one could speak here of a judgment's *possibly* being founded in a presentation, one cannot affirm that such a foundation is *necessary* (and thus one cannot speak of judgment as having a presentational basis in Brentano's sense). For consider the case mentioned earlier involving a turning away from a perceived state of affairs: clearly the latter no longer requires to be presented, in the strict sense, for the relevant conviction to endure. Of course that conviction is still 'referred' to one and the same state of affairs, but this reference [*Bezogen-sein*] is no longer something which is mediated through a presentation of the state of affairs. But nor, certainly, is it mediated by a meaning or intending of that state of affairs. For meaning in our sense is essentially bound up with linguistic expressions. There is, in fact, a whole series of possible intentions to what is objectual,¹¹ of which we wish here to consider only two: acts of presentation, in which the object is 'there' for us, such that we 'have' it as our object and have it, in cases of absolutely complete intuitiveness, in closest proximity; and acts of meaning, in which we acquire a spontaneous directedness to the object which, how-

ever, continues to stand at the remotest distance from us. Those acts which found convictions of the type which are not founded through a presentation – acts which as a whole we may perhaps best designate as acts of ‘knowing about’ – we leave to one side. This we can do all the more since we do not, in general, designate this kind of knowing as a judgment, but only the conviction which develops from the perception of a state of affairs. All that we have wished to show [against Brentano] is that a presentational foundation of such conviction is not necessary. With these remarks we have come to the end of our general discussions on the theory of judgment. We wish to hold on to the following as their result: that the term ‘judgment’ is to be understood in two senses, on the one hand as *assertion*, which relates to its objectual correlate in acts of *meaning* which may or may not be accompanied by intuitions, and on the other hand as *conviction* or *belief* which develops out of more or less intuitive acts of presentation. This implies that we must speak also of the negative judgment in two senses, and thus already the *problem* of the negative judgment has been raised up onto a new level.

II States of Affairs

§ 6 Positive and Negative Position-Takings

From those acts, such as acts of presentation and meaning, in which we *lay hold of* something objectual (either by having it as our object or by being directed towards it), we have to distinguish experiences which, as in the case of conviction or belief, involve our *taking a position* with respect to something. Other examples of the latter with which we are acquainted include striving after something, expecting something, and so on. There is an opposition running through this second class of acts – but not through the first – between positivity and negativity. We not only strive positively after something but may also struggle against it. In both cases [of *Streben* and *Widerstreben*] we have a striving, but the two are, so to speak, of opposite sign.¹² Now we find exactly the same in the case of conviction. So far we have naturally concentrated upon positive conviction; there is however, standing in opposition to this, a negative conviction, having a fully equal status. Let us suppose that someone asserts that a flower is red, and that in order to convince ourselves of this we go to the place where the flower is to be found, and see that it is yellow. Thus after we have approached the flower with the question as to whether it is truly red, there has developed within us a negative conviction in relation to the corresponding state of affairs, a ‘disbelief’ in the flower’s being red. Both positive and negative convictions may relate to one and the same state of affairs; if we search for expressions which would distinguish the two then we could say that the first is a bestowal of conviction [*Überzeugungszuwendung*], the second a privation of conviction [*Überzeugungsabwendung*]. Both are however ‘convinced’ position-takings. The moment of conviction is common to the two (just as the moment of striving is common to positive striving for and to striving against something). It is this moment which separates the two types of conviction from other intellectual position-takings, e.g. from conjecture or doubt. And it is this which allows us to designate them both as judgments, the polar opposition just mentioned being that which brands the one as a positive the other as a negative judgment.

From the point of view of a description of their nature, positive and negative conviction are ranged alongside each other on an equal footing. But a certain difference between the two seems to appear when we take account of the psychological preconditions which must be met if either is to be acquired. If we simply look out at the world which surrounds us we are confronted by a plenitude of states of affairs which we behold, and towards which our convictions are subsequently related. It is clear that only positive convictions could develop in this way. A negative conviction could never arise through a simple *reading off* of a state of affairs from without; such a conviction rather always presupposes that we approach an existing state of affairs with a prior intellectual position-taking relating to a second, conflicting state of affairs. The conflicting state of affairs may be, for example, believed, conjectured, doubted, or merely put into question, but as we behold the other state of affairs the original positive conviction or conjecture, doubt, uncertainty or question becomes transmuted into or finds its answer in a negative conviction. Here we note a peculiarity of the negative judgment, to which we are not yet however in a position to do justice.

Alongside the negative conviction of a state of affairs there is the positive conviction of a contradictory state. The belief that a is not P and the disbelief that a is P , stand, in regard to their logical content, as close to each other as possible. Nevertheless as judgments they are completely different and can by no means be allowed to be identified. Not only that which pertains to the side of consciousness¹³ but also the objectual side are fundamentally different in the two cases: the belief stands opposed to the disbelief, the being P of a to the being not P of a . Now disbelief relative to a given state of affairs is something which above all else deserves the name of a negative judgment. And further, since it has been quite usual in the traditional theory of judgment to call something a judgment not only in virtue of its specific character as a judgment but also in virtue of the specific nature of its objectual correlate, we shall also bring the positive conviction of negative states within the sphere of our investigations. The most intransigent difficulties were encountered, after all, precisely in regard to the case of conviction of (belief in) something negative – which was not, of course, separated in the literature from the case of negative conviction of (disbelief in) something positive. And the treatment of these difficulties will reveal itself as necessary also for our own deliberations. They are difficulties which have their origin in the somewhat primitive conception according to which

the positive judgment is regarded somehow as a *combining* or a *unifying* (a conception which, defensible or not, clearly acquires a quite different sense according to whether it refers to judgment as conviction or to judgment as assertion). According to this conception a true judgment is one whose associated 'act of unifying' corresponds to a factually real unity in the objectual world. It is clear that any attempt to apply this conception analogously to the negative judgment will meet with difficulties. For we would have to conceive such a judgment as a separation, and then we should seek in vain for the real relation which would be reflected in this separating. What after all should it mean – as Windelband rightly asked (1884, p. 169) – to say that in the simple judgment 'blue is not red' one was giving expression to a separation? And if precisely this example may tempt us to regard, say, the relation of *being different* as the real relation which is here in question, then reflecting upon a judgment such as 'certain functions are not differentiable' would immediately convince us of the futility of any such attempt. This is how negation as such came to be conceived as 'no real relation' but merely as a 'relation-form of consciousness' (Windelband, *loc. cit.*). Negation thereby came to be regarded as something purely subjective; according to Sigwart and a series of other, more recent logicians it is an act of rejecting. However whilst it can be admitted that in the case of negative conviction of a positive state of affairs the negativity belongs exclusively to the side of consciousness, every such attempt breaks down on those cases where a positive conviction is directed towards something negative. The possibility of such cases is evident, and it is not the task of logic to re-interpret them as something which they are not, but rather to do justice to them as they are.

§ 7 The Objectual Correlates of Judgments: Judgments and Relations

Just as our treatment of the negative judgment had as its necessary presupposition a clarification of the concept of judgment in general, so now we must investigate the nature of the *objectual correlates* of judgments in general before we can become clear about *negative* judgment-correlates. And here too we shall be able to take these investigations only so far as it indispensable for our own particular aims.

We already know that there exist essential connections between the subject-side of a judgment and that objectual something to which it is

related, connections of a type which imply that not every intentional act is appropriate to every arbitrary object, but rather that there exist on both sides relations of necessary co-ordination. Thus it is evidently impossible that an act of conviction should relate to a sound, a colour, a feeling, or a thing of the external world, just as it is impossible to assert a sound, or a thing, or what have you. Or, passing from the sphere of real objects to the sphere of ideal, i.e. extra-temporal objects, what should it mean to believe or to assert a number, or a concept, or something of that sort? In whichever sense we may want to understand the concept of judgment there are essential laws which exclude the possibility of a judgment ever relating to entities of this type, i.e. to entities of the type which we may quite reasonably designate as (real or ideal) *objects*.

Brentano and his followers of course seem to adopt a different point of view in this matter. According to them any arbitrary objectual something can be judged, i.e. can be 'accepted' or 'rejected', a tree or a sound or what have you. And here we see how necessary were the conceptual distinctions which we made at the beginning of these investigations. For so long as one is operating with such an ambiguous term as 'acceptance' it is of course possible to make a case for its being applicable to all sorts of entities. There is indeed a sense of acceptance or approval which can involve a valuing or an assenting relation to objects: to actions or propositions, for example. But once we exclude all additional meanings, and concentrate only upon those which can truly claim genuinely to refer to judgments – i.e. either to convictions or to assertions – then it becomes impossible to deny that these intentional functions can never, of their very nature, relate to objects such as colours or things or experiences. This is why Brentano and his followers are, in this matter, somewhat isolated.

Since Aristotle logic has been dominated by the view that it is *relations between* objects which are posited in judgments. And indeed, this view is very tempting: for if objects cannot be judged it seems that relations between objects are all that would remain to serve as the correlates of judgments. However widespread this view, however, it can by no means hold its own against a closer analysis. And for this we do not even require any special investigation of relations – a short consideration will provide us with all that we require. Let us take relations such as those of similarity or difference, or of right or left. Now certainly there are judgments in which such relations seem to be believed or, re-

spectively, to be asserted: 'a is similar to b' or 'a is to be left of b'. At the same time however there is one type of judgment – and precisely the most common type – with regard to which we can find absolutely no such relation on the objectual side, namely judgments of the form 'a is P'. Take, for example, the judgment: 'the rose is red'. According to the traditional theory what is judged here is a relation between the rose and red; clearly however this is not at all the case. Of course there are such relations, and they can occur in judgments: 'the rose forms the substrate of the red'; 'the red is inherent in the rose'. Here we have the peculiar converse relations of *being a substrate of* and *inhering in*, as defined for physical things. But these are certainly not posited in the judgment 'the rose is red'. One must not let oneself be misled by the close mutual kinship of the three judgments involved. Certainly it is the same factual material [*derselbe sachliche Tatbestand*] which lies at the basis of each, but they comprehend this factual material in quite different ways and in quite different directions. That it is possible to make all three judgments in regard to the existence of the same underlying factual material lessens not at all the differences between them. Just as the judgments 'a is to the left of b' and 'b is to the right of a' are different, even though it is perfectly identical factual material which underlies them both, so it is with the judgments 'the rose forms the substrate of the red', and 'the red inheres in the rose'. And both are in their turn different in their meaning, though not in their underlying factual material, from the judgment 'the rose is red'. Only in the first two cases do we find relations on the objectual sides of the judgments concerned; considered without prejudice the third judgment displays nothing of a relation.¹⁴ But how are we to understand more closely the nature of the objectual correlate of this judgment, *the being red of the rose*, put forward as an example of the form *being P of a*?

§ 8 States of Affairs as Objectual Correlates of Assertion and Belief

It is already evident that we must sharply differentiate the being red of the rose from the red rose itself. Those statements which hold of the one do not at all hold of the other. The red rose stands in the garden, it can wither; the being-red of the rose does not stand in the garden, nor does it make sense to speak of its withering away. There is a strong inclination to regard this merely as semantic quibbling and to raise the

objection that peculiarities of language are here being confused with peculiarities of the things themselves. We are far from wishing to defend such confusions where they do in fact occur. But one should be rather careful with objections of this kind: in particular they should not be raised until it has been ascertained what 'mere peculiarities of the use of language' really are. There are passages in Kant, for example, [which rest on constructions] which our linguistic usage would no longer permit.¹⁵

Let us suppose that someone infringes one or other rule of linguistic usage. Then one would at most object to him that he was expressing himself in a manner which was not customary, never could one object that that which he said was false, solely in virtue of its irregular expression, when it would otherwise be true, nor that it was true when it would otherwise be false. The meaning of the proposition involved is not affected at all by the mode of expression, i.e. what we have before us here is truly a 'mere difference of words'. The matter is quite different however when we are comparing the two judgments 'the red rose stands in the garden' and 'the being red of the rose stands in the garden'. Here it is not merely *linguistic* differences which we have before us. The first judgment is true, the second is false or even senseless. The being red of a rose cannot *as such* stand in the garden – just as, say, a mathematical formula cannot be sweet-smelling. This implies however that the being red of the rose, as much as the mathematical formula, is something which presents its own demands and prohibitions and is something of which judgments may or may not hold. Does one really wish to apply here the notion of differences of linguistic usage? Does one really want to say that between the being red of the rose and the red rose itself there obtains a 'mere difference of words', that it is merely linguistically irregular to say that the being red of the rose stands in the garden? Would it not be a remarkable kind of linguistic usage which admits expressions like 'the being red of the rose' whilst forbidding their occurrence as the subject of certain judgments? And how, most importantly, could the violation of linguistic usage turn an otherwise correct judgment into one which is false or even senseless? Finally, although the present point requires no further argument, we can appeal to the fact that the proposition 'the red rose stands in the garden' is correct and the proposition 'the being red of the rose stands in the garden' is false, whether expressed in German, French or Chinese. This shows that the entities which serve as the subjects of the two otherwise identi-

cal judgments must be different, in other words that the red rose is something different from the being red of the rose.

Actually we have here nothing more than the corroboration of something which we have already established, namely, that since physical things can never be asserted or believed, and since, on the other hand, the being red of the rose functions as the objectual correlate of the judgment 'the rose is red', it follows that this correlate must be something other than the red rose itself, which is a thing in the external world. Henceforth we shall refer to such correlates as *states of affairs*. This term has been used thus far in our arguments quite without ceremony; it is indeed the term best suited to objectual formations of the form *being P of a*.¹⁶ Thus we have to distinguish between *objects in the strict sense*, whether these be real, (like physical things, tones, experiences), or ideal, (like numbers or propositions, or concepts), and *states of affairs*, as entities of a quite different nature. So far we are acquainted with only *one* peculiarity of states, that they are, in opposition to objects, that which is believed or asserted in judgments.¹⁷ We now wish to supplement this with a further set of determinations.

§ 9 Further Characteristics of States of Affairs

States of affairs stand in relations of ground and consequent

The difference between the relation of ground and consequent and the relation of cause and effect has become part of the stock-in-trade of philosophy today. What must be noted however is that we have to deal here not only with a difference in the relations involved but also with a fundamental difference between the elements which stand in these relations. The movement of one ball causes the movement of another; here it is a physical event which serves as the cause of a second physical event. On the other hand physical things, events, processes and conditions never appear in the relation of ground and consequent. Indeed one can assert quite generally that no object can ever serve as ground or consequent. It is impossible that a physical thing, say, or an experience, or a number, should entail anything or that anything should follow from them. It is at most the *existence* of a thing or of an experience which can function as a ground. But the existence of an object is clearly not itself an object, but rather a state of affairs. It is always and could only be

states of affairs which serve as ground and consequent. That something is the case [*daß etwas so oder so sich 'verhält'*] is the ground for a second state of affairs which follows from it: from the state of affairs that all men are mortal there follows the mortality of the man Caius.

Thus we arrive at a further determination of states of affairs, that they and only they stand in the relation of ground and consequent.¹⁸ Everything which we encounter, either in science or in everyday life, as a connectedness of ground and consequent, is a relation between states of affairs. This holds also of those relations which tend to be collected together under the name of laws of deduction: these are, properly conceived, nothing other than general principles expressing relations between states of affairs. There are profound implications for the construction of logic which develop out of this insight (see below, § 19, note 40); however in this connection our own interest will have to take another course.

States of affairs may suffer modalities

The various different types of laws of deduction which have been distinguished within traditional logic must, if they are to be conceived as relations between states of affairs, have their basis in differences between types of states of affairs. We wish to consider such differences in type from two standpoints. In the first place states of affairs can differ amongst themselves according to their modality. Beside the simple state which is the being *P* of *a* there is also *a*'s being probably *P*, *a*'s being possibly *P*, and so on. Here we cannot go further into the precise nature of these differences of modality. What is important for our purposes is that it is once more states of affairs, and only states of affairs, which can adopt such modalities.¹⁹ There is absolutely no way in which an *object* can be probable, such a predication relative to an object would have no sense, and wherever we find someone apparently speaking of such a probability, say of the probability of a physical thing, then this is nothing more than an inadequate form of expression. In such cases one has in mind the probability of the *existence* of a thing or of certain physical occurrences, that is to say with nothing other than the probability of states of affairs. A probable tree or an improbable number are obviously impossible – and clearly not because we have to deal here specifically with trees or numbers, but rather because the object

form itself excludes such modalities, where the form of states of affairs quite generally and essentially admits them.

States of affairs may be either positive or negative, subsistent or non-subsistent

Viewed from another side, states of affairs can be distinguished as positive and contradictory-negative. This too is an opposition which we could never encounter in the world of objects. Beside the being *P* of *a* there is a being non-*P* of *a*. The two states of affairs are contradictory in relation to each other, the subsistence of the one rules out the subsistence of the other. There is, in contrast, no tone non-*C* by the side of the tone *C*, and no colour negative red alongside the colour red. One does speak, though, of negative position-takings. But positive and negative position-takings, love and hate, for example, whilst certainly opposed to each other, are nevertheless not mutually contradictory. Only when one and the same subject adopts opposite positions relative to one and the same thing can we speak of an inner inconsistency, of a 'self-contradiction' of this subject. We would then be talking of a quite different type of contradiction, however. The relation which interests us here, between positive and negative as logical contradictories, is to be found only in the sphere of states of affairs.²⁰

Positive and negative states of affairs are totally co-ordinated to each other. If there exists somewhere a red rose then with the existence of this physical thing are given arbitrarily many positive and negative states of affairs. The red rose exists, the rose is red, a specific instance of red inheres in the rose; the rose is not white, not yellow, etc. The red rose, this physical unit-complex [*dieser dingliche Einheitskomplex*] is the factual material which underlies each and every one of these states of affairs [*ist der allen diesen Sachverhalten zugrunde liegende Tatbestand*]. In the case of the rose we speak of existence, in the case of the states of affairs based upon the rose we do better to speak of subsistence.²¹ *It must be noted that subsistence is by no means included as an essential moment within the concept of a state of affairs.* Just as we can separate (real or ideal) objects from their (real or ideal) existence and recognise without further ado that certain objects, such as golden mountains and round squares, do not exist (or even, that they could not exist), so we separate also the state of affairs from its subsistence and speak of states,

like the being golden of mountains or the being round of squares which do not subsist or, again, which could not subsist.²² In this respect there is a far-reaching analogy between objects and states; but we immediately notice also a fundamental difference between the two: that wherever a state does not subsist there subsists of necessity the contradictory state which is opposed to it. For non-existent objects, in contrast, we have no correlated objectual existents. The relation of contradictory positives and negatives – with all the laws bound up with it – has its place exclusively in the sphere of states of affairs.

§ 10 The Apprehension of States of Affairs: Judgment vs. Apprehension

So far we have found that the following holds of states of affairs: that they are that which is believed and affirmed, which stand in the relation of ground and consequent, which possess modalities, and which stand in the relation of contradictory positivity and negativity. These determinations are to this extent sufficient, that every entity to which they apply is of necessity a state of affairs. Clearly they do not strictly speaking constitute a definition of states of affairs, but it seems questionable, for such most primitive objectual formations²³ as states of affairs, things, processes, whether definitions are possible at all, and whether, if they were possible, we could achieve anything with their aid. The only thing which can be demanded of us in the context of our present problems is that we remove these formations from the realm of bare opinion and of inadequate imagery and that we bring ourselves as close to them as possible.

This leads us to the question how exactly states of affairs are given to us. Clearly we encounter initially quite peculiar difficulties in this regard. Take once again our example of the being red of the rose. I do say, after all – and everyone would agree with me in this – that I ‘see’ the being red of the rose, and by that I mean not, say, that I see the rose or its redness; I mean, rather, that I see something which is evidently different from the red rose, which we designate as the state of affairs. But there are certain misgivings which present themselves as soon as we attempt to convince ourselves of the rightness of this mode of speech. I see before me the rose, I see also the redness which is to be found inhering in it. But it seems that this exhausts what it is that I see. No matter how much I strain my eyes it seems that I shall not in this fashion disco-

ver a being red of the rose. (Cf. Husserl, 1900/01, II, p. 416.) And still less can I see negative states of affairs, the being non-white of the rose, for example. And yet I mean something quite definite when I say, 'I see that the rose is red' or 'I see that it is not white'. This is not an empty mode of speech, but one resting on experiences in which such states are actually given to us. Admittedly they must be given to us in a different way from the way in which the rose and redness are given. And this is indeed the case. In seeing the rose I 'discern' its being red, this becomes 'apprehended' by me. *Objects* are seen or looked at, *states of affairs*, in contrast, are discerned or apprehended. And one should not allow oneself to be confused, either, by the mode of speech according to which objects too may be apprehended – *as humans say*, or *as animals*. We have here an equivocation whose roots are easily grasped. 'Apprehension' in the sense of conceptual laying-hold-of is something quite different from apprehension in the sense of the discernment of states of affairs. Note that in the just-mentioned cases of apprehension in the former sense the objects are clearly not brought to apprehension in our sense at all; at most it is their being human or their being animal which is apprehended or discerned.

These reflections may be generalised immediately to apply to all judgments effected on the basis of sensual perceptions. That is, whether one is speaking of what is seeable, hearable or smellable the corresponding state of affairs will not itself be seen, heard or smelt but rather apprehended. But nor do we need to restrict ourselves to this group of judgments. Let us take an arbitrary judgment of another type, say ' $2 \times 2 = 4$ '; here too we must distinguish the manner in which the objects which occur in the judgment – 2 and 4, in this case – are given, and the manner in which the whole state of affairs is given. Numbers are of course not sensually perceived, but it would yet be premature to deny them any perceptual or, to choose a more suitable expression, any intuitional mode of givenness. For even numbers can be presented to us. I can make clear to myself in relation to two arbitrary individual objects what the number 2 is; my gaze is then directed towards the object-pair in question, but my intention does not hold short with that; rather, I use it as the basis on which to bring the number 2 to intuitive givenness. Here we cannot go further in investigating these very important cases of intuitional presentation of ideal objects. Husserl has discussed them searchingly (1900/01, II, pp. 600ff); he designated them as cases of 'categorical intuition', and just as the genuine apprehension of states must

be distinguished from sensual presentation, so too must it be distinguished from categorial presentation. It is, after all, immediately clear that the manner in which 2 and 4 are given to us is something quite different from the manner in which we apprehend the identity of 2×2 and 4. We apprehend the state of affairs; the numbers are intuited, could never of their nature become apprehended. We could say quite generally that the entities which are the elements of a state of affairs are perceived, are seen, heard, or grasped categorially. And on the basis of these 'presentations' the state of affairs itself is apprehended in a new and peculiar act. The presentations which lie at the basis of the apprehension differ among themselves according to the type of entity involved. The acts of apprehending built upon them however do not sustain any differentiation of this kind.

Thus we have obtained a further determination of states: they and only they are apprehended in the particular sense discussed by us here. But this should not be taken to imply that a state may not be presented to us except where an act of apprehending is involved. On the contrary, we shall draw quite particular attention to the fact that there is such a thing as a bare bringing to mind of states of affairs which is accompanied by no act of apprehension. I can bring to mind from memory the being red of the rose without needing to perceive the rose itself. Just as the apprehension of the state rested upon a genuine perception of the thing, so this bringing to mind of the state rests upon a mere bringing to mind of that same thing. But in the bringing to mind of the thing in itself we do not yet have the bringing to mind of the state of affairs. We have learned, after all, to separate rigidly things from states of affairs, and we know that to a given thing, as body of factual material [*zu derselben Dingtatbestand*], there belongs a plenitude of subsisting states. Thus on the basis of the bringing to mind of this same red rose I can bring to mind the being red of the rose, the being non-yellow of the rose, and so on.²⁴ It is clear that we have before us once again what Husserl called categorial intuition, that is, an intuitional presentation which is not itself sensual but which in the end finds its foundation in a sensual intuition. That the bringing to mind of a state of affairs is not an apprehension is immediately evident. Yet this bringing to mind nevertheless plays an important role in epistemology, since there our 'understanding' of propositions and therewith, in many cases, the apprehension of states of affairs is frequently explained in terms of such bringing to mind. We cannot here pursue these connections any further; it is im-

portant only that the act of apprehension be separated from all other acts in which we are related intentionally to states of affairs.²⁵

Apprehension is not the bringing to mind of a state, but nor obviously is it the assertion of a state. For it is essential to an act of apprehension that in it the correlate state of affairs is, in the fullest sense, there for us, where in an assertion it is in contrast merely meant. The characteristic difference between these two acts is too immediately apparent for it to be necessary that we go more deeply into it here. What might perhaps loom near, however is a confusion of apprehension with conviction. For in conviction also, in so far as it comes into consideration for us, the state in question is presented. However the absolute difference between the two is shown already by the just-mentioned considerations. Let us suppose that I bring to mind the being red of a rose which I had apprehended at some earlier stage. I am convinced of it, precisely as before; here again we have the conviction of a state which has been brought to presentation, now however there is no apprehension which lies before us. But even in those cases where apprehension and conviction are present alongside each other the difference between the two is unmistakable. I apprehend the being red of the rose; in this apprehension the state of affairs is presented to me, and on the basis of the apprehension there develops in me the conviction of, or belief in, that state of affairs. Conviction is, in this case, founded in apprehension; the former is the position which I take up, my receipt, so to speak, for that which apprehension offers to me. And we become clear concerning other aspects of the kind of difference between the two when we note that the kind of gradations of certainty which lead from conviction to doubt have no place at all with regard to apprehension, and further that apprehension, just like assertion (and in contrast to the condition of conviction or belief) is of a completely punctual nature.

Assertion and conviction both carry the name 'judgment', and we now see that we must distinguish judgment and apprehension in the sharpest possible way.²⁶ Moreover we see that that conviction which develops in relation to a state of affairs which has been brought to presentation – which we earlier recognised as a type of judgment which is distinguished from convictions of other types – may be characterised more closely as a conviction which is founded in the apprehension of a state of affairs. The first determination of states of affairs which we attained was that they are that which is believed and asserted, the final determination which we shall award them is that they are apprehended.

§ 11 States of Affairs and Relations

In the dispute as to whether it is any arbitrary entities or whether it is only relations which may be judged, both parties are in the wrong. Both have ignored this third formation – the state of affairs – which is neither object nor relation, and which alone, for reasons of principle, can furnish the intentional correlate of judgments. The question will now be raised as to how one is to deal with judgments such as ‘*a* inheres in *b*’ or ‘*a* is similar to *b*’. For even should we admit that there is no relation which is judged in judgments such as ‘*a* is *P*’, the situation seems to be quite different in these two cases. It is not difficult to lay such doubts to rest. The being similar of *a* and *b* is something that may be asserted, believed, apprehended, which can take on modalities, etc. It is certainly, therefore, a state of affairs. If one designates both it, and other states of affairs of the same form, as relations, then this is to be committed to the view that there are states of affairs which are relations and states of affairs – like the being *P* of *a* – which are not relations. Accordingly some judgments are seen as having relations, others as having non-relations as their objectual correlates. But even in those case where judgments are correlated with relations this intentional correlation is mediated through these relations’ being *states of affairs*, not through their being *relations*.

There is of course more to be said on this. The term ‘relation’ is by no means unambiguous. Not only left and right, over and under go under this name, so also do being left, being over and under, etc. The two groups are however fundamentally different. Only members of the second group are states of affairs – though states of affairs, which are in need of completion; the former are related to corresponding states as the colour red is related to being red in colour. Neither red nor left and right can be negated or take on modalities, as can being red and being left or right. For certain relations, similarity and inherence, for example, this difference is concealed through the ambiguity in the terms ‘similarity’ and ‘inherence’. These terms may on the one hand mean the being similar and the being inherent (or inhering), and in using them in this way we would be speaking either of the assertion of or of the belief in the being similar of *a* and *b*. But they may, on the other hand, mean that through which the *being* in the state of affairs²⁷ becomes determined as a being similar or as a being inherent. In this sense we speak of *a* having [a certain] similarity with *b*. Just as we could transform the sen-

tence 'a is red' into the new sentence 'a has redness' – where 'redness' here does not at all mean being red but signifies rather nothing more than the substantivisation of 'red' – so we can transform the sentence 'a is similar to b' into 'a has similarity with b' – and here too 'similarity' does not mean being similar (what, after all, should it mean to say that a has a being similar?) but signifies rather the simple substantivisation of 'similar'.

Thus we see that there are relations in two senses: according to the first sense, relations are at the same time states of affairs, and according to the second they are something quite different from states of affairs. Here we do not wish to come to a decision as to which of the two senses should more justifiably be bestowed upon this expression.²⁸ We wish only to draw the following consequence for our own arguments, that if we interpret 'relation' in the second sense, then relations could never be judged, for they would never be states of affairs. We could then divide states of affairs into two categories, those in which relations are contained as objectual elements – as the being similar of a and b – and those for which this is not the case – as the being red of a rose.

§ 12 On Impersonalia*

It follows from the above that the assertion made by Ameseder that 'every positive objective of so-being is a relation' (1904, p. 75) is in neither sense acceptable. But one must go one step further still. Not only are there 'objectives of so-being' (states of affairs of the form *being P of a*) which are not relations; there exist also states of affairs which instead of having two or three objectual elements have only a single element. For such states of affairs it is immediately apparent that we cannot speak of relations; at the same time they show that Meinong's division of states of affairs into those of the form *a exists* and those of the form *a is B* (1910, p. 72) does not correspond to any genuine disjunction. We may take as examples the states *being warm*, *being settled*, and so on, which may in no way be reinterpreted as 'objectives of so-being' (the being-warm of something or other). Such one-

* This section originally appeared in Reinach's *Gesammelte Schriften* (pp. 117–120) as an appendix to "Zur Theorie des negativen Urteils". It has been inserted into the text at this point, along with the note to which it refers.

membered states can be believed and asserted. This is the way in which we obtain the judgments 'it is warm' and 'it is settled'.

We may hereby attain a solution – at one stroke as it were – of the old and much-discussed problem of the nature of impersonal judgments. I dip my hand into a basin and thereby acquire the conviction that the liquid therein is warm. We shall attempt to grasp this situation somewhat more precisely. I touch the liquid and at the same time perceive its warmth. In touching the liquid or, better, in 'feeling though it' (at the same time grasping, in this penetration, that it is liquid) and in becoming aware of the warmth in it and of it, its being warm becomes evident to me, I apprehend this state of affairs. Liquidity and warmth are here sensually perceived, but not the state of affairs: the latter is not 'penetrated' or sensed, but rather apprehended. Of course sensual perception and the apprehension of states of affairs are not absolutely unrelated to each other. It is only a sensual perception which makes the apprehending possible, the latter is *founded* in the former. In every case the apprehending is to be recognised as something completely specific; it is an act of a quite peculiar type, of which it is essential that it relate always and only to states of affairs [cf. § 10 above-Tr.]. In this respect it is similar to conviction or belief, with which however it must by no means be confused. In apprehending the state of affairs, in making it more or less evident to myself, there develops in me the conviction in it. But this conviction, as a position-taking, a condition which may endure for an arbitrary time, is sharply to be distinguished from the spontaneous, temporally punctual act of apprehending. The necessity of this distinction becomes quite specially clear when we note that there may well be conviction in the absence of apprehending. The case of conviction which develops simultaneously with an apprehending but then survives it can already demonstrate this to us. Above all however we are to reflect on those cases where no apprehending is present at all, as when we can bring to mind in memory a house in such a way as to acquire the conviction that it appeared thus and so without thereby being able to apprehend this state of affairs in memory. We do not wish to speak further of such cases of conviction in what follows, and by 'conviction' in the sense of judgment we shall understand only *apprehending* conviction, that is to say, conviction which is founded in an apprehending act. It is now very clear that in the example just analysed we have to distinguish, on the object-side, the object and its property (i.e. the liquid and its warmth) and the state of affairs (the being warm of the liquid) and,

on the other side, the 'side of consciousness', the correlated sensual perception and the apprehending, and then further the conviction which is rooted in the latter.

What it is to perceive warmth, this we know immediately when we bring to mind some example without our having to embark upon a close analysis of this kind of sensation. In our case the warmth is sensed as belonging to some object, the liquid. But now this is not a necessary condition. I go out of my house into the open; I can then perceive pure warmth – I perceive it in all exposed parts of my body. On the basis of this sensual perception I apprehend a state of affairs: the being warm, and I acquire the conviction of this state of affairs: the conviction that it is warm. We are far from wishing to deny that here too the warmth *may* appear as belonging to something other, to the air, for example, which is streaming against us. What is essential however is that this does not need to be the case. Let us think of a case where we have not the slightest awareness of the movement of the air about us, where we perceive, as we go out into the open, pure and isolated warmth. In such a case this is not bound up for us with any further object. It would be a fabrication of the worst kind if one wanted to assume that warmth must be bound up for us with the surrounding space or with the 'totality of existents' or even with 'chaos', which would then all of them have to be perceived at the same time. For what is sensually perceived is warmth, pure and simple; and what is apprehended, in consequence, is the being warm, and thus conviction also relates to this same state of affairs. That the being warm is a state of affairs stands beyond doubt after our earlier investigations. For it is clearly something which can stand in relations of ground and consequent, that possesses a negative counterpart to which it stands in contrary opposition, that can adopt modalities, etc. But it is at the same time a state of affairs which distinguishes itself in a characteristic way from others, e.g. from the being warm of a liquid: for this state of affairs is through and through one-membered. We apprehend and become convinced of a precisely determined being thus and so, but one which is in no way the being thus and so of some object (the air, or something similar), but rather it is a simple unattached so-being.

Much more ought we to fear objections from an opposite direction. It is the warmth – so it might perhaps be argued – which, in our example is perceived; and thus also it is this alone which comes to be apprehended and believed. If I am convinced of the warmth then my judgment relates to this condition, which we call warmth, and it is not at all clear

why we should now require the concept of a state of affairs here at all.²⁹ This objection threatens not only our conception of the individual examples at present under investigation but also the foundations of the theory of judgment earlier attained. For a spatially perceptible condition is certainly not a state of affairs. And if in this case conviction truly related to the condition, then we could no longer maintain the thesis that it lies in the essence of the judgment as such to relate to states of affairs. It is however necessary to proceed with great care at this point. The question arises whether the expression 'warmth' is wholly unambiguous, whether it truly designates the determinate condition in all its uses. Already, before we bring directly into view the phenomena in question, there are certain considerations which are suited to shake this belief. We speak of having pleasure in the warmth and of being pleased about the warmth, and this change of expression is not without significance. It is distinct references of our feelings which are meant thereby. Pleasure 'in' something presupposes that that something in which I have pleasure is somehow present to me. I must sense the warmth, see a colour, hear a melody, or I must at least bring each of these intuitively to mind if I am to have pleasure in them. Concerning pleasure *about* something however the matter is quite different. Facts are related to me. I hear the words of he who is speaking and understand them. I can be pleased about that which he relates, even though it need not in any way be brought to mind by me. One thinks of conversations in which sentence rapidly follows sentence in complex cycles. I understand that which is communicated to me, and I can alternately be pleased or displeased about it. But that it should be intuitively present to me, that is neither necessary, nor, as a rule, is it in fact the case. This immediately takes us one step further forward however. That in which I have pleasure, and that about which I am pleased, the two being grasped in such different ways as the corresponding attitudes develop in me, are themselves completely different from each other. I take pleasure in the warmth, in the rose, i.e. in objects in the widest sense. I am pleased about this: that there are roses, that it has become warm; every pleasure *about* something – like every sadness, anger, etc. – necessarily relates to a state of affairs. Thus if we can speak, now, both of having pleasure in the warmth, and of being pleased about the warmth, this points to a double meaning of the expression in question. Indeed I do take pleasure in the warmth as an object when I let myself luxuriate in its washing about me on all sides. But when, in contrast, I am pleased about

the warmth, then this will in general mean that I am pleased about this: that it is warm (that is, about the state of affairs). Thus 'warmth' means both the condition and the state of affairs. Only thus can we understand how one could come to the erroneous conception that it is not a state of affairs which is judged in a sentence but a condition.

That this is an erroneous conception is quite certain. Warmth in the sense of a condition can never be something which could be believed. This can best be made clear by reflecting on the fact that conviction that it is warm stands opposed to conviction that it is not warm as its contradictory. Now if the first relates to a positive condition, the second must therefore relate to a condition which would contradict it. But what condition could this be? The condition of coldness perhaps? But firstly warmth does not contradict coldness in the logical sense; and secondly it is not at all correct that the two judgments 'it is not warm' and 'it is cold' are equivalent to each other, much less that they are identical in meaning. The condition of warmth has no contradictorily opposite negative condition; only the state of affairs of being warm has the contradictory opposite: not being warm. And similarly it makes no sense to posit, alongside the 'simple' condition, a condition having the modality of probability or of improbability attached to it; conditions do not admit of such modalities, which attach rather to states of affairs. Thus when we have, beside the judgment 'it is warm', also judgments such as 'it is probably warm', etc., such judgments can be made understandable only as relating to states of affairs and not to conditions. All of this is an immediate consequence of our earlier discoveries. What is new is the insight that there indubitably exist one-membered states of affairs and convictions of such states of affairs. Of course the conviction of its being cold, light or dark, noisy or quiet, musty or clear, each of these has equal status with the conviction that it is warm. Each case is such that it is the warmth or coldness, the light or darkness, the quiet or loudness, the clarity or mustiness, as pure conditions, which are perceived, and that on the basis of this the corresponding state of affairs is apprehended and believed. These states of affairs are one-membered. Certainly in some cases there is a second member to which the sensed content is in itself attached, but this is not comprehended within the judgment and thus the conviction is in no way related to it; in other cases however there is no objective second member at all. Thus when I am convinced that it is dark, or that it is quiet, it is normally not possible at all to state what, in an individual case, is dark or quiet.

§ 13 Negative Conviction

We have now acquired the means to answer our initial question. We started with the case of positive conviction directed towards something negative, pointing out the difficulties which have been encountered in regard to this case. These difficulties are unavoidable for the traditional conception, which allows relations to function as the intentional correlates of judgments. This view could be maintained so long – though only in the sphere of positive judgment-correlates – because, on the one hand, many states of affairs could indeed be considered as relations, and on the other hand for those which remained (as, say, the being red of a rose) transformation into a relation, whilst in fact incorrect, still seemed possible in the absence of a closer analysis. The matter is quite different in the negative case; here it is after all only too clear that with the being non- P of a no relation between a and P is judged. Thus it is quite understandable that sensible logicians endeavoured to transfer negation from the objectual side over to the side of consciousness. We have seen that this attempt broke down for the case of positive conviction in something negative. It is now not difficult for us to recognise why this should have occurred. The negative something toward which the positive conviction or belief in the being non- P of a is related is of course neither an object nor a relation but rather a negative state of affairs. Negative states of affairs subsist in precisely the same sense and with precisely the same objectivity as do positive states of affairs. A subjectivising re-interpretation is here neither necessary nor possible. Alongside the negative conviction or disbelief in a positive state of affairs, now, there stands on an equal footing the positive conviction or belief in a negative state of affairs. And both can carry the name 'negative judgment', for a logic which systematically carried through the distinction between judgment and judged state of affairs could scarcely decline to classify judgments according to the characteristics of their correlated states of affairs.

In the light of the discussions so far, negative conviction in positive states of affairs and positive conviction in negative states of affairs seem to be completely parallel to positive conviction in positive states of affairs. If we turn our attention, however, to the preconditions under which negative judgments of these two kinds develop, then we discover certain important differences as compared with positive judgments. We have so far only hinted at the peculiarities which are involved; now

however we must throw a rather stronger light upon them. Positive states – as already pointed out above – can be ‘read off’, e.g. when, on the basis of the sensual perception of a physical thing, there arises simultaneously both the apprehension of and a belief in an appropriate state of affairs. Now a negative state of affairs could never be ‘read off’ and a negative conviction could never arise in this fashion.

To take first of all the case of negative conviction: as already pointed out above, this has as its psychological presupposition an intellectual position-taking relative to some state of affairs, *S*, a position-taking which may be one of positive conviction, of conjecture, of questioning, or what have you. Having taken up such a position we approach a state of affairs, *S'*, which is in conflict with *S*. As we apprehend *S'* and simultaneously grasp this conflict, *S* appears to us under a quite different aspect, an aspect for which we have, as yet, no adequate terminology and about which we can give, at this stage, no more than indications. The *second* apprehended state, *S'*, stands before us in such a way that it can be said to have an evidential character: in this apprehension the state of affairs is evident to us.³⁰ When, now, we grasp the conflict in which the first state stands with this second state, the former acquires that peculiar aspect which we might most reasonably designate as negative evidence. And it is only on the basis of this latter ‘negative’ evidence that the relevant negative conviction or disbelief develops within us.

Let us consider an example of this. In simply scanning the world around us we may clearly come to the positive conviction that some object is red, but never to the negative conviction that it is yellow. The precondition of the latter is that the corresponding state of affairs has somehow been first brought into consideration, whether through our questioning, doubting, or in some other way. What, then, takes place when we move from such consideration of an initial state of affairs to a terminating conviction? We stand before the relevant factual material in the existing world and we apprehend that the object is red. With this state of affairs positively evident to us we grasp that the state which is under consideration, the being yellow of the rose, stands in conflict with it, and thus this second state acquires that peculiar countenance which we have chosen to call negative evidence. Now only does there develop within us the disbelief in this state of affairs.

Negative conviction therefore is subject to two preconditions: it must be preceded by an intellectual position-taking relative to the state of affairs in question; and there must then occur an apprehension of a con-

flicting state and a grasping of this conflict. The first of these conditions refers to the attitude which is the precondition for the occurrence of the judgment. Thus it is of specifically psychological interest. The second precondition is that which must be satisfied if negative conviction is to acquire certainty and justification. Thus it has a specifically epistemological interest; we shall call it the fundament of the negative judgment.

Let us now turn to the case of positive conviction of a negative state of affairs. This too is subject to quite specific preconditions, for if we were to limit ourselves to reading off those states which are given to us by the world of real and ideal objects then such a thing as a negative state would never be presented to us. And here also it is certain intellectual position-takings which are presupposed. I must turn my interest to the negative state as such and, for example, put it into doubt or into question, if I am to be able to judge about it. That we come to such position-takings at all is quite understandable given the existence of negative conviction in positive states of affairs. For the present case is so closely related to the case of positive conviction of a negative state that psychologically the one can very well move in to take the place of the other.

Much more important than this psychological precondition is the fact that here too there is a complex epistemological fundament which underlies the conviction involved. Like the negative conviction of a positive state so also the positive conviction of a negative state presupposes the apprehension of another state. The conviction that 3 is not smaller than 2 can develop only on the basis of the apprehension that 3 is greater than 2. Here already however we can clearly discern the difference between this and the former case. For there it was necessary that a state be apprehended which stood in conflict with the judged positive state. Here in contrast the judged negative state – the being-not-smaller-than of the number 3 – stands with the apprehended state – the being greater than – in a relation of necessary connection of such a kind that the subsistence of the one is directly bound up with the subsistence of the other. In consequence our present task is a quite different one from the above. For there the (positive) state to which the (negative) conviction was referred was negatively evident, in the sense that it stood in conflict with the other, positively evident state. Here the (negative) state toward which the (positive) conviction is referred is positively evident, since of course it stands in a relation of necessary connection to the positively evident state.

Now of course there also exists negative conviction (i.e. disbelief) in negative state of affairs, i.e. a doubly negative judgment. The psychological precondition here is an intellectual position-taking relative to the negative state which is in question. The epistemological fundament however, as in all of these cases, consists in the apprehension of a positive state which underlies the negative conviction which is involved. As in the first case so also here this underlying state must be such that the state which is judged stands in conflict with it, but here the conflict is a quite peculiar relation: the two states of affairs are contradictory to each other.³¹

Naturally we are not dealing here with empirical contingencies, but rather with *a priori* connections among essences. One such connection can be given the following provisional formulation: that every positive conviction of a positive or negative state of affairs presupposes – epistemologically – the positive evidence of that state of affairs [i.e. presupposes that it is apprehended with positive evidence]. Every negative conviction of a positive or negative state of affairs presupposes the negative evidence of that state of affairs. The positive evidence of a negative state of affairs presupposes in turn the positive evidence of a positive state which is necessarily bound up with it. The negative evidence of a positive or negative state presupposes the positive evidence of a conflicting positive state – and in the case of the negative evidence of a state which is itself negative the two states are always contradictory to each other.

Each of these not at all simple relations will require a still more detailed investigation.

III Negative States of Affairs and the Sphere of Assertion

§ 14 The Character of the Assertive Sphere: Simple and Polemical Negative Judgments

We have established above a distinction between conviction and assertion. Conviction or belief develops on the basis of the apprehension of states of affairs. It outlives that apprehension, and it can endure even when the state in question is no longer current. If conviction passes away, then it leaves behind what is generally called inactual knowledge. On the other hand however a state of affairs of which we remain convinced can become re-positing in an act of assertion. We have already seen that at the basis of every assertion there lies a conviction. This thesis can now be made more precise as follows. The conviction which underlies an assertion must in every case be positive; in no case can a negative conviction underlie an assertion. It belongs to the essence of assertion (assertive positing) that that which is asserted is *believed*; thus if there should develop in the sphere of conviction a *disbelief* then it must transmute into a belief in the contradictory state of affairs before an assertion can develop out of it.

As in the case of conviction so also in the case of assertion it is states of affairs which can alone function as objectual correlates, though in the case of apprehending conviction the states in question are presented,³² where in assertion they are merely meant. And this is connected to another important peculiarity of assertion. For in apprehending conviction the state of affairs stands before me as it were in one blow, in its totality; we have no sequence of successive acts of grasping [constituents of the state], but rather one single act, in which the state as a whole is taken hold of. Quite different is the case of assertion. If I say, positingly: the rose is red, then there is here a series of acts in which the elements of the state of affairs are meant successively. The state is not meant in one blow – as it is in mind in one blow in the case of apprehending conviction. It is rather, built up in a series of acts, analogous to the way in which the elements of a melody constitute themselves in succes-

sive experiences of hearing. Clearly these acts of meaning do not appear side by side unrelated to each other – as little as do the successive experiences of hearing the notes of a melody. Just as there the unity of the elements unifies the several experiences into the total hearing of the melody, so the unity of the elements of the state of affairs unifies the acts of meaning into a total meaning of the whole state. In the present case this total meaning is governed by the specific moment of assertion, but it may be governed in other cases by, for example, the moment of questioning. The state of affairs which stood before us in one blow in our apprehending conviction of it now acquires, in this assertive total meaning, a peculiar modification of its form, becoming articulated into the elements now successively constituting themselves. A series of categorical forms which are often referred to as ‘merely grammatical’ – although they extend beyond the linguistic sphere into the region of logic – have their place here. A further development of this point would, however, lead us too far afield.

As in the case of conviction, so also in the case of assertion we have to distinguish the positive and the negative judgment. Side by side with the judgment ‘*a* is *P*’ we have also the judgment ‘*a* is not *P*’. Traditional logical theory tends here to oppose rejection to acceptance, denial to assertion, negation to affirmation, or otherwise make appeal to some opposition of this sort. According to such theories it is one and the same state of affairs which is asserted or affirmed in the positive judgment and negated or denied in the negative, corresponding exactly to the way in which, in the other judgment-sphere, both positive and negative conviction (belief and disbelief) can relate to the same state of affairs.

This view is however by no means as self-evident as it may at first sight seem. The following difficulty seems above all to have been overlooked in this regard. Positive and negative conviction are both of them *conviction*, even though they are of opposite sign. This is what allows us to conceive both as judgments of a single type. But what is it which assertion and denial (or affirmation and negation) have in common which makes both of them judgments? This is clearly a question which cannot be answered immediately, without further consideration. It is of course true from the descriptive point of view that even in the sphere of assertion positive and negative judgments are closely related to each other. Indeed Lotze’s attempt (1880, p. 61) to propose a tripartite division encompassing affirmation, negation, and questioning as standing equally, side by side with each other, breaks down precisely in virtue of this

intimate community of the positive and negative judgment, as compared to acts of questioning. But it thereby becomes all the more urgent for the traditional conception that it give an account of what precisely this relationship is. However this problem may be solved, it cannot be avoided as a problem by the proponents of this conception. That it has not so far been solved need not imply any objection to the view in question. We wish merely to point out that we have here exposed an important difficulty for a view which had at first seemed so clear and self-evident. The only thing which can be decisive in cases such as this is to turn one's attention directly to the phenomena themselves; only thus can we discover once and for all whether denial is properly to be regarded as on an equal footing with assertion.

First of all we must pose once more our familiar question, the question as to whether the term 'negative judgment' has an unequivocal sense in the sphere of assertion at all. And just as we distinguished two types of negative judgment in the case of conviction, so we must now do the same here also, even though the distinction does not perhaps spring so immediately to the eye in this as in the former case.

Let us consider the judgment, 'the king was not energetic', as it occurs in two different contexts. In the first context it is uttered by a historian who is expressing his opposition to the view that the king has been energetic. In the second context it occurs purely descriptively, in the course of a historical narrative. One must not overlook the quite different aspects which are possessed by the judgment in these two cases: in the first case it has the aspect of opposition to the contradictory positive judgment, ('the king was *not* energetic'), in the second case that of simple portrayal, ('in this period the country flourished anew. The king was, be it said, not energetic, but ...'). One may wish to take no notice of such 'trivial' differences. This attitude we can very well accept, but only so long as it is admitted that they are differences. And given the evidence of the situation before us this is something which one cannot avoid: on the one hand we have a polemical taking up of a position against another judgment, on the other hand a simple positing. Now in the first of these cases the traditional conception, according to which the negative judgment is to be regarded as a denial or a rejection, has all the appearances on its side. In the second case, in contrast, an unprejudiced consideration would lead us much rather to speak of a positing or an asserting. In any event it has by now become clear that this whole question, far from being self-evident, demands a much closer investiga-

tion. We begin with an analysis of that which is brought to expression in the word 'not', for it is of course this which outwardly differentiates the negative from the positive judgment.

§ 15 Cognitive Functions: 'And', 'Hence' and 'Not'

Already above we have spoken quite generally of 'words' and of the peculiar meaning-acts directed towards objectual correlates which lie before us with the understanding utterance or words. Husserl speaks here of meaning-bestowing acts, acts which ensure that we do not stay tied to the mere word-sound as such, but rather that the latter acquires 'meaning' [*Bedeutung*] for us. However well-founded is this notion of meaning-bestowing act, and however important it is for the understanding of the fundamental concept of (ideal) meaning as such – of which we will here have nothing further to say – it must be emphasised that objectual meaning and meant objectual correlate cannot be assigned to every word. Words such as 'and', 'but', 'also', 'hence', 'not', and so on, are understood in the course of the understanding utterance of sentences without our being able to say that they are guided by acts of meaning objectual correlates – as are, say, the words 'Socrates' or 'tree'. It is indubitable that when I utter one of these words understandingly in the context of a sentence there is something more than the utterance itself which is involved; but it is equally indubitable that this something more is not a direction towards something objectual in the sense earlier delineated. For what could this objectual something be, which would correspond to 'also' or 'but'? This makes all the more urgent the question as to what it really is which corresponds to such 'objectless' expressions. Here we wish to restrict our attention to 'and' and 'not'.³³ It is strictly speaking only the latter which is of interest to us, but calling in aid the other, more neutral example will be useful to our purposes.

If I say '*a* and *b* are *C*', then in regard to the subject-place I am directed to *a* and to *b*, but not however to any *and*. In spite of this, the directedness towards *a* and *b* does not exhaust everything which is involved here: *a* and *b* are not just meant, they are at the same time *connected together*. It is this connection which corresponds to 'and'. The and-function, therefore, connects; it combines together.³⁴ And indeed it connects together always pair-wise. If someone wishes to combine *a*, *b*, *c* as in '*a* and *b* and *c* are *D*', – then two such connecting functions are required. Of course instead of this one could also say: '*a*, *b* and *c* are *D*',

or even: '*a, b, c* are *D*', but the absence of the word 'and' does not imply that the corresponding function too is absent. It is indeed clear that in these cases also the and-function is doubly present. For *a, b, c*, are not meant without relation to each other; they are rather bound together in a 'connectional meaning act'.

We must separate most strictly the connecting function which we ascribe to 'and', from that which is constituted for us in the connectional meaning-act, i.e. the 'totality' or 'whole' made up of *a* and *b*. These – certainly highly ambiguous – terms should not be misunderstood. Above all, the whole, *a and b*, which is constituted through the operation of the and-function, is not a spatial or temporal togetherness; it is not at all the kind of unity which would be characterised by any material relationship between its constituents, however remote. The most heterogeneous entities can clearly be 'bound together' by means of 'and'. And just as little should this connecting-function be confused with synthetic apperception through which entities brought to presentation are combined into a unity. (See Lipps, 1906, p. 119). For the and-function is to be found in the sphere of meaning, in which, of course, entities are not brought to presentation at all.

It is scarcely possible to determine more closely this connection: one can only issue the invitation to reflect upon it and convince oneself of its peculiar nature. It is not at all something which is brought to presentation in the understanding utterance of the sentence, just as little as are – according to the results of our earlier investigations – the objects themselves. If I say: *a* and *b* and *c* and *d* are *E*, then a series of connecting functions is involved, but the totality which thereby develops is not presented to me. And what holds for this whole of several objects holds also for a whole consisting of only two. Of course I am at each stage free to bring this totality to presentation. And then I apprehend it securely as that which had been constituted through the connectional meaning-act. Indeed without this possibility we could not speak of a constitution effected by means of the and-function at all. However, in the course of speech such a bringing to mind does not normally take place.

Here we find an opposition skew to that which was distinguished earlier between meaning and presenting. There corresponds to 'and' not the meaning of some object, but rather a function, in particular a connecting function.³⁵ This connecting has to be separated on principle from the presentation of that which becomes constituted in it. And thus beside the opposition of meaning and presenting one and the same ob-

jectual something there now appears the quite different opposition between the execution of a function and the presentation of that which is constituted in this execution. Certainly there is such a thing as direction towards the function itself; indeed it is to precisely this that we appeal when speaking of the function. And this in turn must be distinguished from the presentation of the function, as this occurs, e.g. when one attempts to make our current considerations understandable. It is on the other hand possible to direct oneself toward [to *mean*] that which is constituted in the function, as when we speak of 'the totality *a and b*', and it is in turn possible also to bring to presentation this same totality. Here we have once again our old opposition between meaning and presenting. What is new is this second opposition, between the execution of a function on the one side, and the presentation of that which is constituted by the function on the other.

Our principal aim is the clarification not of 'and', but of 'not'. Consideration of the former has been of benefit however, since the relationships associated with it are less complicated than and yet at the same time in several respects parallel to those associated with 'not'. For when I say '*a is not B*' it is again impermissible to speak of a direction towards a *not* in the sense in which one can speak of a direction towards *a*, or towards *B*. Here too it is a function which is involved. In the case of 'and' we spoke of a 'connecting' function; here we have a function which we can designate as a 'negating'. But whilst in the former case there are always at least two objects which become connected together, the negating function is set to work on only *one* objectual something. Its locus of action can be very precisely determined. Neither *a* nor *B* can be negated, but only the *being B* of *a*. In our example therefore the negating function relates particularly to the 'is', and therefore at the same time it relates to the whole state of affairs, *a is B* which becomes constituted, articulated and [thereby] modified in its form, through the execution of the judgment. To this extent the old scholastic thesis is perfectly correct: *in propositione negativa negatio afficere debet copulam*.

Clearly we must make a distinction here also between the *function*, that to which the function is *applied*, and that which is *constituted* or *developed* in this application. For as the *is* in the state of affairs becomes negated, there develops the contradictory-negative state of affairs *a is not B*. It is not altogether easy to picture clearly to oneself the situation here. The negating function itself has to be securely grasped as that which corresponds to the 'not', but so also has the fact that this function

is applied to that element of the state of affairs which finds its expression in the 'is'. This 'is' becomes negated and transmuted thereby into an 'is not'. Thus by means of the negating function there arises the negative state of affairs. This latter is itself in no way brought to mind in the ordinary course of our thinking; it is as though the advance of our meaning acts leaves it behind. But it is at all times open to us to bring it to mind in new acts of presentation and thereby to apprehend it as that which is constituted for us through our act of negation. Thus we have the meaning and the presenting of the negating function, and we have also the meaning and the presenting of the negative state of affairs which has become constituted for us through this function. And finally we have the opposition with which we are here concerned between the execution of the negating function and the presentation of the negative state of affairs thereby constituted.

The expression 'constitution' should not be misunderstood; its use should naturally not imply that negative states of affairs are somehow created or manufactured through the negating function. For we know that negative states of affairs subsist precisely as do positive states, quite independently of whether or not they are presented to anybody or come to be apprehended, believed, meant, or asserted. *That 2×2 is not equal to 5*, this state of affairs, subsists wholly independently of any conscious subject which may grasp it, just as much as does the positive *being identical of 2×2 and 4*. Thus precisely as in the case of positive states, so also negative states are apprehended (though on the basis of the apprehension of positive states), and in this apprehension is founded the belief or conviction in them. If states thus believed are subsequently brought forward in acts of assertion, then in the case of positive states the latter are built up from acts of meaning of objectual elements. In the case of negative states, in contrast, this building up requires the execution in the meaning-sphere of a function which negates certain meant elements. This therefore is the sense of the expression 'constitution': not that states of affairs in themselves are generated through the function of negation, but rather that by means of this function they are built up in and for the act of meaning.

§ 16 The Moment of Assertion

Let us turn once more to our original question. Since according to our exposition there occurs in the negative judgment a negating or denying,

so one could say also that the negative judgment is to that extent itself a denial [*Verneinung*], and that we ourselves have overturned our original scruples in regard to this thesis. But this is completely to misconstrue the situation. For the division of judgments into affirmations and denials involves, after all, much more than the claim that there exist judgments with and without denials. What one wants to claim is that in the notion of denial the essence of the negative judgment *as a judgment* is completely captured, that it is sufficient to characterise something as a denial in order for it to qualify thereby as a judgment – and it is precisely this which we must call into doubt. This doubt finds complete corroboration in our analysis in terms of functions. It is not true that the notion of denial captures all that pertains specifically to judgments in the negative case; for there exist formations in which a denial is to be found which are not judgments at all. Consider the case where in response to the judgment ‘*a* is not *B*’ we say: ‘*a* is not *B*; that I doubt very much.’ A denial is certainly present in this response, but one cannot seriously speak of the presence of any judgment ‘*a* is not *B*’ which is somehow revoked in the second half of the sentence. A genuine, complete assertion is clearly not to be found in the antecedent clause. Thus we have here a case of denial which is not a judgment. And such examples can be multiplied: ‘Is *a* not *B*?’, ‘Suppose *a* were not *B* ...’, etc. Everywhere we can find denial in the absence of judgments.

Now one may very well say that ‘denial’ was not meant in this sense. In the sentence ‘*a* is not *B*; that I doubt very much’ and in the other cases mentioned there is no denial, no negating act, present at all. Something further must be added in order that the sentence be turned into a judging denial. With this we can do nothing but agree. But what is it which is to be added? If we compare our sentence with the judgment: ‘*a* is not *B*’, then we see very clearly what this should be. What is there merely repeated, put forward without being honestly asserted, is here truly asserted. Thus it is the moment of assertion which makes the negative judgment, just as much as the positive judgment, into a judgment at all.

We shall therefore say that there are assertions in which no negating function is to be found – these are the so-called positive judgments. And there are also assertions in which the copula of the state of affairs, and thereby the state of affairs as a whole, is negated. In the negating function a negative state of affairs becomes constituted, and it is the negative state thus constituted which is put into question in the negative

question, assumed in the negative assumption, and finally asserted in the negative judgment. On the other hand there is no 'act of affirmation', and just as little is there an 'act' of negation in relation to which we should have to seek the essence of the negative judgment. Much rather do both the positive and the negative judgment present themselves as assertions; and the negative is distinguished from the positive judgment only in this, that in the former the assertion relates to a negative states of affairs which becomes constituted in the execution of the negating function. It is this negating function which makes the negative judgment into a *negative* judgment – and it is the moment of assertion which makes it into a judgment at all.³⁶

We spoke at the beginning of the difficulty for traditional theories of judgment of exhibiting the feature which makes purported acts of affirmation and of denial into judgments. Such difficulties do not arise for our conception. Positive and negative judgments are judgments in virtue of their possession of the specific moment of assertion. The term 'positive judgment' does not somehow imply the presence of a special act of affirmation or a special affirmation-function, but merely the absence of the negating function. A welcome corroboration of this is provided by the fact that where our language exhibits a 'not' as the expression of negation, there is, in the case of the positive judgment, no special particle which would give expression to a corresponding function of 'affirmation'. The traditional conception of positive and negative judgments is unable to provide an explanation of this peculiarity of language.

§ 17 The Pure Logic of Emphasis

Our conception throws immediate light on the simple negative judgment. But how does it fare in the case of the polemical negative judgment which we distinguished above? If I turn against someone who has asserted the being *B* of *a* with the words: '(No.) *a* is *not B*', then it seems hardly possible to deny that here a rejection or a denial plays an essential role. And indeed we do not wish to deny this at all. But we have to insist that the various factors involved here are kept strictly apart.

What strikes us first of all about the polemical judgment is what we shall call its accentuatedness or emphasis. For in contrast to the simple negative judgment the 'not' here is emphasised. It would be very super-

ficial thinking to want to consign this emphasis exclusively to the sphere of linguistic utterance. For whilst, certainly, there is such a thing as spoken emphasis which is related purely to the sounding of words, this emphasis is only the expression of emphasis in our initial, logically significant sense. What purely phonic emphasis achieves in speaking is achieved in printed or written sentences by the use of bold or italic print or underlining of the word in question. Each of these different modes of expression gives expression to one and the same thing, and it is this identical thing which is our present concern. This finds support also in the fact that the spoken emphasis of one and the same word can serve to give expression to the logically significant emphasis of different things. Consider the judgment '*a is B*', which may on one occasion contradict the assertion '*a was B*' and on another the assertion '*a is not B*'. Through the emphasis of the same word 'is', what is emphasised is in the first case the current temporal moment which the 'is' expresses, and in the second case the positivity of 'is', as something which is opposed to the 'is not'. This second, logical emphasis is surely something primitive, not further reducible. It does not contribute to the constitution of the element which is emphasised; but it must also be very strictly distinguished from any 'heeding' or 'apperceiving' of that element, for such acts have their place not in the sphere of meaning an object but in that of presentation. Here we shall not be able to pursue the important problems of emphasis nor investigate the laws to which it is subject, but shall set forth only that which is indispensable for our purposes.

There is, first of all, emphasis which occurs in the simple meaning of an object: '*the rose (not the tulip) is red*'. And then there is also emphasis which relates to what we called functions; '*a and b (not a alone) are C*'. Here it is a connection which is emphasised; in the execution of the and-function that which is constituted, i.e. the specific moment of connectedness of the totality, undergoes an emphasis. And similarly we find that besides simple negation there is also an emphasised negation: here what is emphasised is the negativity of the negative state which is constituted in the execution of the negating function. All judgments carrying emphasis presuppose the existence of something against which this emphasis is directed. Emphasised negation, in particular, is necessarily directed against another contradictory judgment or sentence which is rejected by the judging subject.³⁷ Thus the polemical negative judgment is distinguished in two respects from the simple negative judgment: it presupposes some contradictory positive judgment (or a

contradictory positive sentence), against which the polemically judging subject is directed and which he rejects; and – connected closely with this – it is such that an emphasis is built into the execution of its negating function through which the negative character of the state of affairs is set into relief relative to that opposing positive judgment. The rejection is directed against the alien judgment, the emphasis relates to the negative state of affairs posited by the subject himself.³⁸

Through this distinction the situation which was at first problematic is now clarified. The polemical negative judgment, too, must indubitably be characterised as an assertion; this is not affected at all by the fact that, thanks to the emphasis which is applied, the negating function emerges more strongly here than in the simple negative judgment. There are, admittedly, other formations which are not judgments and yet which are such that the negating function plays the same dominant role (although for such formations the prior rejection of something contradictory is missing). Consider for example the assumption: ‘Suppose *a* were *not B*’. If we ask what distinguishes the assumption from the corresponding judgment, then we can point only to the moment of assertion on the one side and to the moment of assumption on the other. That this situation has been misunderstood is very understandable. It was easy, first of all, to overlook the moment of assertion as something additional to a negating function made prominent through emphasis, and then – what is even more important – it became easy to mistake the rejection of the contradictory positive judgment which precedes the negative judgment for the negative judgment itself.

Thus we see that also in the case of the polemical judgment it is the moment of assertion which makes up the judgment-character as such. And herewith we have broken with the old logical dualism which wanted to split up the unified assertion into two quite different acts, both of which would then – heaven knows why – carry the name ‘judgment’. Thus we can agree completely with Theodor Lipps when he says (1906 p. 168): “The negative judgment is, like the positive, an act of acceptance” – is, in our terminology, an act of assertion.³⁹

At the same time we have discovered amongst negative assertions – which is what we may call all of those assertions in which a negating is to be found – a fundamental distinction: that between simple and polemical negative judgments. The logicians have in the main treated only of the polemical negative judgments, which are so much nearer to hand because they are so much more common; in scientific contexts it tends

to be almost exclusively negative judgments of this kind which occur. Ideally speaking, however, every polemical negative judgment corresponds to a simple negative judgment and conversely.

The very same distinction can be carried through also with respect to positive judgments. For in addition to the simple judgment '*a* is *B*' we have also the polemical judgment '*a* is *B*' directed against a contradictory negative judgment or a contradictory negative sentence, the positivity of the corresponding state of affairs being brought into prominence through the emphasis on the copula. In this case everything is related in a way quite analogous to the case of the negative judgment, but here, in contrast, it is the simple positive judgment which in actuality occurs more frequently. Thus the distinction between simple and polemical judgments can be established for judgments in general (in so far, of course, as they belong to the sphere of assertion and not to that of conviction).

§ 18 *Sachverhaltselemente* (The Elements of States of Affairs)

The meaning of 'not' is not exhausted in its giving expression to the negating function. Functions of other types can also be bound up with it, without the judgment involved becoming branded as a negative judgment. A theory of the negative judgment must however make reference to them, if only to ward off the possibility of their being confounded with that which is genuinely negative. One needs only to compare two judgments such as '*a* is not *B*' and '*a* is – not *B* (but rather *C*)' in order to disclose immediately a fundamental difference. At first one may well express this difference by saying that in the first case the 'not' relates to 'is' and, in the second case to '*B*', so that in the former it is only the copula which is affected, in the latter the predicate-constituent of the judgment. But clearly this is not an account with which we can be content. The question immediately arises whether the two are 'affected' in the same way. And this is indubitably not the case. For in the first case a negating occurs; the 'being [such and such]' in the state of affairs is negated and a 'not being' is thereby constituted. In the second case, in contrast, there is, no possibility of talking of *B* becoming negated such that in this negation a *non-B* would be constituted. There is no such thing as a negative object, which would somehow become constituted in a negation.

We have an identical situation in the case of the judgment 'Not *a* (but rather *c*) is *B*'. Here too we have a 'not'; and here also there can be no suggestion that a negating occurs in which some kind of *non-a* would be constituted. There is clearly some function which lies before us here, but not a negating function; rather it is a 'retraction' or 'dislodgment' of something which has been intended in the flow of speech.

We spoke earlier of the way in which in an assertion the state of affairs is built up successively out of its elements. Now normally this building up goes through successfully without disturbances; the elements of the state of affairs follow upon one another, supplementing each other in a fashion which is similar to that of the tones in a melody. But there are cases where an element which interposes itself is then retracted – and these are the cases in which 'not' functions in the manner just discussed. In regard to the genuine negative judgment there can be no talk of its being a dislodgment or retraction.

There are very many different types of elements of states of affairs, both essential and inessential. For after all states of affairs, as these constitute themselves in assertion, cannot be simply stuck together, as it were, out of arbitrary elements: they are rather subject to definite laws of constitution. In particular, when the building up of a state has once begun, this cannot be arbitrarily broken off or brought to an end but demands the addition of definite elements, elements prescribed by laws relating not to content but to form, quite parallel to the situation which we encounter in the building up of a melody. We cannot, for example in the case of a state of affairs which has begun with 'the rose is', arbitrarily break off at this point; some element or other, perhaps of the form of [a predicate] *B* must join up to complete it, and that element is, to that extent, a necessary element of the state of affairs. In the judgment 'the car has travelled quickly', in contrast, the 'quickly' is not a necessary element but rather one which is inessential to the formal constitution of the state of affairs. Now elements of states which are retracted through the above-mentioned use of 'not', when they are necessary elements, require to be replaced by others of the same form: Not *a* – but rather *c* – is *B*; *a* is – not *B*, but rather *C*. In the case of inessential elements of the state there is, in contrast, the possibility of retraction without replacement: The car was – by no means quickly – driven.

Of course those judgments in which a retraction function occurs are not to be designated as negative judgments, for in them there is no negating to be found, and nor – what comes to the same thing – is a

negative state of affairs asserted in them; nothing more is involved than the retraction of an element, its exclusion from the state which is in the course of being built up. In the judgment 'a is – not B, but rather C,' a *positive* state of affairs, the being C of a, is asserted, and the fact that within this act of asserting there occurs the retraction of a statal element cannot alter this at all.

The concepts which we have introduced in this section have their place exclusively in the sphere of assertion, not in that of apprehending conviction. This holds above all for the concept of function. Whilst in the assertion 'a is B and C' we posit, thanks to the connecting function, one single state of affairs, in the sphere of apprehending conviction it is *two* states of affairs which come to presentation, for in this sphere there is no connecting, (no connectional *meaning*). And we have an analogous situation for each of the remaining functions. Each of them arises only in the sphere of assertion. Certainly the application of such functions is not arbitrary: they must find their support and justification in the states of affairs themselves and in the relations between them. Only when a negative state of affairs obtains is a negating function permitted to be activated within an assertive meaning act. Only when there are states which stand in definite relations of foundation or of opposition do the functions of 'hence' and 'but' acquire a justification.

The distinctions between being emphasised and unemphasised, between the simple and polemical negative judgment – and between both of these and the judgment which involves merely the rejection of a statal element – have their place only within the sphere of meaning and not in that of apprehending. Once this has been clearly seen, it can no longer be doubted that the division of the judgment into apprehending conviction and assertion carries with it a division of the theory of judgment into two parts, each requiring quite separate treatment.

IV The Locus of Negation

§ 19 Subjective and Objective Aspects of Negative Judgments

We wish briefly to express our view concerning some principal problems which have developed around the negative judgment in the historical development of logic, and thereby once more to throw into light the most important of our results. One thing which is much disputed is the locus of negation: is it a 'real relation' or something 'purely subjective'? In regard to such an ambiguous question a one-sentence answer cannot be supplied. If it is a matter of whether negation is to be sought for on the 'side of consciousness' or on the objectual side of the judgment, then we must say that it is possible to speak of negativity on both sides. In the sphere of apprehending conviction there is disbelief, i.e. negative conviction, and in the sphere of assertion there is the negating function. Both are 'subjective' insofar as they belong to the side of consciousness. However beside (negative) disbelief there is also (positive) belief in something negative (in a negative state of affairs); and further the negative state of affairs to which assertion is related is constituted in the function of negation. Here, clearly, we have negativity on the objectual side of the judgment, and it is to that extent 'objective'.

But talk of the alleged subjectivity of negation has another, quite different sense, one which has been confounded with the above. Even someone who admits that a negative entity can function as the objectual correlate of conviction and assertion may yet go on to say that this negative entity is nothing 'real', that even though it is not something which is located on the side of consciousness, it is yet something which is essentially dependent upon consciousness and which to that extent does not possess any objective being. We must, however, reject such an opinion in the sharpest possible way. Certainly there is no real 'relation' which is posited in the negative judgment – but just as little need this be the case in the positive judgment. Judgments, both positive and negative, refer much rather to states of affairs. These states divide into positive and negative and both of these in turn divide into subsistent and non-subsistent. If a state subsists then its subsistence is independent of

all consciousness; there is absolutely no justification for offering an explanation of precisely the negative state of affairs as being dependent upon consciousness. The denial of objective subsistence to all states of affairs whatsoever, that is the absurd position of absolute epistemological scepticism; for states of affairs are, after all, what is apprehended and judged. He who does not subscribe to this scepticism is not allowed, either, to deny the subsistence of negative states, for the objective subsistence of both are bound together by laws. This is stated with full force in the fundamental principles of logic: of two contradictory states of affairs, either the positive or the negative state must subsist. And: if a positive state does not subsist, then of necessity the contradictory negative state does subsist.⁴⁰

§ 20 Negation of the Predicate and of the Copula: The Doctrine of Infinite Judgments

The question concerning the locus of negation is contestable in yet another dimension, distinct from those so far discussed. Distinguished logicians have declared that, in the judgment, negation attaches not to the copula, but rather to the predicate, that is, in the case of the judgment: 'a is not B', not to the *being B* but rather to *B* itself. We hold this conception to be completely erroneous. In the sphere of apprehending conviction, first of all, it finds absolutely no foothold. For if, on the basis of my discerning the being red of a rose, I apprehend that the rose is not white, and my conviction relates to this latter state of affairs, then we have no function at all, no 'not', which could act upon anything, whether predicate or copula; a simple negative state of affairs is apprehended by us. It is only in the sphere of assertion that a negation-function is to be found; but there it acts upon the copula, not somehow upon the predicate 'B'. This becomes all the more clear when we reflect upon a case where 'not' truly relates to the predicate, for example in the assertion: 'a is – not B, but rather C'. Here it is indeed the predicate-element which is 'affected', but this affecting is a retracting from the state of affairs, not a negating.

Once it has been recognised that the negating-function can relate only to the copula, then it also becomes untenable to talk of 'limitative judgments' or of *propositiones infinitae*, where negative objects are supposed to function as predicates, or subjects, of positive judgments,

(‘the rose is non-red’, ‘non-smokers take their places in this section’, etc.). Here the logician has allowed himself to be misled by the [form of the] linguistic expression which is involved, for there just is no such thing as a negative red or a negative smoker. If we remove the ellipses which lie before us here then our judgments read: ‘the rose is something non-red (i.e. something that is not red)’ and: ‘the non-smokers (i.e. those who do not smoke) ...’. In both cases it is states of affairs which are negated, though certainly states of affairs which are not themselves asserted in the judgments in question but which have rather suffered peculiar transformations – of a type not here further discussed – in their subject- and predicate-places, respectively.

§ 21 Concluding Remarks

Let us now cast one final glance at the thesis according to which the negative judgment always has as its presupposition an executed or attempted positive judgment, and that it presents itself to us essentially as a rejection of this positive judgment, a thesis which has been so much discussed especially since Sigwart’s 1904 (see also Erdmann, 1907, p. 504 ff.; Bergson, 1907, p. 311 ff.; and Maier, 1908, p. 272 ff.). Here there are, it seems to me, all kinds of observations – both correct and incorrect – mixed up together.

Recall, first of all, our discovery that every apprehending negative conviction (every disbelief) and every apprehending positive conviction (or belief) in something negative presupposes the apprehension of a positive state of affairs. Here there can be no talk of the negative judgment’s presupposing a positive *judgment*, for the apprehension of a positive state of affairs is not identical with conviction or belief in it. Or recall, secondly, the fact that both negative conviction and positive conviction in something negative have as their psychological presupposition certain intellectual position-takings. Only in the case of negative conviction is this position-taking directed towards a positive state of affairs. And moreover whilst this position-taking *can* be one of conviction, and thus also a judgment about the positive state involved, it may just as well be one of conjecture, doubt, etc. (See also Windelband, 1884, p. 177.)

Thus the thesis that every negative judgment presupposes a positive judgment must be restricted to one case only, that of negative convic-

tion – and even there it need not *necessarily* apply. But what is wholly to be rejected in the sphere of conviction is the stronger claim according to which the negative judgment is precisely a judgment *about* such an attempted or executed positive judgment (Sigwart, 1904. p. 159). For of course a negative conviction does not relate to a judgment at all, but to a state of affairs.

This second claim reveals that a covert move has been made from the sphere of conviction over into the sphere of assertion. For in the latter sphere there are indeed, as we know, negative judgments directed against contradictory positive judgments which they deny. Certainly the objectual correlate of the negative judgment is in this case too the positive state of affairs; but nevertheless there is here a good sense in which we can say that the negative judgment presupposes a positive one, against which it is directed. We would take exception to this only in pointing out that it is not the negative judgment in general which is here involved, but only the negative *assertion*, and even then only the *polemical* negative assertion.⁴¹

The simple negative judgment, as we have seen, does not have as its presupposition a positive judgment which it rejects or denies. And negative judgments of this type play such an important role, especially in descriptions and narratives, that it is a clearly thoroughly one-sided conception of the negative judgment to suppose, with Kant and many others, that the negating judgment ‘serves solely to guard us from errors’ (A 709).

Notes

¹ From its very beginning logic has met with great difficulties in its treatment of the negative judgment, difficulties which have still by no means been successfully resolved: extensive differences persist within the most diverse schools of thought. Only part of these difficulties touch on the negative judgment as such; for the other part they result from the fact that the positive judgment has not yet successfully been determined in an unequivocal way. And as long as the concept of judgment itself suffers from equivocation and unclarities, so also will the treatment of the negative judgment. In the following paper the attempt will be made not to solve the problem of the negative judgment in all its aspects but to bring this problem nearer to a solution in certain directions. First of all however the scope of our present problems dictates that we begin with some considerations of judgments in general.

Since I must limit myself, in what follows, to the exposition of what is most necessary to my subsequent goal, I have almost completely dispensed with discussions of the relevant literature.

- ² It is clear that consenting relates to the judgment not only in the sense of the act of judgment but also in the sense of the judgment-content. It is not necessary, however, that we should carry through here the somewhat difficult differentiation involved.
- ³ This is the case also, perhaps to an even greater extent, with the term 'approval' ('*Billigung*') which is employed by Windelband to designate the judgment: 1884, pp. 167 ff.
- ⁴ That it won't do to characterise the two cases as cases of 'mere utterance of words', only the first of which rests on an associated *conviction*, which is absent from the second, will be shown by the discussions which follow.
- ⁵ Brentano himself of course speaks of a presentation 'in the widest possible sense' (1889, p. 15).
- ⁶ '*Vorstellung*' in the sense of 'imagination'-Tr.
- ⁷ [*Dasein*] Clearly one cannot confuse this existence or 'being there' with that which is involved in an object's being there expressly face-to-face: we cannot speak of e.g. that which belongs to the background of perception as being there in this latter sense. Cf. Theodor Conrad, 1911, p. 57.
- ⁸ On this see Lipps, 1906, pp. 113 ff., and Husserl, 1900/01, II, p. 129f.
- ⁹ Here we ignore so-called 'intuitive word-presentation', since we are concerned exclusively with meaning or intending objects.
- ¹⁰ We clearly do not wish to conceal the fact that wherever an assertion is made by an empirical consciousness there will usually be much more involved than acts of meaning of a particular kind.
- ¹¹ Thus, to mention only one example, we have spoken here of meaning as something which occurs with the understanding *utterance* of words but not of those related occurrences involved in the understanding *hearing* of words. The latter cannot themselves be designated as acts of meaning since they do not exhibit any spontaneous 'direction towards' but rather a receptive 'taking in'. Nor, however, are they acts of presentation, since that to which this understanding is related is not, in the pregnant sense, 'there', to the subject, or at least need not be there.
- ¹² Cf. Lipps, 1906, p. 230f.
- ¹³ This expression for the judgment as such distinguished from the objectual something to which it is related is readily understandable. It would clearly be more correct to speak of the *intentional* side of the judgment.
- ¹⁴ Instead of depending upon the relations of subsistence and inherence (for physical things) one may attempt to appeal to the more general relation of belonging together, assigning this to our third judgment. Thus Marbe (1910, p. 5) holds that the judgment 'the rose is red' refers to the relation of belonging together which holds between the redness and the rose. But once more we have to object that the judgment 'the rose belongs to the redness' is *different*, in its meaning, from the judgment 'the rose is red'. The former is, for example, reversible ('the redness belongs to the rose'), the latter not. And if one wishes to designate such differences of meaning as trivial, still this triviality of meaning-differences does not make them meaning-*identities*. We are of the firm conviction that if we are to arrive at a solution to the problems which here concern us we must pay the most careful attention to displacements of meaning of this kind, however insignificant they may be in other contexts.
- ¹⁵ Reinach's original reads: Kant spricht davon, daß er irgendein Problem 'vor' unbee-rechtigt halte, heute verbietet uns das der Sprachgebrauch. – Tr.
- ¹⁶ A controversy has arisen with regard to this concept; for the literature see Meinong, 1910 p. 98 ff. In his treatise on "Appearances and psychic functions" 1907, Stumpf remarks that already three decades earlier Brentano had emphasised in his logic lectures that there was a specific judgment-content [*Urteilsinhalt*] corresponding to the judgment, that it was something distinct from the presentational content (or *Materie*)

and that it was expressed linguistically in 'that'-clauses or in substantivised infinitives. Stumpf himself, as he reports, already in his lectures in 1888 began to employ the expression 'state of affairs' [*Sachverhalt*] for his specific judgment-content. The details of how Brentano and Stumpf developed their concepts of judgment-content and state of affairs are unknown to us. The concept of judgment-content as found in the work of Marty, esp. in his 1908, differs in all essential points from *our* state of affairs.

We appeal rather to Husserl's *Logical Investigations* 1900/01 in which the peculiarity and significance of the concept of state of affairs was first clearly and vigorously made prominent in the literature. Our determinations overlap in part also with those pertaining to the concept of *objective* employed by Meinong and his school; however, there are considerable divergences. The most fundamental objection which must be raised against Meinong seems to me to be that his concept of objective runs together the two completely different concepts of proposition (in the logical sense) and state of affairs. It is insufficient to designate the proposition, with Meinong, as a particular kind of objective 'which lies before us and is comprehended, being iterated, where possible, and being at the very least as it were formulated in words' 1910, p. 100. For the moment, however, we must wait for the results of later discussions for the demonstration of this thesis. In what follows we limit ourselves to a short indication of those passages where Husserl's and Meinong's accounts differ from or coincide with those given here.

¹⁷ Likewise, though without the distinction within judgment in general of conviction and assertion, Husserl 1900/01, I, p. 12, II, pp. 48, 378, 416f., and Meinong, 1910, pp. 44, 46, etc.

¹⁸ See Meinong, 1910, pp. 21, 216, etc., and cf. Husserl already in his 1900/01, I, pp. 242, 36f., etc.

¹⁹ Meinong, 1910, p. 80ff.; Husserl, 1900/01, I, 131., 16.

²⁰ Husserl and Meinong too speak respectively of 'contradictory states of affairs' and of 'contradictorily opposed objectives': 1900/01, I, p. 91f., 1910, p. 93.

²¹ This coincides with Husserl's terminology, 1900/01, II, p. 598. Meinong too speaks of subsistence of objectives, but he speaks also of subsistence in the case of *objects* such as numbers, shapes, etc., with respect to which we should prefer to speak of an (ideal) *existence* (cf. 1910 pp. 63, 74). The fact that Meinong in certain circumstances is prepared to speak also of the truth and falsity of objectives arises as a consequence of his running-together propositions and states of affairs. States of affairs subsist or fail to subsist. Propositions are true or false.

Husserl, who had used the designations 'true' and 'false' in the first volume of his *Logical Investigations* in application to states, allowed them to fall away as he carried through the differentiation between propositions and states. But then we still, in the second volume, find that the expression 'validity' is so applied; this is another term which is better avoided, since it too has its primary application in the sphere of propositions. He reaches complete clarity concerning the terms 'truth', 'subsistence' and 'being' only at pp. 597f.

²² That we tend in everyday speech to understand under 'state of affairs' only 'factual objectives', i.e. subsisting states of affairs (Meinong, 1910, p. 101), seems to me to be no adequate reason not to retain a term which – as Meinong himself declares – has the advantage of bringing with it a 'living meaning'. (1905, p. 33).

²³ By 'objectual formation' and 'entity' (*Gegenständliche Gebilde* and *Gegenständlichkeit*) we understand here not only objects but also states of affairs.

²⁴ Whether there is, besides the *bringing to mind* of a state of affairs, also a bare *perception* of a subsisting state of affairs, i.e. one which lacks an accompanying apprehension, is a question the discussion of which would take us too far a field; it would however have to be answered in the affirmative.

- ²⁵ Given these arguments we cannot at all agree with Meinong when he claims that 'objectives' can be grasped only through judgments and assumptions (1910, p. 131 ff.). There is e.g. a (categorical) bringing to mind, a meaning, an apprehending, and a whole series of other acts in which states of affairs are grasped.
- ²⁶ It is accordingly inadmissible to regard apprehension, with Meinong, as something which is by nature true judgment (Meinong, 1904, p. 18). A 'true' conviction which has been built up on the basis of an act of apprehension is not itself an apprehension. And on the other hand not every apprehension need be 'true'. If I discern from afar the approach of a cyclist then speaking purely descriptively this is an apprehension, even should it be the case that in reality it is not a cyclist at all who is approaching but rather, say, a cow.
- ²⁷ [i.e. the being-something-or-other] – which is of course not to be confused with the subsistence of the state. The two are so fundamentally different that the account of the objective which is given by Ameseder (1904, p. 72) and Meinong (1910, p. 61) as something which 'is being and has being' can, in our opinion, lead only to confusion. Furthermore not every state of affairs allows itself to be portrayed without artificiality as a 'being'. Consider the states of affairs – discussed in § 12 below – which correspond to the judgments 'it is raining' or 'it is freezing'; [Reinach's own examples are: 'es wird getanzt' and 'mich friert'].
- ²⁸ Ameseder 1904 suggests the designation *Relate* for relations in the second sense (p. 72). See further Husserl 1900/01, II, p. 609, and Meinong 1910 p. 57f.
- ²⁹ Thus Maier, in his *Psychologie des emotionalen Denkens*, defends the view that the *Impersonalia* ['it is warm', 'it is raining', etc.] relate to conditions (and/or processes).
- ³⁰ By 'evidence' here we do not understand only the ideal case of absolute self-givenness but every givenness of states of affairs in acts of apprehension.
- ³¹ It must be noted that the present discussion relates merely to *immediate* apprehension and to evidence which is directly attained. In the case of those negative judgments which are arrived at on the basis of deductive inferences the situation is quite different.
- ³² Where we have to deal with the apprehension of 'relations' (in the sense of relational state of affairs) than it is of course unnecessary – as Brunswig, 1910, has exhaustively shown – that either of the members standing in the relation be in any way brought to presentation. Rather it may be grasped in a quite peculiar experience which Brunswig designates as a 'direction towards' [*Richtung auf*], an experience which is neither a presentation nor a meaning act in our sense.
- ³³ To the different words 'not', 'non', 'nicht', and so on, there corresponds, of course, an identical function.
- ³⁴ Instead of speaking in each case of 'the function which is executed with the utterance of the word "and"' it is admissible to speak, more briefly, of 'the and-function'.
- ³⁵ Already in his logic lectures for the summer semester of 1906 Pfänder had spoken of 'cognitive functions' [*Denkfunktionen*] with particular reference to 'and'.
- ³⁶ We ought, briefly, to draw attention also to the following. Just as apprehension grasps the apprehended state in its subsistence [*Bestand*], so assertion brings forward the asserted – positive or negative – state of affairs in its subsistence, it fixes or establishes this subsistence. One must be on one's guard against confusing this fixing of the subsistence of a state with the *predication* of subsistence of a state.
- ³⁷ Those judgments and sentences are called *contradictory* to which are correlated contradictory states of affairs: there is an analogy here to the way in which we distinguish sentences and judgments according to their *modality*, even though modalities are, properly speaking, inherent only in the corresponding states of affairs.
- ³⁸ Here the necessity of our earlier distinction between 'rejection' or 'denial' of a judgment and 'negative judgment' becomes very clear, for we have the two of them together, side by side.

- ³⁹ Only thus, also, can it be understandable why for every judgment – in our now standard sense – there is an underlying *positive* conviction. Were the negative judgment a ‘denying’, then it would have to arise out of a *negative* conviction in the state of affairs which it denied.
- ⁴⁰ It will be seen that these principles relate to states and their subsistence; the same holds for the other fundamental principles of traditional logic. These have normally been related to judgments, e.g.: two contradictory judgments cannot both be correct. This principle is certainly incontestable, but it is a derived and not a primitive principle. A judgment is correct if the state of affairs corresponding to it subsists; and two contradictory judgments cannot both be correct *because* two contradictory states of affairs cannot both subsist. The law pertaining to judgments thus obtains its foundation from the corresponding law which relates to states of affairs. Attempts have been made from other quarters to relate this law not to judgments but to *propositions*. Two contradictory propositions, it is now said, cannot both be true. We acknowledge freely the difference between judgment and ‘proposition in itself’; but just as the proposition must be separated from the judgment, so also must it be separated from the state of affairs. A proposition is true when the state of affairs which is correlated with it subsists. And two contradictory propositions cannot both be true *because* two contradictory states of affairs cannot both subsist. Thus here too the propositional law leads back to a law which relates to states of affairs. At the same time this provides an example which may indicate the sense of our claim above, that the major part of traditional logic will prove to have its foundations in a general theory of states of affairs.
- ⁴¹ And clearly in this too there is nothing peculiar to the negative judgment as such, since there are of course *positive* polemical judgments in a precisely corresponding sense.

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Wholes, Parts and the Objectivity of Knowledge

§ 1 Ontology Presupposed in Epistemology

Ontology has frequently been understood as the theory of the ultimate classes of existents or entities and of characteristics of and structures that belong to all entities merely as such.¹ Understood in this way acts of human cognition, for example, like all other events, obviously fall within the domain of ontology. There are thus also certain special or 'regional' ontologies, which attempt to exhibit the essential characteristics and structures peculiar to the members of each of the ultimate classes of entities – of minds (and mental acts), physical bodies, and so on. But even given these special ontologies it is still not strictly true, as Gustav Bergmann has often said, that "... epistemology is but the ontology of the knowing situation."² There is a normative or criteriological aspect to epistemology that is not reducible to a mere ontological analysis of cognition. It nonetheless seems plausible to suppose that an adequate ontology of cognitive acts would be a *necessary* condition for the satisfactory execution of the normative inquiries in epistemology.³ The intent of this paper is to cast some light upon the relationship between ontology and the theory of knowledge, by explaining Edmund Husserl's use of certain general ontological structures to clarify the manner in which individual cognitive acts can have an objective 'content' – or in short, to clarify how knowledge as commonly understood is possible. In the process, some important but lesser known aspects of Husserl's philosophy will be explained. Our first step must be an explanation of how Husserl understood the problem of the objectivity of knowledge.

§ 2 Three Aspects of the Objectivity of Knowledge

Certain time-worn philosophical questions about knowledge arise from the fact that the experiences – cognitive and otherwise – of each person are a part of his and only his life, and exhibit characteristics peculiar to him alone. My present perception of that tree out of this window and from this chair is indelibly *mine*. It has features that in all probability will never be combined in just *this* way again – which is completely assured if we include its temporal locus among those features. As a particular event it is non-repeatable even within my own life stream. It also could not, of course, be a part of the life stream of any other person. No one else could have had or can have *that* particular experience, although they might have one very like it. Further, it is possible that the ‘object’ of my perception might not exist at all, or might actually be very different from what it appears to be. Given simply that I see that tree, even in a very clear and determinate manner, it does not follow by any rules of generally accepted logic that the tree seen exists in actuality.

With few exceptions, the points just made are conceded today in philosophical accounts of human cognitive experience. But, once conceded, they pose difficulties about other aspects of human knowledge that seem equally obvious, or even more so, before philosophical reflection sets in; and in so doing they threaten to undermine the very possibility of knowledge itself. One may reason as follows: My perception of that tree, as has just been said, is logically distinct and seems separable in essence from every other experience of mine, from every experience of any other person, and from the very ‘object’ of that perception itself. But, this being so, it is well on the way – along lines familiar to readers of Hume – to closing in upon itself entirely and becoming a minute Spinozistic substance or Humean ‘perception’, wholly self-sufficient and therefore ineluctably alone. It has nothing in common with anything else and hence is incapable of communication; and it carries within it no inherent reference beyond itself, for it has no necessary connections with any other thing. Perhaps only some loose-jointed causal creed relates its involvements with other things, and that faith can and has been shaken by plausible lines of argument. The cognitive event as an individual entity (or substance) thus becomes utterly ‘subjective’.

But how, then, are we to understand certain other obvious features of

knowledge that presuppose, precisely, the involvement of the cognitive experience with things other than itself. These are the 'objective' features of experience. The tree presents itself to me as something that was there before and remains after I see it. It is hard to imagine what our experience would be like if this were not so, and unclear even what it would mean to suppose a *tree* to be produced or annihilated with the mere act of looking at it. My perception lays claim to a certain *transcendence* toward an object (that tree) which is independent of the particular experience of it, if not of all experience whatsoever. Moreover, my many experiences of the same tree are subjected to a rigorous *order* or *lawfulness*. The parts of the tree, and the perspectives which it exhibits from various approaches, dictate a determinate succession of possible experiences in relation to the tree. There are also certain obvious general conditions of my seeing the tree at all, and these must be respected as I undertake to examine the tree visually and otherwise. I cannot arrive at a perception of this tree – or of this tree from *that* angle – by just any arbitrary set of previous experiences. As Kant and many others have noticed, in such matters all roads certainly do not lead to Rome. Any variability in the routes that bring you exactly there is rigorously confined within abiding necessities. And finally, my perceptions of the tree can be verified or falsified by the perceptions of a second person. Thus *my* 'object' can also be *his* 'object', even though our experiences differ both individually and qualitatively. It is similar with other types of cognitive experiences. He can also verify my memory, check my inference, evaluate my hypothesis. With reference to most types, at least, of cognitive acts, a certain community or identity is presupposed between my cognitions and those of others. That presupposition is even a condition of our cognitive disagreements.⁴

Thus there are three apparently objective aspects of acts of knowledge: transcendence toward an independent object, conformity to general order or law, and a certain community of what is cognised. They, together with the subjective aspects previously stated, confront us with the problem of how – in Husserl's words – we are to understand '... the relationship between the subjectivity of knowing and the objectivity of the content of knowledge.'⁵ The problem of the objectivity of knowledge may be viewed as the problem of how to reconcile these three objective aspects of knowledge with the subjectivity of cognitive acts, and a solution to that problem constitutes a necessary (if not sufficient) condition of any account of the possibility of knowledge.

§ 3 Husserl's First Problem: The Objectivity of Formal Methods in Arithmetic

The first aspect of the problem of the objectivity of knowledge that presented itself to Husserl was the one concerning the rigorous order or lawfulness of knowledge. This came about in the course of his mathematical studies at Berlin under Karl Weierstrass. Much in the mathematical methods of those times (the 1870s) could not be reduced to general rational procedures, but at critical points depended upon the blind (even when accurate) instincts and tact of individual mathematicians – who often held quite divergent theories about the techniques by which they nevertheless obtained identical results. Weierstrass and others of course regarded this as a deficiency in mathematical knowledge, and one which both required and admitted of a remedy.⁶ It was generally presupposed that the domain of number was itself rigorously ordered, and that knowledge of that domain must possess a corresponding rigorous order. The task was only to find this order, to realize it in practice, or at least to show how it could, in principle, be realised.

Husserl's early ambition was to carry out this task. He first undertook clarification of the concept of number by an intuitionally based analysis of the simpler objects that fall under it (i.e., of the smaller numbers, 2 through 12). Here he was satisfied with his results. Then, setting out from the clarified concept, he endeavoured to give a rational reconstruction of the path leading from it to the most remote truths of numerical analysis or general arithmetic. But he found this reconstruction to be impossible. The building-blocks at his disposal at the time – various sorts of 'representations' of numbers – simply were not always what was used in the epistemic progressions that occur in arithmetical practice. The employment of the artificial symbolisms and formal techniques so pervasively and accurately used in the advancement of arithmetical knowledge (in the solution of equations, for example, or in the ordinary adding up of a column of figures on paper) clearly was not a matter of *representing* or *thinking about* numbers and number relations at all, but consisted to a very large extent of a mere rule-governed manipulation of sense-perceptible symbols.⁷ At this point in his career he found himself unable to explain how such formal procedures or 'calculations' yielded their uniformly and objectively correct results. What is the order in the mental processes of the working mathematician, focused almost entirely upon things other than numbers and number rela-

tions, that nevertheless allows those processes to eventuate in a grasp of truths about numbers and number relations?

But this question soon broadened into a realization that arithmetical thinking is not peculiar in this regard. He therefore found himself involved in a more general epistemological inquiry⁸ concerning how ordinary as well as scientific thinking – both of which largely deal in highly partial or extrinsic determinations, or even mere symbols, of the subject matters at issue, instead of with the matters themselves – nevertheless can result in an accurate grasp of truths about *die Sachen selbst*, and in many cases even a grasp of those very *things* themselves? What are the laws of cognitive experience that account for this? It is in this form that the problem of the objectivity of cognitive experience in general first addressed itself to Husserl.

Moreover, it is the objective order⁹ in the *discursive* aspect of cognitive experience that remains uppermost in Husserl's concerns at least until 1900. Thus, in the "Introduction" to the *Prolegomena to Pure Logic* he describes the problem of the objectivity of knowledge as "the cardinal question of epistemology", and then proceeds to state that it "... coincides in essence, mainly, if not entirely", with the question about the theoretical foundations of logic viewed as a technology (*Kunstlehre*) of the advancement of knowledge, and especially about the relationship of such a technology to psychology.¹⁰ The technology in question is simply an applied logic – that is, one which furnishes criteria and techniques for distinguishing acceptable from unacceptable concepts, theories, derivations, and assertions, and also develops methods for originating acceptable ones. Hence in 1900 the question of the objectivity of knowledge is for Husserl mainly the question: What must we know, and what must therefore be the case, in order that such an applied logic should be possible? In particular, can the possibility of an applied logic be explicated – as was widely assumed at the time – solely by reference to truths established in the science of psychology, and by reference to the corresponding facts and empirical laws of the mental processes of human beings?¹¹

§ 4 The Problem Generalised

In order to bring out the more important details of the overall problem of the objectivity of knowledge as Husserl eventually came to un-

derstand it, and especially to show how the questions of *transcendence* and *community* emerged to stand along with his earlier question about epistemic order or *law*, we shall examine a number of his earlier statements concerning that problem. His first published statements on the general problem of the objectivity or possibility of knowledge occur in the 1894 paper, "Psychological Studies in the Elements of Logic."¹² This paper was the first publication resulting from his move toward the reform of logic that he knew to be necessary by 1891;¹³ and it is, as he later said, a "first sketch of the *Logical Investigations*, especially of Investigations III and V",¹⁴ published six years later.

Upon first approach the paper is quite puzzling in its overall form. It appears simply as two separate "Psychological Studies." Each study is devoted to the analysis and clarification of a single distinction between very general types of contents or elements in the human mind. The first study explores the contrast between concrete and abstract *sensa* (or 'primary contents' as they were then often called), and the second study deals with the contrast between the intuitive and non-intuitive consciousness of an object. There is no explanation given in the paper of what, if anything, the one contrast or study might have to do with the other; nor, with regard to the former study, is there so much as a mere statement as to why the contrast between concrete and abstract "contents" is of any theoretical interest at all. However, the final section of the paper is a discussion of why the contrast between the intuition and the mere 'representation' of an object is of significance to a theory of knowledge. With what is said there – along with a knowledge of the specific failure in the *Philosophy of Arithmetic*, and of what these two "Studies" later became (the IIIrd and the Vth and VIth Investigations, respectively) – the motivation and structure of the paper becomes somewhat clearer. Simply put, by exploring some aspects of what is given in sensation, Husserl here begins to lay the foundation for an account of how that which is sensibly given functions as the foundation of a theoretical grasp of that which is not – and often cannot be – sensed or, more generally, intuited.

As has already been mentioned, the problem with which Husserl had been left in his attempt to reconstruct arithmetical thought was the problem of how the intuitive contents of the calculating arithmetician could possibly function, as they most certainly do, in the apprehension of non-intuited – and usually non-intuitable – numbers and number relations. How can the mathematician trust himself to ordered but 'blind'

operations upon groups of sense-perceptible symbols, with complete assurance that his symbolic operations will pick out numbers satisfying the relations with which he is concerned: e.g., the number which is the sum of numbers a , b , and c , or which satisfies the equation $\frac{b^2}{a} = x$? The enterprise of the *Philosophy of Arithmetic* foundered upon this question.

Now in the final section of "Psychological Studies", by contrast, the question has become quite general in form. The 'representative' or non-intuitive function in thought is said, without reference to any specific discipline or subject matter, to be

... truly an occasion for astonishment. In and for itself it is certainly a fact most worthy of consideration that a psychical act can reach out beyond its own immanent content to another content which in no way is really encountered (*bewußt*). And yet it seems that we do have consciousness (*Bewußtsein*) of these latter contents in a certain manner. For – and this again is a fact most worthy of attention – while we are engaged with the representing contents, we believe ourselves to be employed about the represented objects themselves. In the flow of conceptual thinking it is in most cases optical and acoustical sequences of words that do the representing alone or almost alone. The contents meant enter into consciousness either not at all or only in a quite rudimentary fashion. Occasionally wholly different contents, which stand in a distant relation to the contents meant, will act as a surrogate for them, as when at the mention of London merely the shape of England indistinctly comes to mind.¹⁵

But, Husserl notes, this is not only true of *thought*. The situation is little improved in ordinary conversations about things actually present in a common external environment: "What a sparse show of representing intuitions turns up to the unprejudiced observer in these cases! Here and there such intuitions are wholly lacking; and where they occur they are as a rule dim, decimated, often ungraspably fleeting, and inadequate even in the typical aspects of the intuitions intended."¹⁶ Yet it seems to those involved in such conversations as if the objects referred to directly coalesce with the words occurring. Words and objects seem immediately 'together' for those who understand what is said. These are facts about cognitive experience which, according to Husserl, even the best of psychologists and logicians have failed to consider, much less understand.

And yet here lie great, unsolved puzzles. We stand close to the most obscure parts of the theory of knowledge ... *I refer to the possibility of knowledge in*

general. Scientific knowledge – the kind of knowledge which will first come to mind – is totally based upon the possibility of being able purposively to choose such thinking, with certain precautions, over thought more closely bound to intuition. But how then is insight into the relevant subject matter possible in science? And how with such a manner of thought does one even come to mere empirically correct results?¹⁷

After these remarks about scientific thinking in general, Husserl turns once again to his own discipline, mathematics. It has stood for centuries as a model of exact science. But the seemingly interminable controversies between its practitioners "... over the meaning of its elementary concepts and the grounds of the validity of its methods stand in striking contradiction to the fact that its procedures supposedly carry rational insight for everyone alike." Community of technique and of results seems unperturbed by divergent or even contradictory interpretations of how the technique works and is justified. But surely this means that the technique's "... status as a *rational* procedure is – and of this there can be no doubt – mere delusion." On the other hand, the *results* of the use of mathematical techniques are not only agreed upon. They are ordinarily *correct*, and in general provide us with the truth about numbers and number relations – and, in their applications, about things of all types. Yet there existed no satisfactory account of how this all comes about, even though the problem really is one which concerns "... all of science and ordinary thought as well."¹⁸

This prompts Husserl to ask whether we should

... revert to Hume's scepticism as our basis, and then extend it farther than its great author did, to take in even mathematics and all *a priori* science? Vainly we turn, for the resolution of such doubts, to the old logic or the new. They leave us totally in the lurch. Logic or the 'theory of science' (*Wissenschaftslehre*) must concede, if it will be honest, that all science is a mystery to it. This is where we stand today ...¹⁹

Such remarks make it clear that by 1894 Husserl's initial concern with objective order in arithmetical thinking had led him to the completely general question about how an epistemic act or process transcends itself in a correct grasp of truths and objects which are not really 'present' in it and belong equally to all members of the community of inquirers. This 'astonishing' transcendence was forced upon his attention in virtue of the fact that it is precisely what as a rule is going on in the specifically arithmetical treatment of numbers and number relations. However,

having been confronted with it in arithmetic, he then discovers it on all sides.

Of course it is not as if Husserl were the only philosopher who ever concerned himself about the mind's cognisance of entities other than its own 'contents'. With all of their differences, Locke, Hume, Kant and Husserl pose and answer what is fundamentally the same problem. Closer to our time, H. H. Price raised the same set of issues – and in a very similar terminology to that of Husserl. Price held²⁰ that "... thinking is cognition in absence," and that: "The objects which we say we are thinking about are not directly present to our consciousness ... The problem or paradox then is this: How is it that merely by operating with symbols one can be in cognitive contact with absent objects or events? ... How can we be aware of anything beyond the symbols themselves?"²¹ His book *Thinking and Experience*, published in 1962, attempts a solution for this 'problem or paradox'.

§ 5 Transcendence Becomes the Main Issue

Husserl's writings for a decade and a half following 1894 show an increasing emphasis, at least in terminology, upon the issue of transcendence in contrast to that of orderedness or law. By 1901 he had come to see that the attempt in *logical* theory to clarify the meanings of terms such as 'concept' and 'object', 'truth' and 'proposition', 'fact' and 'law' – for the purpose of elucidating the possibility of the order and community in scientific thinking – raises questions which largely coincide with the 'basic questions of epistemology'.²² All cognition is of course directed upon objects or states of affairs that exhibit an identity over against the many real or possible acts of thought (of one or of many persons) directed upon them. To this community or sameness of objects must be added the further fact that all cognitive activities fall into general types subject to ideal (non-inductive) laws that determine whether or not the given activity can or must hold valid of relevant objects. But we must ask, then, how

the 'in itself' of the objective comes to presentation and, thus, to a certain degree may become subjective again; what it is for the object to be 'in itself' and 'given' in knowledge; how the ideality of the universal, as concept or law, can enter into the stream of real psychological experiences and become an epistemic holding (*Erkenntnisbesitz*) of the one thinking; what the *adaequatio rei et intel-*

lectus involved in knowing amounts to in the different cases, depending upon whether the knowing 'grasp' takes in an individual or a universal, a fact or a law; and so on.²³

In this statement of 'the basic questions of epistemology' the problem of transcendence by the cognitive act toward an object that is *an sich* is clearly the dominant element. Indeed, of the four separate questions listed, only the third bears upon the aspects of community or of law or orderedness in knowledge. In the 1907 lectures on *The Idea of Phenomenology*, it is flatly declared that '... transcendence is both the initial and the central problem of the critique of cognition.'²⁴ While this obviously marks *some* shift from the 1900 statement already quoted, according to which the 'cardinal question of epistemology' concerns the theoretical foundations of applied logic, the change is in fact less than the two statements may suggest. The community and order or lawfulness in thought and knowledge is, after all, one type of transcendence of individual acts of thought. It is simply not intentional transcendence toward an *object*. On the other hand, transcendence toward an object is something that is shareable – since many persons can grasp the same object – and occurs only through a complex act or series of acts rigorously ordered in a definite manner. Association between the three aspects of the general problem of the objectivity of knowledge as outlined above is so close, on Husserl's view, that any apparent shift from one to another can only be a matter of emphasis.

A more balanced expression of the three aspects of the problem of objectivity in their interconnection is given by Husserl in 1910 in his *Philosophy as a Rigorous Science*:

How can experiences as consciousness present or make contact with an object? How can experiences be mutually legitimated or corrected by means of each other, and not merely replace each other or confirm each other subjectively? How can the play of a consciousness whose logic is empirical (*das erfahrungslogische Bewußtsein*) make statements objectively valid for things that are in and for themselves? Why are the rules of play, so to speak, of consciousness not irrelevant for things? How is natural science to be comprehensible in absolutely every case, in its claim at every step to posit and to know a nature that is in itself – in itself in opposition to the subjective flow of consciousness?²⁵

The issues of *transcendence* ('... things that are in and for themselves,' '... a nature that is in itself'), *law* ('... the rules of play ... of conscious-

ness,' 'experiences ... mutually legitimated or corrected by means of each other'), and *community* (as this occurs in a 'natural science') are all brought out in this passage; and here once again the related questions are marked as the ones which epistemology is supposed to answer. But Husserl observes that "... up to the present, despite all of the thoughtfulness employed by the greatest scholars in regard to those questions, that discipline has not answered in a manner scientifically clear, unanimous, and decisive," as would befit a "rigorous science."²⁶

But it must be added that for a correct appreciation of Husserl's central problem one must not take his question about transcendence to be merely the question about the *intentional direction* or vectoral character of an experience with regard to a specific object. It is not the question: What is it about a given experience that makes it to be an experience of or about one object rather than another? This is not, of course, to say that Husserl had no interest in the question about mere intention, or even that it was of little importance to him. It was indeed of great importance to him, and he dealt with it early and often.²⁷ However, the *mere* directionality of an experience does not involve transcendence, although transcendence may be associated with it. Intentional direction is a *wholly immanent* matter, on Husserl's view. That an experience is directed specifically upon one object rather than upon another is simply a matter of what *its* parts and properties are. What he found 'truly astonishing', on the other hand, was that an object transcendent to the experience or experiences directed (more or less mediately) upon it could nonetheless be, and could be known to be, *accurately* conceived of in that experience, and could in many cases even be 'bodily' present to consciousness precisely *as* it was thought of – and, moreover, could be known to be so present. How all of *this* is possible is what constitutes the problem of epistemic transcendence for Husserl.

§ 6 The Beginnings of a Solution

If knowledge is to have the objective characteristics commonly assumed and is therefore to be possible – or, indeed, even if it is not – then that must rest at least in part upon the general nature of the cognitive act as such. That is, it must rest upon the type of event or entity such an act is. I have found no evidence that the early Husserl clearly put this *general* point to himself and then directed his researches accordingly. However, it is a fact that the writings produced from the early 1890's to 1901 pro-

ceed to work out a categorial or ontological analysis of the cognitive act (and sequences of such acts), and to show the possibility of objective knowledge by reference to that analysis. Further examination of the "Psychological Studies in the Elements of Logic" will show that it not only provides a general statement of the problem of the possibility or objectivity of knowledge as this was conceived by Husserl, but also lays down the framework of a solution to it by initiating the treatment of the cognitive act as a complex *whole* exhibiting *necessary connections* between its *parts* as well as in relation to other acts – necessary connections which are, moreover, treated as open to rational insight (*Evidenz*).

The first "Study" seems to have been intended merely as an analysis of certain general structures of independence and dependence involving *sensa*²⁸ and their elements. For example, intensity and quality are considered as elements in the *sensa* associated with tones:

If we represent the quality to ourselves as wholly suppressed, then the intensity is also suppressed. And this is not a mere fact, but rather is a rational (*evidente*) necessity. Likewise in the converse case. Also, a change of intensity ineluctably signifies a certain modification of quality, even though the generic type of the quality remains the same. Here we simply are not dealing with a totality [of *sensa*] in which the one term can be varied while the other remains identically the same – instead of merely generically so, as in the case just mentioned. The two terms interpenetrate. They exist within each other, not outside of each other. Again, the cessation of the intensity conditions a total annihilation of the quality; and this is no mere fact, but rather is a rational necessity.²⁹

Other cases of necessary dependence are provided, for example, by extension in relation to color, by shape in relation to both, and – to take a very different type of case – by a judgment in relation to the representation which serves to provide the object which the judgment is *about*. In general, Husserl concludes, a sense content or *sensa* element is dependent if and only if "... we have *a priori* insight (*Evidenz*) that change or suppression of at least one among the contents given with (but not included in) it *must* yield a change or suppression of that content itself."³⁰ Further, if an element in a whole of sense contents is dependent in relation to some other element in the whole, then it is *abstract* in relation to that whole. Otherwise it is *concrete*.³¹ Thus 'abstract' and 'concrete' are defined by reference to dependence and independence.

It is especially important to notice that *being abstract*, as thus explained, has nothing essentially to do with *being known*. *Abstract* is pre-

sented as an ontological, not as an epistemic concept, even though it is here analysed in its application to the sense contents of cognitive acts. In a note published in 1897 Husserl states that the analyses of his first psychological study "... must be given an objective twist, in a very obvious manner," to provide a 'law of objects in general'; and that "... the crucial distinction [dependence/independence] here is not one restricted to contents, but rather is one which applies to objects in general. This makes it *metaphysically* significant. But the same then is also true of the remaining distinctions connected with it which are dealt with in that study."³²

While this statement was published three years later, in the "Psychological Studies" itself, not only – as we have noted – does Husserl not define 'abstract' epistemically, but he also *explicitly* rejects definitions of that term by reference to special abstracting acts or ways of noticing a content or object.³³ In the second "Study" he further rejects the commonly held view that the abstract cannot be intuited, insisting that the concepts of *independence* and *dependence* (and of *concrete* and *abstract*) contain no reference at all to the contrast between intuitionality and non-intuitionality.³⁴ He was perhaps led to this view by a conviction that he had himself intuited the abstract elements of *sensa* in their dependencies and independencies upon one another, as recorded in the first study.

Thus, by 1894 Husserl believed himself to have found inspectable and necessary (though non-analytic) connections between certain abstract elements within whole cognitive acts. Indeed, the case of judgment and founding representation, noted above, shows that he already recognised such 'material *a priori*'³⁵ connections between psychical elements other than sense contents. However, he here deals systematically with sense contents only; and, although he clearly knew that there were other components of cognition – principally the psychical acts of which he learned from Brentano – he apparently did not yet see how they were all to be fitted together.

That this is so is well brought out by an admission also contained in the 1897 publication referred to above. He there states: "Since the appearance of my essay, I have become aware of the essential distinction between abstract contents (as parts of intuitions) and abstract *concepts* – a distinction which, unfortunately, I did not notice at all."³⁶ He then explains that the first study in fact dealt with abstract *contents* only, and that its claims hold true when so understood. However, to fail to see

that a concept is not a sense content – even an abstract one – demonstrates how unclear Husserl remained at the time about the cognitive act as a whole. The unclarity had deep roots, no doubt; for already in the *Philosophy of Arithmetic* of 1891 he had distinguished between the concept as an *abstractum* underlying a general name and concept as the ‘thought correlate of the name.’³⁷ Notwithstanding the residual obscurities, however, the essential points disclosed in analyses of sense contents in the first “Study” stood firm in his later views and were later extended to the remaining components of cognitive acts – which, after all, do also belong among ‘objects in general’. The first “Psychological Study”, though narrow in scope, brings to light a general ontological structure – necessary synthetic connections between parts of a whole cognitive act – which eventually will serve as one of the two main supports for Husserl’s solution to the problem of the objectivity of knowledge.

The second main support emerges in the second “Psychological Study”. As the first study exhibits a ‘material *a priori*’ structure between elements of one cognitive act, so the second study exhibits such a structure between two or more whole cognitive acts of certain specific types.

A basic concept in this second study is that of an intuited ‘content’ (object) being also an *immanent* content. On Husserl’s view, when I see this sheet of paper before me under normal conditions, the whiteness that I see (intentional ‘content’) in the paper is *also* present as one abstract element in the *sensa* which I ‘use’ to see the white paper. Of course the qualities of the perceived object and those of my *sensa* customarily diverge to some extent, and are only more or less similar. (I still *see* a white sheet of paper at dusk, when my *sensa* exhibit some shade of grey.) When and insofar as the qualities of our *sensa* and of our object as intended are identical we have, on Husserl’s view, an *intuition* of the object; and this means that “... the object itself is actually put before us in such a manner that that object is *itself* the subject of psychical activity.”³⁸ In such a case “... the intended content becomes immanent content,”³⁹ and “... an immanent object of the act simultaneously appears to us as that which is intended by the act.”⁴⁰

By contrast:

Certain psychical experiences, in general called ‘presentations’ (*Vorstellungen*), have the peculiar character of not including their ‘objects’ in themselves as im-

manent contents (and thus as present *in* consciousness). Rather, in a certain manner which must still be more precisely characterised, they *merely intend* their objects ... The phrase 'merely intend' here signifies precisely that a content is a content not given *in* consciousness, but one aimed at, meant, or consciously referred to, by means of some contents that are given in consciousness. These latter contents are consciously used as surrogates of the former; and, indeed, they are so used without the intervention of conceptual knowledge of the relationship which obtains between the surrogates and the intended object. Such presentations we will call 'representatives' (*Repräsentationen*).⁴¹

From these words we see that the second "Psychological Study" is also to some extent a study of acts by reference to their sense contents, for the distinction between intuition and mere representation of an object is drawn in terms of a difference bearing upon the sense contents involved in the two cases. However this is not to say that the distinction is *merely* a difference of *sensa*. Husserl insists that such a view would be quite wrong. The 'witness of inner experience' makes it clear that in addition to *sensa* "... there exists in the two cases in question a different manner of psychical engagement with or in the [sense] content."⁴² The component of the single cognitive experience other than *sensa* is, no doubt, the still unclarified Brentanian 'act'.

However, the distinction between intuitive and non-intuitive cognitions in terms of a difference of internal quality or structure is only one part of Husserl's findings in his second study: one which lays the foundation for what is yet more important with regard to the problem of the objectivity of knowledge. For having distinguished acts of intuition from acts of presentation, he then discovers an essential relationship between them as they bear upon the same object. This is the relationship of "fulfillment" of the presentation by a correlative intuition of the same object:

If a representative goes over into its correlative phenomenon, e.g., into an intuition immediately intended by it, then the immediate psychical experience of the fact that the intuited is *also* the intended shall be designated as consciousness of the *fulfilled* intention. Of the representative we say, more simply, that it has found its *fulfillment*. This latter term will be used by us in general to designate the direct or indirect correlate of a representative.⁴³

It is commonly the case, however, that a 'representative' does not lead *directly* into a full intuition of the relevant object, but only to another representative with a more intuitive content than the original one. In

many cases, a *full* intuition of the represented object is not possible at all. The concept of a^n (a to the n th power) directly leads to the concept of a product of n factors of a . The latter concept then extends the intention onward, as it were, toward the specific number in question. If a and n are not *very* small numbers, the number referred to by ' a^n ' can only be approached through a series of representatives, but can never be fully brought to an intuitive grasp. Even in such a simple case as this we really have a series of presentations referring more or less directly to a corresponding intuition. Husserl calls

... the correlative phenomenon most nearly adjoined to the representative its *proximate fulfillment*. The *ultimate* fulfillment of any representative is the *intuition* proper to it. To say that it is a *pure* intuition expresses the fact that a content bears no representative function whatsoever. In contrast to this, we speak of an intuition which is *impure* or representational where a presenting content, in virtue of the identity or similarity of its content with what is presented, temporarily serves us as a provisional replacement of the latter. In such a case ... we then are turned to the presenting content itself in precisely the manner characteristic of pure intuition. An impure intuition is said to be *incomplete* when the immanent content of the representative consists of a part of that which it represents.⁴⁴

Now of course the relation of fulfillment mentioned here is a very complicated one, and Husserl did not have its details worked out fully until several years after 1894. But it is clear that for every 'representative' of an object there will be an associated possible sequence of more or less intuitive experiences of the *same* object, converging more or less directly upon the case where the object *itself* is present in experience – or, conversely stated, where experience has transcended itself toward its object. Husserl holds the *possibility* of such a transcendence (the precise nature and extent of which varies with the kind of object) to be essential to every representative act: "... each representative ... points to an intuition that corresponds to it, but is not necessarily actual."⁴⁵

The complexity of cognitive acts together with the essential relationship of (possible) fulfillment discovered by Husserl to hold between them and corresponding intuitions directed upon the same object are the principal elements in his solution to the problem of transcendence. In 1901 he holds recourse to the cognitive act as a "... web of partial intentions, fused together in the unity of a single total intention" directed upon the object, to be the *only* way in which we can "... understand how consciousness reaches out beyond what is actually lived through

(*das wahrhaft Erlebte*): [how] it can, so to speak, mean beyond itself, and the meaning can be fulfilled.”⁴⁶

Now it will be recognized that this distinction between intuitive and non-intuitive (or ‘representative’) cognitions coincides in extension with Hume’s distinction between impressions and ideas. Hume also held to an essential correlation of every idea with some impression or group of impressions. Moreover, his ‘principle of the priority of impressions to ideas’ did not in general disallow the emergence of ideas from other ideas. For: “As our ideas are images of our impressions, so we can form secondary ideas which are images of the primary ... Ideas produce the images of themselves in new ideas ...”⁴⁷ The result is a sequence of possible ideas corresponding and more or less directly related to each impression, quite as is the case with Husserl’s representatives and their intuitions.

So these two philosophers cover the same ground and divide it into two parts along the same lines. Beyond such obvious and superficial similarities, however, they have different concerns and hold *widely* divergent views. It will be useful to discuss one of their main differences here, where we are concerned to explain the general lines along which Husserl worked toward an account of the possibility of objective knowledge. This is the difference over the role of *complexity* in the cognitive act. For Hume, the complexity of an idea – or, in general, of a ‘perception’ or experience – is always something inherently problematic, for which an explanation is required. That is mainly due to the fact that, for him, “... the mind never perceives any real connexion among distinct existences.”⁴⁸ For Husserl, on the other hand, complexity as such is not a problem. Not only connections, but even necessary connections, are simply *found* by him. He is a *radical empiricist* in the sense stated by Wilham James, and so “... must neither admit into ... constructions any element that is not directly experienced, nor exclude from them any element that is directly experienced.” Hence, “... *any kind of relation experienced must be accounted as ‘real’ as anything else in the system.*”⁴⁹

The complexity of the representations and intuitions (and series thereof) that constitute the flow of both scientific and ordinary thinking is, precisely, but one application of the ontological schema of whole and part applicable to objects in general: “Every object is either actually or possibly a part, i.e., there are actual or possible wholes that include it.”⁵⁰ Husserl is at peace with complexity both inside and outside of the stream of cognitive events. Moreover, he has discovered certain *neces-*

sary relationships (relationships of foundation⁵¹) between parts and parts, as well as between parts and wholes. Such necessary relationships also govern elements in that cognitive flow. With this we have the general framework for an explanation of how cognition transcends itself – or, in the misleading language of the “Psychological Studies”, for an explanation of how an intended object becomes an *immanent* content. We likewise have a framework for the explanation of the necessary order within, and the possibility of a community of content between, the many particular cognitive acts which occur in the course of human events. We shall now very briefly summarise the main elements in the explanations that Husserl ultimately devised.

§ 7 Wholes, Parts and Properties in the Objectivity of Knowledge

Community: The complexity of cognitive acts in general breaks down into wholes of various types, depending upon the number, type and manner in which those wholes contain other acts and elements that are not acts. As a helpful analogy, one can think of the complexity of sentences in a formal or a natural language: extending from those that have parts, to be sure, but no sentences as parts, to those that have many different sentences as parts, combined in several different ways. A similar complexity is to be found in cognitive acts, where the experienced elements involved may in general be either dependent or independent (abstract or concrete) as explained above. However no such element, being individual, can be repeated or shared. What can be repeated and shared on Husserl's view are of course the ‘significational species’ or essences (concepts, propositions, theories) that enter into cognitive experiences as their intentional *qualities* or determinations. The familiar ontological schema of the subject and its predicates (the individual and its qualities and relations), together with that of whole and dependent part (moment), provides the solution to how the conceptual content can be had by many persons at many different times.⁵²

Law: The connections between the ideal singulars (universals or species) embedded in cognitive acts of various types and in their parts dictate relevant necessities and possibilities for the acts in which they are embedded. Discussing the forms of *fire*, *heat*, *snow*, and *cold* in his *Phaedo*, Plato had Socrates say: “... When snow ... is under the influence of heat, they will not remain snow and heat; but at the advance of

the heat, the snow will either retire or perish ... And the fire too at the advance of the cold will either retire or perish; and when the fire is under the influence of the cold, they will not remain as before, fire and cold."⁵³ Now the same general ontological structure of necessities and possibilities determined for subjects by their properties also governs within and between cognitive acts. The forms of the thoughts *that all men are mortal* and *that Socrates is a man*, along with their truth, necessitate truth in the possible thought *that Socrates is mortal*. 'And', 'extra', and 'across', taken in their normal senses, of necessity do not, in 'and extra across', express a thought capable of truth or falsity – if indeed they express any thought at all. A perception of a chair at any moment admits of only a restricted range of subsequent experiences which may constitute experiences of the same thing (the chair seen). To return briefly to Husserl's problem in the *Philosophy of Arithmetic*, an essential correlation between arithmetical symbol systems and the number series makes it possible for the 'blind' use of the former to found – in Husserl's special sense – a correct and justified conceptual apprehension of the latter. And so on.⁵⁴ The necessities and possibilities in the relevant individual cognitive events follow from the qualities and relations embedded in those events.

Transcendence: The part/whole structures and ideal law connections (*wesensgesetzliche Verbindungen*) already referred to as making community and law possible for the various cognitive acts of one or many persons also function in Husserl's explanation of the possibility of the transcendence of such acts toward an object intended. The complexity in an act of representation of a table, for example, is to be understood in terms of intentional qualities and *sensa* (noetic and hyletic data) correlated to the parts and aspects of just *such* an object. The act is said by him to be capable of transcendence toward an object that is *an sich*, in virtue of the act's essential correlation with an intuition of the appropriate type: one in which *that* object is 'bodily' present in the manner dictated by its nature, the qualities of the object coinciding in the greatest possible measure with the qualities of the *sensa* present in the intuitive act.⁵⁵

A very great part of the power of Husserl's philosophy resides in his insight that the problem of the possibility of knowledge is essentially (though not wholly) a problem in general ontology. He saw that knowledge must be treated as one segment of being, if its *objective* characteristics are to be explained. Perhaps it is correct to say that we cannot

settle the fundamental ontological questions about *complexity* (whole and part), about *substance* and *quality*, and about *necessary relation* by talking about ideas or experiences only, or about words and their complications only; for in all such talk we shall only assume positions on those questions as they concern, precisely, 'ideas' or words themselves. To overlook this was, I believe, one main error in the historical turn of philosophy to subjectivity as that turn was in fact executed, and in all its offspring – including the linguistic turn in whose shadow we still live today.

Notes

- ¹ Aristotle, *Metaphysics*, Book IV.
- ² Bergmann, 1964, pp. 126 and 304.
- ³ On the relation between normative and theoretical analyses in general, see Husserl, 1900/01, pp. 74–89.
- ⁴ In Sellars, 1968, p. 91, the importance of this point is recognized as follows: "... The 'content' of representings – individual contents, general contents, state-of-affairs contents, etc. – must be construed as *ones* in *manys* in order to do justice to the inter-subjectivity of thought, the fact that different persons, and the same person at different times, can represent the *same* even though the representings (the acts) are numerically different. Thus one and the same content must be capable of existing 'in' – in some sense of 'in' – many representings." Cf. pp. 62–63.
- ⁵ Husserl, 1900/01, p. 42.
- ⁶ Schuhmann, 1977, p. 7.
- ⁷ Husserl, 1891, pp. 257–258.
- ⁸ Husserl, 1900/01, pp. 41–42.
- ⁹ Husserl, 1900/01, pp. 64–68.
- ¹⁰ Husserl, 1900/01, pp. 56–57.
- ¹¹ Hence, as we have said, it is first and foremost the objective order in the *discursive* aspect of cognitive experience that Husserl seeks to clarify. It is also necessary to emphasise that it is clarification, not justification, that he seeks. When he asks how knowledge is possible, the 'how' is not a generally or specifically sceptical 'how'. Rather, he is inquiring only about the means – the nature of the specific structures and processes – through which subjective experiences succeed in cognitively grasping independent and publically accessible objects. He does not doubt that they ordinarily *are* grasped, or that they do exist. In this respect his conception of the task of the theory of knowledge differs in emphasis (at least) from that of other important philosophers. A general sceptical 'how' is not *the* question to be answered in the theory of knowledge, and a *general* scepticism is regarded by him as a demonstrably absurd position. See Husserl, 1900/01, pp. 135–145.
- ¹² Husserl, 1894, pp. 297–320.
- ¹³ His explicit statement on the need for reform is in an 1891 letter to Stumpf, quoted in part on pp. 41–42 of Biemel, 1959. The statement on reform is not quoted by Biemel, and the letter as a whole is, unfortunately, still unpublished. On Husserl's move toward reform in logic see Willard, 1979.

- ¹⁴ Husserl, 1956, p. 295.
- ¹⁵ Husserl, 1894, p. 315.
- ¹⁶ Husserl, 1894, p. 316.
- ¹⁷ Husserl, 1894, p. 316, italics added.
- ¹⁸ Husserl, 1894, p. 316.
- ¹⁹ Husserl, 1894, p. 316.
- ²⁰ Price, 1946, pp. 83–84.
- ²¹ Price, 1946, pp. 84–85.
- ²² Husserl, 1900/01, p. 253.
- ²³ Husserl, 1900/01, p. 254.
- ²⁴ Husserl, 1958, p. 28.
- ²⁵ Husserl, 1910, p. 87.
- ²⁶ Husserl, 1910, p. 87.
- ²⁷ Husserl, 1913, § 86.
- ²⁸ We use the word 'sensa' here in the meaning made common by C. D. Broad (1960, pp. 180–183). See also H. H. Price (1932, pp. 2–5), for elaborations of a view identical with Broad's. The sensa of Broad and others are identical with the primary contents, sense contents, or sensate matter of which Husserl speaks.
- ²⁹ Husserl, 1894, p. 299.
- ³⁰ Husserl, 1894, p. 299.
- ³¹ Husserl, 1894, p. 301.
- ³² Husserl, 1894, pp. 319–320; cf. the opening paragraphs of the IIIrd Logical Investigation.
- ³³ Husserl, 1894, pp. 302–303; cf. Husserl (1900–1901), pp. 443–448.
- ³⁴ Husserl, 1894, pp. 305f.
- ³⁵ See Husserl, 1900/01, pp. 455–458 on this terminology; and for the positivistic reaction, Schlick, 1930/31, pp. 277–285.
- ³⁶ Husserl, 1894, p. 320.
- ³⁷ Husserl, 1891, p. 78.
- ³⁸ Husserl, 1894, p. 304.
- ³⁹ Husserl, 1894, p. 305.
- ⁴⁰ Husserl, 1894, p. 307.
- ⁴¹ Husserl, 1894, p. 307.
- ⁴² Husserl, 1894, p. 313; cf. p. 320.
- ⁴³ Husserl, 1894, p. 308.
- ⁴⁴ Husserl, 1894, pp. 308f.
- ⁴⁵ Husserl, 1894, pp. 308; cf. Husserl, 1913, final paragraph in § 142.
- ⁴⁶ Husserl, 1900/01, p. 701. (Cf. p. 513 of Volume II of the first German edition, where the statement is seen to be essentially the same as of 1901.)
- ⁴⁷ *A Treatise of Human Nature*, Book I, Part I, Section I, next to last paragraph.
- ⁴⁸ *Treatise*, Appendix.
- ⁴⁹ James, 1947, p. 42.
- ⁵⁰ Husserl, 1900/01, p. 436.
- ⁵¹ Husserl, 1900/01, pp. 474–475.
- ⁵² Husserl, 1900/01, pp. 329–332; cf. Husserl 1913, § 15.
- ⁵³ Stephanus pagination, no. 103.
- ⁵⁴ Husserl, 1913, §§ 2, 6, and 7.
- ⁵⁵ Husserl, 1913, §§ 142 and 143; cf. Husserl, 1900/01, pp. 701, 761f. For further elaboration see § 5 of ch. 5 of my *Studies in Husserl's Early Philosophy* (to appear).

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Criteria of Abstractness

The Ontologies of Husserl, Frege and Strawson against the Background of Classical Metaphysics

In analytic philosophy someone tends to be called a Platonist if he assumes that there are abstract objects. It has been asked what conditions must be fulfilled for someone to be correctly called a Platonist, or for a theory to be Platonistic in this sense. In the ontological discussion devoted to this question, a discussion which has sometimes resembled shadow-boxing, the question '*what is an abstract object?*' has been almost completely neglected. It is this question which will occupy us in the present essay.

We begin with a rather unassuming answer. That abstract entities at least include those entities designated by the italicised portions of the following sentences:

(a) Properties (Attributes)

Bravery is a virtue.

Being courageous is something different from *being foolhardy*.

(b) Relations

How does *friendship* differ from *love*?

(c) Propositions

Given *that the Earth is smaller than the Sun*, it follows *that the Sun is bigger than the Earth*.

Did Pythagoras prove *Pythagoras' Theorem*?

Xanthippe doubted *the courage of Socrates*.

What you just asserted is not true.

(d) Genera and Species

The whale is a species of *the genus mammal*.

(e) Types

The word 'is' occurs more than once on this page.

The St. Matthew Passion is performed every year in many places.

(f) Numbers

4 is an even number.

(g) Classes

The class of centaurs is empty.

We wish to call expressions like those italicized in (a) to (g) – so long as they are used in the way there exemplified – *abstract singular terms*. That the caveat is needed is easily shown by an example like ‘The whale has just rammed the ship’ where, unlike in (d), the expression ‘the whale’ is not used to designate an abstract entity. *Abstract singular terms* are distinguished from other singular terms in that they stand for abstract objects.¹ Thus we do not know what an abstract singular term is until we know what an abstract object is.

We have presented a list and said: These and their like are called abstract entities. This answer naturally does not determine what is peculiar to all abstract entities, does not determine what they have in common. Plato’s Socrates would therefore hardly have been pleased by it. We wish in this essay to investigate some more exacting answers: those of Plato, Husserl, Frege and Strawson.

§ 1 Non-perceptibility (First Platonic Demarcation-Proposal)

Two answers were put into the mouth of the ‘Friends of Forms’ by Plato in his characterisation of their position in the ‘Battle of Gods and Giants’²:

Str: You make a distinction between becoming and being, do you not?

Th: Yes, we do.

Str: And you say that with the body, by means of perception, we are in touch with [κίνησις] becoming and with the soul, by means of thought, we are in touch with being. And being, you say, is always unchanged and the same, whereas becoming is different at different times.

Th: Yes, that is what we say.

Can we not simply follow the ‘Friends of Forms’ and ascribe non-perceptibility³ as well as unchangingness to all and only the abstract entities? We shall divide this question into sub-questions and begin with

the discussion of the proposal to define abstractness by means of non-perceptibility. (This Platonic demarcation-proposal was, incidentally, accepted by Husserl in the *Logical Investigations*: the disjunction 'ideal/real' is there put forward as exhaustive, it being asserted of real objects that they are defined directly as possible objects of a simple [i. e. sensual] perception.⁴) If this criterion is to be acceptable then non-perceptibility must be something which all abstract entities have in common, but it must also be something peculiar to such entities alone. Thus we might once more sub-divide our problem and ask first of all:

Does it hold for all abstract entities that they are not perceptible?

Is non-perceptibility a *necessary* condition of abstractness?

Some abstract entities do indeed seem to be perceptible. When someone says he has heard the word 'sympathiser' a great deal recently, it is not that he wants to say of a single utterance of the word that he has often heard it. He could of course have this in mind if, for example, he were continually to replay the tape-recording of a particular speech and if the word 'sympathiser' were to be used precisely once in the speech in question. If we exclude such cases from consideration, however, then we would appear to have to interpret such a statement in such a way that in it a word-type, that is, an abstract entity, is described as perceptible. This would then leave us with a great number of counter-examples which would prohibit an affirmative answer to the question above.

Of course there is an objection that suggests itself immediately at this point:

(a) NN. has often heard the word/work α

means, after all, nothing more than

(a') NN. has heard many utterances/performances of α .

Thus the property of perceptibility is really only ascribed to concrete objects: to occurrences (tokens) of a type. But in this form the objection is still not convincing. If it is correct that (a) has the same sense as (a'), then this of course also holds the other way round: (a') means nothing other than (a). The paraphraseability of a sentence by another sentence does not prove that the one, rather than the other, is a mere *façon de parler*. If, then, we want to show that the advocate of the thesis of the non-perceptibility of abstract entities is not refuted by examples like (a), it is not enough to refer to their paraphraseability by (a').

Consideration of a number of analogous cases will help us to move a little further forward here. What could be objected to someone who, by appeal to sentences like

(b) The lion has 4 legs,
thinks he can show that at least one abstract entity (a species) has 4 legs? How is the hypothesis to be refuted that, according to 1 *Corinthians*, 13, 4–7:

(c) Love is patient and kind ... it trusts in everything, it endures everything,

at least one abstract entity (a virtue) has moral properties? Here too it would not be enough to point out that (b) and (c) amount to

(b') All (well-formed) lions are four-legged,

(c') Whoever (really) loves is trusting and patient.

We can however ask our fictitious opponent what the expressions 'patient' and 'four-legged' in (b) and (c) mean. It is certainly not the case that an animal species is just another simply four-legged thing alongside Leo and Lenny and that love is simply something that is patient alongside Jesus of Nazareth and Francis of Assisi; for otherwise it would have to be possible to amputate a leg of a species or cause a virtue to explode in rage. So the terms 'four-legged' and 'patient' in (b) and (c) have to have a different sense than the sense they have in 'Leo is four-legged' and 'St. Francis is patient'. But what then, is the sense that they have when they are used not as concrete but as abstract general terms? The expressions 'four-legged' and 'patient' are not simply equivocal like 'tenor' and 'taste'; for the sense they have in (b) and (c) is systematically related to the sense they have in sentences about Leo and St. Francis. Our opponent could therefore argue that these terms are used neither unequivocally nor equivocally but analogically,⁵ and he could meet our request for an explanation of their meaning as follows:

'has four legs' in (b) means nothing other than 'is a species whose well-formed exemplars have four legs'; 'is patient' in (c) means nothing other than 'is a virtue such that the people who possess it are patient'. If he answers in this vein he removes the sting from his claim that there are patient or four-legged abstract entities; and indeed if we had known that this was what he meant we would not have considered him an opponent.

The same can be said *mutatis mutandis* for (a). Examples like this do not refute the assumption of the non-perceptibility of all abstract entities. The type α is not one noise among others; it would otherwise have to be possible to measure e.g. its phonetic intensity. The term 'is perceived' must therefore have a different sense from the sense that it

has in sentences like 'the explosion was perceived also in the surrounding districts'. Its mode of use is analogical: it must mean in such sentences as much as: 'is a word/work of which at least one utterance/performance is perceived'. Understood in this way (a) turns out to be no more than an apparent counter-example. As long as we meet with no objection more cogent than this against the thesis that non-perceptibility is a necessary condition for abstractness then we can confidently hold on to it.

For this reason, then, we proceed to the second sub-question associated with the proposed criterion:

Does it hold for all non-perceptible entities that they are abstract?

Is non-perceptibility a *sufficient* condition of abstractness?

Here everything now depends upon how the determination

(P) *X* is not perceptible

is to be understood. Let us assume for the moment that what is meant is nothing other than:

(P 1) *X* can be perceived by no human being.

The question now immediately arises whether those things whose presence we discover with the aid of, say, a microscope or some other instrument, can be designated as perceptible? If not, then (P 1) quite certainly does not offer a sufficient condition for abstractness, for we do not want to classify viruses as abstract entities. If however instruments are admissible, how powerful are they allowed to be? It seems that any drawing of boundaries here must be arbitrary. We can however avoid the problem of arbitrariness if we admit all physically possible instruments and interpret (P) as signifying

(P 2) *X* can be perceived by no human being, irrespective of the instrument he may have at his disposal.

But (P 2) does not give us a sufficient condition of abstractness either. Why should there not be concrete objects which cannot be perceived by any *human* being, whatever instrument he might make use of, — perhaps because they are too small or too distant or both? Here one might object that it is at least conceivable that such entities are perceptible by some sensory being or other, and that they are, for this reason, not abstract. This objection understands (P) in the following way: (P 3) *X* cannot be perceived by any conceivable sensory being, irrespective of the instruments he might have at his disposal.

If an entity fulfils this condition, then this can only mean that it is incapable of occasioning any changes at all. For otherwise a sensual being

would be conceivable of which it were true that the entity in question were responsible for the occurrence of certain sensory experiences on the part of this being (as my hand, say, is responsible for the fact that it appears to me, when I observe it in daylight, as though I were seeing a hand). (P 3) is in fact a condition in which the concept of perception no longer essentially occurs, for it can be re-expressed as:

(C) *X* is incapable of occasioning changes in other objects.

If non-perceptibility is to be more than a mere pseudonym for the criterion (C), then it has to be understood in the sense of either (P 1) or (P 2). It follows that non-perceptibility is not a sufficient condition for abstractness and thus *a fortiori* is not a defining characteristic of the concept 'abstract'.

In (C) we have already reached what is fundamentally a proposal for delineation put forward by Frege. Before discussing Frege's proposal however, we wish to consider the second basic concept of the 'Friends of Forms', the concept of unchangeability. It has in specific circumstances the same extension as (C), namely when precisely those entities which are capable of occasioning changes in other entities are themselves changeable.

§ 2 Unchangeability (Second Platonic Demarcation-Proposal)

Can we align ourselves with the 'Friends of Forms' at least to this extent, that we define abstractness by means of unchangeability? We begin once more with a subsidiary question:

Does it hold of all abstract entities that they are unchangeable?

Is unchangeableness a *necessary* condition for abstractness?

In what follows we want to use the concept of 'change' in the wide sense of the Aristotelian and Thomistic concept 'μεταβολή' ('[trans-]mutatio'), i.e. it is to cover processes of the following kinds:

- 'substantial' changes (coming to be and passing away, γένεσις καὶ φθορά, *generatio et corruptio*),
- quantitative changes (αύξεισις καὶ φθίσις, *augmentum et diminutio*),
- qualitative changes (αλλοίωσις, *alteratio*)
- movements (φορά, *loci mutatio*).

It now begins to seem as if the above question is to be answered in the negative: biologists, for example, are accustomed to speaking of the origin of species, of their development, of their division, of their regres-

sive development and of their becoming extinct. Does this imply that a negative answer has to be given to the question above? An example will again help us to focus our attention:

(d) The species A will soon become extinct.

Here too the temptation has to be resisted to argue that since (d) means the same as:

(d') The last specimen of the species A will soon die
the property of changeableness is really only to be ascribed to concrete objects (the exemplars of a species). We have shown above that such arguments are inadequate. For one can after all turn them against their proponents and say that with (d') one predicts an early end for the species.

Nor, however, can we invalidate (d) as a counter-example by pointing to the difference in the use of 'become extinct' (or 'die out') in (d) and in sentences about individual animals. For this general term cannot be used meaningfully of individual animals. We – and in particular the biologists amongst us – have at our disposal a *system of specific predicates* with which to species and other biological relationship-groups we ascribe a history, predicates which cannot meaningfully be applied to *members* of the species. For this reason (d) should count as a genuine counter-example and the question above should receive a negative answer: some abstract objects are not unchangeable. If, however, unchangeableness is not a necessary condition of abstractness then the latter can of course not be defined by means of unchangeableness.

That we take seriously talk about the change of a natural species, does not mean that we want to deny that such change is logically dependent on change in at least one member of the species. It is logically impossible that the species A should become extinct without a change occurring in one or more of its members. This logical dependence is one-sided; for one does not contradict oneself when one says: 'This member of the species A has just quenched its thirst; it belongs to a species which has not changed at all in the last 100 years.'

Husserl did not adhere to his initial answer to the above question. In the *Logical Investigations* he still speaks without any qualification of "the atemporal 'being' of the ideal"⁶, implicitly rejecting the view that there could be 'ideal entities' which change. One of his most important students, Roman Ingarden, then pointed out that a literary work such as e.g. Goethe's *Römische Elegien*, in contrast to this or that copy of the work, are neither 'real' nor 'ideal' entities:

If the literary work were to be an ideal object ... it would be inconceivable for it to come into being at a given time ... as is actually the case. In this respect the literary work differs radically from such ideal objectivities as, for example ... the number five or ... the essence 'redness'.⁷

In *Experience and Judgment* Husserl then confirms⁸

the distinction between *free idealities* (such as logico-mathematical formations and pure essential structures of every kind) and *bound idealities*, which in the sense of their being, carry reality with them and hence belong to the real world.

We will return to this distinction and the peculiar terminological twist given to it by Husserl in § 5 below.

In making such a distinction we lend support to the idea that there exist abstract entities that are unchangeable. But is this justified? Is the concept of 'free ideality' not in fact empty? In our talk about properties ('essences') and numbers ('mathematical formations') we do after all use sentences such as:

(e) The colour of these gloves will soon turn dark grey.

(f) As of this morning the number of Philip's children is no longer two but three.

Because of the symmetry of the relation of paraphraseability (pointed out many times already), we cannot counter these apparent examples of changes in qualities and numbers by reference to their paraphraseability by:

(e') These gloves will soon be dark grey.

(f') As of this morning Philip has not two but three children.

Consider first the example (e). Let us assume that Joanna has white hair and wears white gloves. Then we may assert that Joanna's hair colour is identical with the colour of her gloves. Does it follow from this and (e) that

Joanna's hair colour will soon turn dark grey?

Since such an astonishing prognosis certainly does not follow from these premisses, this inference does not have the same form as, for example:

Lenin will soon return to Russia,

Lenin is identical to Ulianov,

So Ulianov will soon return to Russia,

which is of course a valid inference. Thus (e) does not attribute a change in the same entity of which it is said in the second premiss that it

is identical with Joanna's hair colour. We can make our statement about this entity more precise in the following way:

Joanna's hair colour at T is identical with the colour of her gloves at T.

The entity identified in this way – a property – cannot possibly be the entity of which it is said in (e) that it will change. But what, then, is it? The answer is contained in (e'). We are therefore allowed to affirm that sentences like (e) do not falsify the assumption of the unchangeability of properties.⁹

Example (f) will cause us still more difficulty. From

The number of Alexander's girl friends is the same as the number of Philip's children

and (f) it does not of course follow that

As of this morning the number of Alexander's girl friends is no longer two but three.

Frege has shown that a statement such as (f) just as little contains a report about the *growth* of a number as the statement 'Ten years ago the King of Sweden was an old man; now the King of Sweden is a young man,' forces us to assume that an old man has become a young man. In the same way in which we speak here about the past and the present monarch as of two different people, so in (f) we speak about two different numbers:

(The number of Philip's children at T) is two,

(The number of Philip's children at T') is three.

On this interpretation the temporal indication is a component of the singular term with which the number in question is specified.¹⁰ The number so specified cannot possibly be the entity which in (f) is said to have changed. But what, then, is the entity of which this is said in (f)? The corresponding question for (e) was easily answered, for in (e) that entity is named in the subject-place which must change qualitatively if (e) is to be counted as correct. In the case at present under discussion however the paraphrase (f') does not help us at all; for no contradiction is involved in assuming that Philip did not change this morning with respect to *any* of the modes of change distinguished (following Aristotle and Thomas) above, yet (f') is true.

If one holds this assumption to be consistent then one must clearly reject the following explication of the concept of change, that

(*) If one makes a true assertion with the sentence '*Fm* at T' and a false assertion with '*Fm* at T'', then *m* has changed.

For, if we substitute 'has two children' for '*F*' and 'Philip' for '*m*', we get a true proposition when we substitute yesterday's date for '*T*' and a false proposition when we substitute today's date for '*T*'; all the same we do not want to say that Philip has changed.

That this explanation (*) is not acceptable¹¹ can be seen from an example discussed by Plato and Aristotle. Let us assume that Socrates is as tall as Theaetetus at *T*. Now if Theaetetus grows between *T* and *T'*, whoever states at *T'* that Socrates is as tall as Theaetetus will make a false assertion. Does it follow that Socrates has changed? Hardly.¹² What is first said and then denied of Socrates is a *relational* determination. From a change in such a determination it follows – we might say – only that one of the relata of the relation must have changed. Which one of the two has changed is not something that can be determined from the change in the relation alone. This suggests the following (partial) explication of change:

(CH) If one makes a true assertion with the sentence '*Fm* at *T*' and a false assertion with the sentence '*Fm* at *T'*' and if with *F* one ascribes to object *m* a particular relation to one or several other objects, then at least one object has changed.

Before drawing any conclusions from these remarks in regard to our example (f), we should like to present some further observations on the contrast between our explication of the concept of change and the formula (*). Is this formula valid when restricted to non-relational substituends for '*F*'? This question, too, is to be answered in the negative, if such 'purely temporal' terms as 'past', 'present' and 'future' (McTaggart's A Series) are non-relational predicates. We apply these determinations to events. Events can change: if we say that a controversy is ever more passionate, then we ascribe to this event a (qualitative) change. Is this the case also when we say of an event that it was first of all present and then past? The condition specified in (*) is here fulfilled: for if one replaces '*T*' in

m is present at *T*

with '*T'*', then (for a sufficiently large interval between the two instants) one obtains a false from a true assertion. In cases of this kind '*Fm* at *T*' is true and '*Fm* at *T'*', false independently of what happens between *T* and *T'*. And for this reason one is scarcely inclined to speak of a change in *m*.

Are purely temporal terms one-place predicates? The thesis has been propounded that such terms express temporal relations to a psychical or

linguistic event (the occurrence of a sense-datum, according to Russell, or the utterance of a sentence according to Reichenbach and others). The implication of such views is that the sentence taken as our example would say of *m* that it takes place *simultaneously with* some specific psychical or linguistic process.¹³ One who accepts this thesis may no longer, for example, assert that everything that is past was at some time or other present; for events that have come about before the development of life are certainly past, but they were never contemporaneous with the appearance of a sense-datum or with the utterance of a sentence. This is not the place to investigate whether consequences of this sort refute the thesis of the (covert) relational character of purely temporal determinations.¹⁴ What is certain (and this is all that matters here) is that with a predicate of the form 'is contemporaneous with (earlier than, later than) *n*' one does not correctly ascribe to an event *m* at *T* a determination that can correctly be denied to the same event at *T'* (in one and the same reference-system). And consequently our formula (CH) is not applicable at all.

It is now important to note in regard to this formula that the entity that, under the conditions given in the antecedent of (CH), must have changed, need not be amongst those entities that are explicitly mentioned in the sentence in question. If, for example, it is true that Phaidon is at *T* sitting immediately next to Socrates, and if it is false that he is sitting immediately next to Socrates at *T'*, then it does not follow from this that either of the two named persons has changed: for it could of course be the case that Simmias has interposed himself between them. And correspondingly for all determinations with which one refers implicitly to some third relatum (cf. also say 'A is more popular than B'): one of the relata of the relation must have changed if such a determination alters; but it need not be one of those elements of the relation mentioned *expressis verbis*.

Our discussion of the concept 'change' makes it understandable why the Aristotelian-Thomistic classification does not make room amongst the modes of change for any *mutatio* in the category of relatives: if at one time but not at another a relational determination pertains to an object, then this object need not *itself* have changed.¹⁵

Let us return now to the example that provoked our reflections of the sense of the term 'change'. What, on the basis of (CH), can we counter to anyone who takes the following position with respect to (f)?:

If, since this morning, the general term 'is identical with the number of Philip's children' no longer applies to the number two, then the latter must have changed – even if this change is not a quantitative one.

This claim cannot, be it noted, be invalidated by putting a temporal specification into the general term:

'is the number of Philip's children at T'
and saying that this term does indeed always apply to the number two. For this would allow us to transform *every* predicate which holds of a particular object into a term which would always apply to it: Philip is not always ill, he changes in this respect, but by suitable substitution for 'T' we can make out of 'is ill at T' a general term which is at all times applicable to Philip.¹⁶ Now however we can point to the fact that the determination whose change is asserted by (f) is a relational one: being the number of something. And along the lines of our explication of the concept of change we can again put the question: *what* is it that has changed if as of this morning it is no longer true of two that it is identical with the number of Philip's children (or if, as of this morning, it is no longer true of Philip that he has two children)?

It would be possible to use a sign with the inscription 'Philip's children' as a nameplate; the bearer of this nameplate would not be any of Philip's children but a discontinuous concrete object whose parts might from time to time gather behind the sign but are normally dispersed. (The fact that the parts of the bearer of the name 'The Milky Way' are separated by huge distances does not affect the expression's status as a name.) The object bearing that name, then, is not to be confused with the abstract object for which the singular term '(x:x is a child of Philip's)' stands, i.e. with the class of Philip's children.¹⁷ The *elements* of this class are persons only; *parts* of the concrete object, however, are not only persons but arms and legs and so on.¹⁸

We can now say that the discontinuous object called 'Philip's children' increased in size this morning, that it used to consist of two children and now consists of three children. What has changed here is not a number (and nor, either, is it Philip).

Thus the concept of 'free ideality'¹⁹ is not empty: abstract entities of certain categories *are* unchangeable, for example properties and numbers.

At the beginning of this section we convinced ourselves that unchangeability was not a necessary condition of abstractness; 'bound

idealities' - natural kinds (species), for example are changeable. The question which still remains:

Does it hold of all unchangeable entities that they are abstract? Is unchangeableness a *sufficient* condition for abstractness?

can now be given an affirmative answer. Thanks to the foregoing discussion of the concept of change we can make this thesis more precise. An entity *m* is always an abstract entity when it satisfies for *every* substitution of a general term for '*F*', the following condition:

If an assertion of the form '*Fm* at *T*' is true and an assertion of the form '*Fm* at *T*' false, then '*F*' ascribes to *m* a relation to another object which changes between *T* and *T*'.

Let us summarize the result of the discussion so far. We have not succeeded in specifying what holds of *all* and *only* abstract entities. We therefore have at our disposal no *definition* of the concept 'abstract' explained at the beginning of our essay by means of a list. We have however obtained a standpoint from which we can group the various categories of abstract entities there presented: the distinction between 'free' and 'bound idealities', a distinction which we shall seek to state more precisely at the end of the present essay.

§ 3 Non-Reality (Frege's Criterion and the Category of Individual Moments)

In Frege's ontology the totality of all objects²⁰ is subdivided into three 'realms', of which the third embraces at least some categories of what we have called abstract entities. The three realms are:²¹

- (I) what is subjective and real,
- (II) what is objective and real,
- (III) what is objective but not real.

The term 'objective' in this table (which may have been inspired by Lotze's *Metaphysics*)²² is to mean as much as 'capable of being the common property of several thinkers'.²³ Frege's definition of realm (I) and with it his conception of the opposition 'subjective/objective' is something which, in the present context, we can afford to neglect. It is based on the assumption going back to Descartes and Locke that states of consciousness are epistemically private: only I can *know* that the pre-

sent state of my consciousness is of such and such a kind. (Wittgenstein has shown in the *Philosophical Investigations* how questionable this assumption is.)²⁴

Let us concentrate instead on Frege's conception of the opposition 'real/unreal'. Frege explains the term 'real' (*wirklich*) in the introduction to the *Grundgesetze* as follows:²⁵

Def. 1. "*fähig, unmittelbar oder mittelbar auf die Sinne zu wirken*" (capable of directly or indirectly acting upon |bringing about effects in| the senses).

This explanation echoes Kant's *Critique of Pure Reason*²⁶:

"...Whatever is connected with perception in accordance with empirical laws is actual (*wirklich*), even though it is not immediately perceived";

"everything is real (*wirklich*) which stands in connection with a perception in accordance with the laws of empirical advance".²⁷

The field of what we can 'immediately' perceive is defined by Kant – one looks in vain for an explanation of the notion of immediate perception in Frege – in terms of the 'constitution of our sense organs'. If our senses were 'finer' then this realm would have a larger extension.²⁸

A further explication of 'real' is to be found in Frege's article 'Le Nombre Entier':

Def. 2. "*kann eine Wirkung hervorrufen und erleiden*" (can bring about and undergo an effect) (Frege, 1895, p. 212).

If we take this as our basis then it will come about that realm (I) will contain more entities than it would on the basis of Def. 1. For why should there not be something which can bring about and undergo an effect but have neither a mediate nor an immediate effect on *our* sense organs?

We shall, in any case, remain with Def. 2.²⁹ What is it that belongs, in Frege's eyes, to realm (III), understood according to this definition? Numbers, of course, which are over and over again expressly characterised as elements of this realm.³⁰ Propositions, too, (which in Frege are called 'thoughts') are consigned to realm (III).³¹ But Frege lists as exemplary elements of (III) also objects which, at first sight at least, do not seem to fall under the concept 'abstract' in the sense of our initial list. In § 26 of the *Foundations of Arithmetic* Frege writes:³²

I distinguish what I call objective from what is handleable or spatial or actual. The axis of the earth is objective, so is the centre of mass of the solar system,

but I should not call them actual in the way the earth itself is so. We often speak of the equator as an *imaginary* (*gedachte*) line; but it would be wrong to call it a *fabricated* or *invented* (*erdachte*) line; it is not a creature of thought, the product of a psychological process, but is only recognized or apprehended by thought. If to be recognized were to be created, then we should be able to say nothing positive about the equator for any period earlier than the date of its alleged creation.

We should bear in mind first of all that it makes sense to say of the objects here put forward as examples that they are subject to change. The axis of the Earth moves relative to the Sun, for the planet of which this is the axis is after all not always at the same distance from the centre of the Sun. Since the latter, in which the centre of the solar system lies, is moving towards the constellation Hercules, this point too changes its position. And even if thinking about the equator is not an excogitation or generation of this line it is still true that before the origin of our planet there was no equator that could be 'grasped by thinking'; to this extent it too came into being.

Now the fact that we ascribe changes to these entities in the ways specified is still not an argument against their being classified as abstract. For it will be remembered that we did not accept unchangeability as a necessary condition of abstractness, but preferred rather to take e.g. talk about changes in natural species, at its face value. But let us look more closely at the cases in question here. The changes described are in each case *logically dependent* on the change of *one particular* object; in the terms of Aristotelian-Thomistic metaphysics, we have to do with changes '*per accidens*' (*κατὰ συμβεβηχός*). In regard to the change in position of e.g. a centre of a mass one could say with Aristotle:³³ "something that is without parts can move at most *per accidens*, as when, for example, the body moves ... to which it belongs ... As the man who sits in a ship it cannot however move *per se* (*καθ' αὐτό*)". And the origin of the equator could also have been characterised by Aristotle as a '*generatio per accidens*',³⁴ It is a peculiarity of the class of entities to which Frege's examples belong that the changes described are in each case logically dependent on the change '*per se*' of a particular object: as a first approximation one can say that their *identity* stands and falls with the identity of a specific non-abstract object. Frege did not draw attention to this trait of his examples, though it is of great interest for the working out of an ontology within the framework of linguistic analysis.

We now wish to arrange Frege's examples in three concentric circles: only with the innermost of these will we reach that peculiarity of them with which Frege himself is concerned. The region bounded by the outermost circle we have just described (roughly) by means of the thesis of dependence of identity: we might call it the region of *individual moments*. With the aid of this expression (borrowed from Husserl³⁵) we can formulate our thesis thus: that Frege's examples

the axis of the Earth

the centre of mass of the Solar System

the equator of the Earth

all fall under the category of individual moments.

We can begin the discussion and justification of this thesis with a characterisation of the just-mentioned singular terms: each is the result of the composition of a functor 'the Φ of ()' – i.e. of an unsaturated expression that becomes a singular term through saturation by a singular term – with a non-abstract singular term. (It will become clear in the course of this section why we have been careful to use this latter expression rather than merely 'singular term' or 'concrete singular term'.)

The singular terms that interest us here share the property of being composed of a functor and a designation of a non-abstract object with:

(A) many descriptions that clearly serve to refer to concrete objects, e.g. 'the highest mountain on Earth', 'the mother of Abel'.

And they share this semantic property also with

(B) many descriptions which unquestionably stand for abstract objects in the sense of our original list,

e.g. for

- *properties*, such as 'the magnitude of the Earth', 'the direction of the Champs-Élysées', 'the colour of this dustjacket',
- *propositions*, such as 'that which Philip has proved',
- *species and genera*, such as 'the breed of dog to which Fido belongs',
- and *types*, such as 'Otto's' favourite poem'.

When is it, now, that we use a description of the form 'the Φ of (non-abstract object)' to make reference to an individual moment? Precisely then, according to our demarcation proposal, when the following condition is fulfilled:

(M) If the $\bar{\Psi}$ of some non-abstract object x is identical with the Φ of some non-abstract object y , then *either*

(1) x is identical with y ,

or (2) x circumcludes (a part of) y ,

- or (3) y circumcludes (a part of) x ,
 or (4) x is continuously adjoined to y .

Ignoring, for the moment, the conditions (2), (3) and (4), which certainly stand in need of comment, it is clear how the limitation of the range of values of the variables ' x ' and ' y ' is motivated. From the fact that the predecessor of $(7 + 2)$ is identical with the predecessor of 9 it follows that $(7 + 2)$ is identical with 9. But we do not want to say that the predecessor of $(7 + 2)$ or, in other words, that the *number* 8, is an individual moment.

It is now easily seen that the entities designated by the terms in groups (A) and (B) do not fall under the concept of individual moment explicated by means of (M): Abel's mother is identical with the mother of Cain, but it is clear that none of the consequences presented under (M) follow from this. And the magnitude (direction, colour) of m can be identical with that of n ; that which has been proved by m can be identical with that which has been proved by n ; the breed of dog to which m belongs can be the same as that to which n belongs; and finally the favourite poem of m can be the same as that of n , – even though m and n fulfil none of the conditions presented in the consequent of (M).³⁶

Thus the functional descriptions given under (A) and (B) do *not* refer to individual moments. But it is precisely to entities of this sort that we do in fact refer by means of the three examples of descriptions given by Frege.

Let us make clear to ourselves why we could not have been satisfied in (M) merely with condition (1). We want to conceive the centre of gravity of a solid body categorially as an individual moment. Now the middle point of a body m can be identical with that of a body n even though m and n are different, provided m is a constituent within n or conversely. Reason enough to take account, via conditions (2) and (3), of the possibility of circumclusion (περιέχελν).³⁷ Further we want to conceive the equator categorially as an individual moment. One can refer to the equator by means of the functional descriptions 'the boundary of the Northern hemisphere' and 'the boundary of the Southern hemisphere': the boundary of the Northern hemisphere is therefore identical with that of the Southern, and yet we have to deal here with *two* hemispheres. Which is why we have included in clause (4) of our definition the possibility of 'continual adjunction' (συνεχές)³⁸.

The logico-semantic difference worked out here, between the de-

scriptions in groups (A) and (B) on the one hand and the terms presented by Frege in § 26 of the *Grundlagen* on the other, now suggests (though it does not, of course, compel) a terminological decision, that *individual moments* are *neither abstract* (in the sense of the usage exemplified by our list) *nor concrete entities*. What is important is not the terminology decided upon; but only the avoidance of conceptual confusion, a danger which is reduced if we accept the categorial *trichotomy* (abstract, concrete, neither abstract nor concrete).

One consequence of our decision is that the opposition between abstract and concrete singular terms can now no longer be accepted as exhaustive: some singular terms ('the axis of the Earth', for example) are neither abstract nor concrete. This consequence is unavoidable since in our classification of terms we have been following Quine in his view that 'the division of terms into concrete and abstract is a distinction only in the kinds of objects referred to'.³⁹ A further consequence of deciding in favour of the trichotomy is that the extension of the concept 'abstract object' can now no longer coincide with the extension of Frege's concept 'objective but non-real object'. This is not, of course, an objection, either against our choice of terminology or against Frege's conception of his realm (III).

It now becomes clear that we were right to select as the range of variables in (M) not concrete but 'non-abstract' entities. The functor 'the length of ()', for example, can just as well be saturated by means of the concrete singular term 'the Champs-Élysées', as by the non-abstract singular term 'the equator of the Earth'. The description which results stands for an individual moment neither in the first case, nor in the second, for whether our variables take concrete objects or individual moments as values, the length of m can be identical with the length of n even when m and n are separate. This is taken account of in our determination of the value-range in (M).

In the next section we shall discuss the origin of the term 'individual moment' and at the same time criticise its traditional application. Before this however we must implement a further circumscription of Frege's example. For it is certain that there are individual moments which Frege would not count as belonging to realm (III). The headache of m can be identical with the headache of n only when m and n are identical. Thus m 's headache is an individual moment. But according to Frege it clearly belongs to realm (I): i.e. to that which is subjective and real.⁴⁰

The second of the three concentric circles in which we want to ar-

range Frege's examples comprises a class of entities which can be designated as *boundaries*. The concept *boundary* we wish to have so understood that under it there fall not only the boundaries of solid bodies, that is surfaces, but also boundaries of boundaries of solid bodies, i.e. lines and also boundaries of boundaries of boundaries of solid bodies, i.e. points.⁴¹

Even with this concept, however, we have not yet succeeded in comprehending that peculiarity which motivates Frege's choice of examples. Obviously, entities like the surface of the Earth are perceptible. But so too – in opposition to a widely held prejudice – are lines (as distinct from stripes) and even points (as distinct from spots): the boundaries of coloured surfaces are such perceptible lines; and if a square area is divided into four differently coloured squares then one sees a point at the centre. One cannot, of course, see lines and points without also seeing the coloured surface whose boundaries they are. (They are to this extent only *per accidens* perceptible.⁴²) But those objects by means of which Frege illustrates his conception of realm (III) in the *Grundlagen* are in principle non-perceptible: in calling the equator an 'imaginary' line we give to understand that it is not possible to speak sensibly of a perception of such an entity, or in other words that its non-perceptibility is not something resulting from the crudeness of our sense-organs or from shortcomings in our instruments.⁴³ In this there does in fact lie a similarity between abstract entities such as numbers and those individual moments that might be designated as *intelligible boundaries*.⁴⁴

§ 4 Individual Moments and Properties: Husserl's Aristotelianism

In the foregoing section we have employed a Husserlian term, attempting to give it a precise sense by means of (M). From the point of view of conceptual history it is noteworthy that in the *Logical Investigations* Husserl designates the individual moments of an object as 'abstract parts':⁴⁵ an individual moment has this in common with the 'concrete parts' or 'pieces' of an object, that it is 'as precisely and individually singular as the whole object';⁴⁶ it is distinguished from the pieces through its 'non-self-sufficiency' (*Unselbständigkeit* or dependence). Husserl is clearly employing the term 'abstract' in a way quite different from that in which we – in conformity with Quine and other analytic philosophers

– have so far used it. The number 9, for example, is *not* whilst the surface of the earth is an abstract entity in the terminology of the *Logical Investigations*.⁴⁷ This usage has a long prehistory, reaching back as far as late antiquity: Boethius translates by means of the word ‘abstractum’ the expression ‘ἐξ ἀφαιρέσεως’ which Aristotle used to designate logically (dependent) entities.⁴⁸

In order to avoid confusion however we shall now revert to our previous usage. Still we wish to remain for a while with Husserl’s *theory* (and the Aristotelian tradition in which it stands). For it is a theory that departs at one important point not merely terminologically but also substantially from that which has so far been affirmed in the course of this essay. Our assertion, that the colour of an object *m* can be identical with the colour of *another* object *n*,⁴⁹ Husserl would not have been able to accept. He argued, that if we survey a plurality of red objects comparing them amongst themselves we can judge with self-evidence:

In all these cases individual aspects differ, but in each the same species is realised: This red is the same as that – specifically is treated as the same colour – and yet again this red differs from that one – i.e. individually treated, it is a different objective, individual feature.⁵⁰

Comparison of two concrete, separated phenomena of the same quality, e.g. green, evidently shows that each has *its own* green ... The green of the one is ... as much separated from the green of the other, as are the concrete wholes in which these ‘greens’ are.⁵¹

In Husserl’s eyes, then, an expression such as ‘the red of this book-cover’ satisfies condition (M), since this red is necessarily numerically different from the red of another, separate object, and this is true even where it is the same nuance of red which is involved in each case. On this theory, exemplars of the species ‘red’ are not red objects (things) *m*, *n*, etc., but individual moments of these objects: the red of *m*, the red of *n*, etc. Only on this basis is it understandable why Husserl calls properties species. A proposition of the form

m is red

is accordingly more complex than it seems:

There is something which is an individual moment of *m* and which is an exemplar of the species red

would be its explication within Husserl’s theory.

This theory is indebted, even in its choice of examples, to the second

chapter of Aristotle's *Categories*.⁵² With the help of the characterisations:

(A) said of a subject

(B) in a subject

Aristotle makes there a distinction between objects in which the opposition between 1. 'Man' and 2. 'Socrates' (ο τις άνθρωπος) corresponds to a similar opposition between 3. 'the colour white' and 4. 'the white of this manuscript page' (τὸ τι λευκόν). That is to say, by means of the characteristics (A) and (B), Aristotle distinguishes between

1. Species and genera of objects in the category 'substance' (second substances): Combination of characteristics A and non-B.
2. Instances of 1. (first substances): Combination of characteristics non-A and non-B.
3. Species and genera of objects in other categories, e.g. in the category 'quality': Combination of characteristics A and B.
4. Instances of 3.: Combination of characteristics non-A and B.

The characteristic (B), which is of most importance here, is explained by Aristotle as follows: 'By "in a subject" I mean what (a) is in something not as a part, and (b) cannot exist separately from what it is in'.⁵³

We can paraphrase this with the help of Husserl's theory of part and whole:⁵⁴ that by 'in a subject' is meant what is (a') not in something as a piece (concrete part) but (b') as a moment (abstract part). The objects which fall under 4. are then just those that Husserl designates as individual moments.

When the distinction between 3. and 4. is understood in this way it normally reminds Anglo-Saxon interpreters of Aristotle not of Husserl but of Stout. In his essay "The Nature of Universals and Propositions" he asserts:

Of two billiard balls, each has its own particular roundness separate and distinct from that of the other, just as the billiard balls themselves are distinct and separate. As Jones is separate and distinct from Robinson, so the particular happiness of Jones is separate and distinct from that of Robinson.⁵⁵

Is the just described conception of individual moments plausible? Are there really such things as 'individualised qualities' or 'quality-individuals'?

G.E.M. Anscombe in her essay on the Aristotelian concept of substance interprets the second chapter of the *Categories* in precisely the way just presented here. But she does not seem to find the assumption

of 'individualised properties' to be an illuminating one, describing Aristotle's example as 'slightly obscure to us', and she therefore substitutes for it one of her own: 'the surface of my wedding-ring',⁵⁶ i.e. a functional description standing for a *boundary*.

Particularly illuminating in this context is something Wittgenstein said in the *Blue Book*:

We use the phrase 'two books have the same colour' but we could perfectly well say: 'They can't have the *same* colour, because, after all, this book has its own colour, and the other book has its own colour too'. This ... would be stating a grammatical rule – a rule, incidentally, not in accordance with our ordinary usage.⁵⁷

To this the advocates of individualised properties could of course say: Certainly, we use sentences such as 'These two books have the same colour'; but that only shows that everyday language admits a lax use of 'the same'. Strictly speaking we only want to state with such a sentence the complete similarity of numerically different colours. We would after all never dream of inferring from the assertion that the two people in the next room are wearing the same pullover that we have there four arms in two sleeves.

Wittgenstein could also have reminded us of sentences such as 'This book has the colour I am thinking about for our new car'. But such examples do not refute the friends of individualised qualities either. Strictly speaking, they could say that we only have complete similarity of two colours in mind when using such sentences. And indeed we do not normally take the assertion that someone has got his father's nose as a reference to a transplantation.⁵⁸

But now there is one decisive disanalogy between the lax use of 'the same' in our talk about concrete objects and the application of this expression in our talk about colours. If, for cases of the first type we occasionally reject an interpretation of 'the same' in the sense of numerical identity, then we have *empirical* grounds for this. It is our eyes that tell us that the proposition '*m* and *n* are wearing the same pullover' interpreted according to the pattern 'the Φ of *m* = the Φ of *n*' is on occasion false. But in cases of the second type – that is, for example, in talk about colours – the advocate of the individualised properties thesis has to bring *a priori* reasons into play: for whether the colour of *m* is equal to, or is not equal to, the colour of *n* is not supposed to be a merely contingent matter. But what can he oppose to our claim, if we insist upon it,

that '*m* and *n* have the same colour' means just what it appears to mean at first sight, namely that the colour of *m* is (numerically) identical with the colour of *n*?

We do not need to be particularly impressed by Husserl's recourse to the testimony of self-evidence. Husserl himself knows only too well that

this testimony, when appealed to, must be conceptually apprehended and asserted, and will thereby lose much authority and permit of well-founded doubts. Different people ... read different things into it or out of it.⁵⁹

In the *Logical Investigations* there is indeed also an (indirect) *argument* for the thesis in question: what sense would talk about the spread of colour over a whole surface have,⁶⁰ if the thesis were false? But is such talk only understandable when one assumes that there are things such as colour-individuals? It needs to be noticed first of all that even if an affirmative answer were to be given to this question this would not amount to a defence of the general assumption of individualised qualities: we do not speak about the spread of a form, e.g. of squareness, over the whole of a surface, Husserl however accepts 'individual forms' as instances of 'geometrical species'.⁶¹

Consider more closely the contrast just hinted at between properties such as red on the one hand and those such as squareness on the other. Frege already saw what is decisive here:

We can, for example, divide up something falling under the concept 'red' into parts in a variety of ways, without the parts thereby ceasing to fall under the same concept 'red';

the concept of square in contrast is a

concept which isolates what falls under it in a definitive manner, and which does not permit any arbitrary division into parts.⁶²

(Thus e.g. some parts of a square book cover that has been torn up by a child might be triangular.) In the same way the property of, say, being rough differs from the property of being 20cm² in size: those things that exemplify the first attribute can, those that exemplify the second cannot, be divided into parts in a variety of ways without the resulting parts ceasing to possess the property in question. Thus to say that the colour red is extended across a whole surface is to say that all perceptible parts of this surface are red. And it is not at all clear why, for the interpreta-

tion of such modes of speech, the assumption of individualised qualities is required.

There is of course something which is spread over the surface of a body and which is just as individual as the object on which it is to be seen. It is what in Latin is called '*pigmentum*' (not: '*colour*') and is bought at a paint-dealer's ('*pigmentarius*'). In usages such as "A bit more red here!" we use 'red' as a mass-noun.⁶³ Such a mass is naturally not a moment of the thing, which it is to be made out, but a *piece* of the thing – that is, a part which is just as concrete as the thing as a whole.⁶⁴

In the absence, now, of a more convincing argument for the conception of individualised qualities,⁶⁵ we can safely stick to the strict interpretation of 'the same' (in the sense of '=') in sentences like '*m* has the same colour as *n*'.

We began our explication of the concept 'individual moment' in the last paragraph with entities which we have classified as boundaries. That boundaries *also* fall under this concept was expressly acknowledged by Husserl in *Experience and Judgment*, § 32a:

Up to now, as examples ... of determination by dependent moments, we have always chosen qualitative moments ... But are there also dependent moments of other kinds?

Let us consider, for example, the edge of a material thing or the total surface by which it is circumscribed as a spatial *Gestalt*; these are certainly dependent moments and not pieces: we cannot take away the surface or the edge from a thing so that it falls into two independent parts. On the other hand, the circumscribing surface is certainly not a quality of the thing. It follows that not every dependent moment of a thing belongs to the thing as a quality.

We have sought in this section to show that Husserl (like Aristotle before him and Stout after him) developed an *indispensable* concept – the concept of a class of entities that are neither concrete nor abstract – on the basis of a *dubious* model, i.e. the construction 'individualised quality' of whose analytical usefulness we have not been able to convince ourselves.⁶⁶

§ 5 Sense-Determinate Singular Terms (Strawson's Demarcation-Proposal)

With this, the final attempt at demarcation of abstract entities to be considered in this essay, the concept of reference, and of its vehicle, the

singular term, will come into the centre of our attention. For Strawson believes⁶⁷ that he is able to give a criterion for abstractness by means of the separation of a specific type of singular term. Since he employs, in this attempt, a terminology different from that used here, we shall have to provide some preliminary lexicographical remarks.

The most important distinction, for us, in the Strawsonian ontology, is that between *particulars* and *non-particulars*. As exemplary particulars Strawson puts forward material bodies and persons.⁶⁸ But individual moments, too, are in his eyes particulars, – more precisely: dependent particulars.⁶⁹ All the entities that fall under the various categories presented in our initial list are conceived by Strawson as non-particulars.⁷⁰

We shall therefore use the expression '*individual entity*' as an equivalent within our framework of Strawson's 'particular'. An entity can be designated 'individual' if and only if it is either a concrete object or an individual moment or, more briefly, if it is a non-abstract object.

Strawson's term 'non-particular' we shall therefore reproduce as '*non-individual entity*', a determination which may be ascribed to an entity if and only if it is abstract.

So much for nomenclature, now to matters of substance. If a speaker makes a statement with a simple assertoric sentence involving a singular and a general term, then with the aid of the singular term he indicates *which* entity it is that he characterises, correctly or incorrectly, with the aid of the general term. The singular term, we can say, describing its role in an abbreviated way, serves the function of reference. That an unequivocal answer to the question 'which?' should be forthcoming, the sense of the singular term involved usually operates *in conjunction with* the context in which the assertion is made. Occasionally however we make the reference unequivocal by explicitly mentioning the spatio-temporal position of the intended referent.⁷¹

Are there singular terms which do not contain any such localising component and whose sense *alone* guarantees unique reference? Let us call singular terms which fulfil this condition 'sense-determinate'; then an abbreviated form of our question would be: are there sense-determinate singular terms? Since this question is decisive for the further course of our discussion we must keep distinctly before our eyes the conditions under which a singular term deserves the title 'sense-determinate':

- A it contains no mention of the spatio-temporal position of the referent;
- B one cannot use it in different situations to make reference to different entities without the sense of the term being a different one in each case;
- C an attempt to use the term to make a unique reference cannot miscarry for the reason that there is more than one equally admissible candidate for the status of referent.

Our question is, therefore: do we have expressions at our disposal which fulfil these conditions?

Let us make clear, first of all by means of examples, why this question is to be answered positively. (In each case we have included in parenthesis an alternative designation of the same object that is not sense-determinate, because it fails to satisfy at least one of the requirements A to C). One can refer by means of a sense-determinate singular term to, amongst other things:

- properties, e.g.
wisdom
(* the property for which King Solomon is famous)
- propositions, e.g.
that the sum of the angles of a triangle is equal to two right angles
(* the theorem whose proof Philip has known since yesterday)
- types, e.g.
'Schmidt'
(* the name of the current Chancellor of West Germany)
- numbers e.g.
Nine
5 + 4
(* the number of member-countries of the E.E.C.).

Is it now perhaps true that one can refer *only* to abstract entities by means of sense-determinate singular terms? Is the possibility of a reference of this kind perhaps a sufficient condition of abstractness? The thesis runs, more formally as follows: that

- (i) if we can refer to an entity with a sense-determinate singular term, then the entity in question is abstract.

Do not superlative terms such as 'the quickest 100 meter-runner of all time' constitute a counter-instance? Such a term contains no spatio-temporal specifications, one cannot use it at different times to make

(correct) reference to different objects; and yet it does not designate an abstract entity. This is all certainly true, but it does not by any means refute (i). For condition *C* is not fulfilled. There could after all be two persons of whom it was true that, had the one not existed, then the other would have been *the* fastest runner of all time.

(i) is logically equivalent to Strawson's claim that if an entity is non-abstract ('individual') then we *cannot* refer to it with a sense-determinate singular term.

In Strawson's own words:⁷²

It is a necessary condition of a thing's being a particular thing that it cannot be referred to by a singular substantival expression, a unique reference for which is determined solely by the meaning of the words making up that expression.

This is first of all manifestly true for concrete entities. If one makes reference to such an entity by means of a singular term containing no mention of a spatio-temporal position, then it is always possible either that another entity may be denoted by the same expression (with the same sense) but employed in a different spatio-temporal context, or that the existence of a number of different entities competing for the title of this term should cause the reference to miscarry. And because this is true of concrete entities, then it must also be true of individual moments. For of course they too can only be identified unambiguously when the concrete objects are designated whose moments they are. Thesis (i) can therefore be accepted.

But is its converse also correct? That is, is it true that:

if an entity is abstract, then we can refer to it by means of a sense-determinate singular term?

Should this thesis also hold, then we would finally have discovered a sufficient *and* necessary condition for abstractness. But is the converse of thesis (i) really true?

We often refer to propositions by means of nominalised sentences. Sometimes these nominalised sentences contain no singular terms at all ('that all swans are white'), or they contain only a sense-determinate singular term ('that 2 is a prime number'). Sometimes however a singular term occurs in such a nominalisation which is *not* sense-determinate. An example would be: 'that all letters on this page are legible'. This nominalised sentence is itself a singular term that is not sense-determinate; for we can use it in different contexts to refer to different proposi-

tions without its sense being in each case different. Referring to *the* proposition that is meant by a particular speaker (or author) in a particular situation is something we cannot do by means of an expression fulfilling conditions *A*, *B* and *C*. Anyone who wanted to accept the converse of (i) would therefore have to classify *this* proposition as a *non-abstract* entity. Some propositions would then be counted as abstract, others not, and this consequence is sufficient reason to reject the converse of (i), as indeed does Strawson himself.

Strawson does not in fact give a necessary condition of *abstractness*; he says rather that⁷³

it is a necessary condition for a thing's being a general thing that it can be referred to by a singular substantival expression, a unique reference for which is determined solely by the meaning of the words making up that expression.

Thus instead of the converse of (i) Strawson advocates the thesis that (ii) if an entity is general, then we can use a sense-determinate singular term to refer to it.

The German expression which fits most closely in our framework with Strawson's term '*general thing*' is Husserl's '*allgemeiner Gegenstand*'⁷⁴. This can be explicated as follows: *Y* is a general entity if and only if another entity *X* can stand to it in the ('unreal')⁷⁵ relation '*X* is an instance (an exemplar, occurrence, case) of *Y*'. (ii) allows us to avoid the awkward consequence yielded by the converse of (i) of having to classify certain propositions as non-abstract entities. For we can now say that whilst certainly all general entities are abstract ('non-individual'), not all abstract entities are general – propositions, for example, are not so.⁷⁶ Is this suggestion plausible when our explication of '*general entity*' is taken as basis?

This question will be answered by the author of the *Logical Investigations* with a clear 'No.' For Husserl in this work classifies propositions or, as he puts it, '*judgements in the ideal, logical sense*'⁷⁷ – as '*general entities*' ('species'), conceived there as properties of acts (of asserting, surmising, etc.)⁷⁸ This conception is hardly plausible. If someone asserts that *p*, then his act exemplifies the property of being an assertion: it exemplifies also the property of being oriented towards the proposition that *p*; but this proposition itself is surely not a property of his act. Husserl himself later went on to reject this conception. In *Experience and Judgment* he writes:⁷⁹

Since any number of affirmative acts, of no matter how many subjects, can affirm one and the same proposition,⁸⁰ so it is a great temptation to think that the proposition belongs to the various acts ... as their general species, just as, say, the general essence redness belongs to the many red things ... Against this one has to say that certainly the proposition is general insofar as it points to an infinite number of positing acts in which it is intended;⁸¹ but it is not general in the sense of the generality of a species, i.e. the generality of an extension ... It has nothing individual falling under it.

Propositions are accordingly abstract entities ('idealities' in the terminology of *Experience and Judgment*); but they are not general entities. And, if numbers are classes of classes, then propositions are the only abstract entities from our list that are not also general entities.

With this it would have been shown that the objection that we brought against the converse of (i) does not refute Strawson's thesis (ii). But the latter might still be false, and we shall see that it is in fact so.

Musical and literary works are (as *types*) general entities.⁸² Now we normally refer to them by means of singular terms that are not at all sense-determinate, - e.g. with a designation such as 'Chopin's Funeral March'. More than one person might be called Chopin and have composed one or more funeral marches. Strawson nevertheless sees no serious difficulty for his conception:

We have ... an easy remedy here. We can regard the pattern of sounds in question as a general thing for which there might (perhaps does) exist a general description the meaning of which uniquely determines its reference.⁸³

But would such a description really achieve what Strawson expects from it? A description of the sequence of sounds made audible by every pianist who correctly plays Chopin's Funeral March in no way designates precisely one *work*. It could be that a work composed yesterday by a Martian (living in a cultural community that has never come into contact with our own) consists of the same sequence of sounds. Would it not be legitimate in such circumstances to speak of *two* works? Each of these works would, despite being indistinguishable *qua* sequence of sounds, be a different general entity, for the first was, after all, composed more than 100 years before the other.⁸⁴

Since it is always logically possible that a work is not qualitatively distinguished from another work but only through the positional property of having originated in this particular place and time, we cannot refer to such an entity by means of a sense-determinate singular term. Certain⁸⁵

general entities in the category *type* are therefore counterexamples to thesis (ii).

And these are not the only counterexamples. *Natural kinds* are general entities.⁸⁶ Now it could be that a certain species has a *Doppelgänger* somewhere in another corner of the universe. Let us suppose for example that there exists on Mars a species which, after an interval of a few thousand years, recapitulates the entire history of mankind, from the creation of Adam to the last judgment, with personnel qualitatively indistinguishable from Adam and his descendants. Then this species, assuming an absence of antideuvian contact between Mars and Earth, is not mankind: only Adam and his descendants are men.⁸⁷

Once more, therefore, we can say that since it is always (logically) possible that a species be qualitatively indistinguishable from another species, but only through the positional property of having originated at this particular time and place, then we cannot refer to such an entity with a sense-determinate singular term. Natural kinds, too, are counter-examples to thesis (ii).

Our criticism of (ii) can now however be given a positive application. Those instances counting against it are 'bound idealities'. Already in § 2 we introduced this concept, taken from Husserl's *Experience and Judgment*, fixing its extension to the class of changeable abstract entities. There we left open what the 'boundedness' of literary and musical works and of natural kinds and the 'freedom' of, say, properties and numbers amounts to. We can now say that:

An entity is a *free* ideality if and only if we can refer to it by means of a sense-determinate singular term. An entity is a *bound* ideality if and only if it is abstract and we cannot refer to it by means of a sense-determinate singular term.

The boundedness of bound idealities is a boundedness to spatiotemporal regions: their position in space and time is essential to their identity.⁸⁸ This reminds us of Husserl, when he says:⁸⁹

Free idealities ... are bound to no territory. Bound idealities are bound to earth, to mars, etc.

Let us look back at what has been achieved in this section. Our altercation with Strawson, too, has brought to light no definition which would apply to all and only abstract entities. But still, thesis (i) gives a sufficient condition of abstractness, and the concept of sense-determinate

singular terms has allowed us to clarify the ontologically significant difference between 'free' and 'bound' abstract entities. But there is a shadow falling even over this partial result:⁹⁰ for we understand properly what a sense-determinate singular term is only if we understand the expression 'sense'. That it is capable of a sufficiently clear explication has simply been presupposed in the present essay.⁹¹

But even if this explication should be provided, we would still have achieved no *definition* of abstractness either in the present or in the previous sections. This does not however signify that we do not at bottom know what it is about which we are speaking when we employ the expression 'abstract entity'. For who would say that a correct answer to the question 'what is an abstract entity?' can be given only by means of a statement of what is common and peculiar to all abstract entities? Who would say that the provision of examples and their classification into groups (e.g. under a heading such as 'freedom vs. boundedness'⁹²) is not an acceptable conceptual clarification?

In fact it is no one less than Socrates (as depicted by Plato) who would make such a claim. He employs definitions such as 'bachelor = *df.* unmarried man'⁹³ or 'square = *is* equilinear rectangle'⁹⁴ as prototypes of every conceptual clarification. And so it is that Wittgenstein, in his first attempts to break the hold of this prototype, in each case uses a specific Platonic text as his foil. He is alluding to the *Theaetetus* when he says:

The idea that in order to get clear about the meaning of a general term one had to find the common element in all its applications has shackled philosophical investigation; for it has not only led to no result, but also made the philosopher dismiss as irrelevant the concrete cases, which alone could have helped him to understand the usage of the general term. When Socrates asks the question, 'What is knowledge?' he does not even regard it as a preliminary answer to enumerate cases of knowledge.⁹⁵

The considerations put forward in the present essay seem to suggest that the same thing holds of abstract entities which Wittgenstein in the *Investigations* says of languages, games, and numbers:

that these phenomena have not even one thing in common which makes us use the same word for all, – but that they are *related* to one another in many different ways.⁹⁶

If this supposition is correct, then our rather modest answer to the question 'what is an abstract entity?' would yet remain fully adequate to

the subject-matter. The listing of examples was therefore not at all merely an instrumental aid, properly to be replaced by the specification of necessary and sufficient conditions for the application of the concept 'abstract entity'.

Notes

- ¹ Thus Quine, whose classification of terms we adopt, writes (1974, § 39, p. 218): "The division of terms into concrete and abstract is a distinction only in the kinds of objects referred to."
- ² Plato, *Sophistes*, 248 A and cf. e.g. *Phaedo*, 78 Cff.; *Parmenides*, 130 A, 135 A; *Timaeus*, 27 Dff., 48 Ef.
- ³ See ch. 4 of Kühne, 1979 (from which the present essay is adapted) for a discussion of the positive notion of 'access to the Forms' at issue here.
- ⁴ 'Perception', in the present essay, always means the same thing as 'sensual perception'. But this supplement is perhaps completely superfluous: see ch. 4 of Kühne, 1979.
- ⁵ Cf. Aristotle, *Metaphysics*, IV 2, 1002 b 32 ff. If one calls the skin colour of a man healthy – this is Aristotle's favourite example – one means that it is a symptom/sign of a healthy organism; if one calls a meal healthy one means that it keeps the organism healthy; but it is the organism which, in the central meaning of the word, is called healthy. Thomas of Aquinas speaks in such cases of an '*analogia multorum ad unum*'. Cf. *Summa Theologica*, Ia, q. 13 a 5; *Summa Contra Gentiles*, I, Ch. 34. The Aristotle-scholar G.E.L. Owen employs in his 1960 the expression 'focal meaning'.
- ⁶ Husserl, 1913/22, vol. II/1, P. 123 f. Eng. trans. vol. I, p. 351 f.
- ⁷ Ingarden, 1931, p. 11 of Eng. trans. See also the remarks on the category type in § 5 below.
- ⁸ Husserl, 1948, § 65 (cf. Eng. trans., p. 267).
- ⁹ To this extent we can agree with Aristotle when he says (in *Categories* V, 4 a 10–21): "One and the same colour cannot be at one time dark and at another time pale; ... one and the same man can however become at one time dark and at another time pale."
- ¹⁰ Frege, 1884, § 46; 1904, p. 108 of English trans.; 1898/99, p. 173 f.
- ¹¹ Cf. Russell, 1903, § 442: "Change is the difference, in respect of truth and falsehood, between a proposition concerning an entity and a time T, and a proposition concerning the same entity and another time T', provided that these two propositions differ only by the fact that T occurs in the one where T' occurs in the other." See on this also Geach, 1969, p. 71 f.
- ¹² Plato, *Theaetetus* 155 BC; Aristotle, *Metaphysics* XIV 1, 1088 a 34–5: "... without changing, a thing will be now greater and now less or equal if that with which it is compared has changed in quantity."
- ¹³ See on this Gale, 1968, pp. 16–22 and the literature there given.
- ¹⁴ Kühne, 1979, ch. 6 contains some further discussion of Reichenbach's theory.
- ¹⁵ Cf. Aristotle, *Metaphysics* XIV 1, 1088 a 29–35; XI 12, 1068 a 11–13. We cannot at this point go into the consequences of the above investigations for the problems associated with the Aristotelian category of place.
- ¹⁶ See Dummett, 1973, p. 492.
- ¹⁷ Cf. ch. 1 of Kühne, 1979.
- ¹⁸ In Leśniewski's Mereology and in the 'Calculus of Individuals' developed by Leonard and Goodman, the part-whole relation is employed in place of the set-theoretic 'e'. Cf. Goodman, 1972, pp 178 ff., and the references there given.

- ¹⁹ Cf. our discussion of Husserl's *Experience and Judgment* at the beginning of this section.
- ²⁰ Thus we are here excluding from consideration functions in the Fregean sense (and thus also Frege's 'concepts'). Cf. ch. 3, § 4 of Kühne 1979 for a discussion of the question of how one can talk about non-objects without objectualising them.
- ²¹ Cf. Thiel, 1968, Ch. 8.
- ²² Cf. e.g. Lotze, 1879, p. 209. The 'being' of something is either (I) its 'becoming presented by us', (II) its 'being able to bring about an effect', or (III) the 'mere validity of a truth'.
- ²³ 1892, Eng. trans., p. 62, n.
- ²⁴ On this see Peter Hacker's valuable 1972.
- ²⁵ Frege, 1893, p. xviii, Eng. trans., p. 66.
- ²⁶ A 231 = B 284 (2nd Postulate of Empirical Thought).
- ²⁷ A 493 = B 521.
- ²⁸ A 226 = B 273.
- ²⁹ But cf. ch. 4, § 2 of Kühne, (1979, on a modification of this definition in Frege's 1918/19).
- ³⁰ Cf., besides the passage already cited: 1884, 26f.; 1894, p. 321 of Eng. trans.; 1903, p. 56.
- ³¹ 1897, p. 149.
- ³² Frege, 1884, Eng. trans. (slightly amended), p. 35.
- ³³ Aristotle, *Physics* VI, 10, 240b 8–10, 18–20.
- ³⁴ Cf. on this also *Metaphysics*. VII, 8, esp. 1033a 29–31.
- ³⁵ On this see the section to follow.
- ³⁶ As an abbreviation for 'm and n fulfil neither (1) nor (2) nor (3) nor (4)' we can say: 'm and n are *separatae*'.
- ³⁷ Cf. Plato, *Parmenides* 138 AB *et passim*.
- ³⁸ In the sense of Aristotle's relational explication of the concept of the continuous: *συνεχῆ μὲν ὄν τα εἴχοντα ἐν* (*Physics* VI, 1, 231a 22).
- ³⁹ See n. 1 above, and Ch. I, esp § 3 of Kühne, 1979.
- ⁴⁰ Cf. e.g. 1918/19, p. 17 of Eng. trans.
- ⁴¹ Aristotle, *Topics* 141b 20ff; Euclid, *Elements* I, Def. 3, 6; XI, Def. 2. On the significance of the category of boundary in Plato and Hegel see the appendix to Kühne, 1975.
- ⁴² The thesis which suggests itself at this point – that surfaces, too, are not *per se* perceptible – is, in the light of the phenomenon of visual after-images, at least questionable. (After-images are of course not 'objective' entities in Frege's sense).
- ⁴³ If someone on a journey were to puzzle over the fact that although he can find the rivers and streets corresponding to certain lines on a map but nothing (however much attention he pays) that would correspond to the line designated as 'equator', then he betrays a misunderstanding of a cartographical symbol.
- ⁴⁴ Entities of this type are not, we might note, captured within the conceptual framework of Husserl's *Logical Investigations*: they are not 'real' because they cannot be perceived, and they are not 'ideal' since they are no kind of 'species'.
- ⁴⁵ Husserl, 1913/22, Vol. II/1, p. 266, Eng. trans., vol. 2, p. 467.
- ⁴⁶ *Ibid.* p. 154, Eng. trans., vol. 1, p. 376.
- ⁴⁷ *Ibid.*, pp. 218f, Eng. trans., vol. 1, pp. 428f, vol. 2, pp. 435ff.
- ⁴⁸ Cf. Aubenque, 1971.
- ⁴⁹ See above, pp. 408ff.
- ⁵⁰ Husserl, 1913/22, p. 109, Eng. trans., vol. I, p. 340.
- ⁵¹ *Ibid.*, p. 155, Eng. trans., vol. I, p. 377.
- ⁵² Cf. also Plato, *Phaedo*, 102 B–D.

- ⁵³ *Categories*, I a 24–5.
- ⁵⁴ *Logical Investigation III*.
- ⁵⁵ Stout, 1921, p. 386. D. Ross alludes to this passage in his 1923, p. 24, n. 1. The just-presented interpretation of cb. 2 of the *Categories* is no longer free from controversy: cf. Owen, 1965, Allen, 1969, and Jones, 1972.
- ⁵⁶ Anscombe and Geach, 1961, pp. 7–10.
- ⁵⁷ Wittgenstein, 1958, p. 55.
- ⁵⁸ Stout says of this distinction between quality-species and quality-individual that “for the most part it is neither needful nor useful to take note of the distinction in ordinary thought or express it in ordinary language.” (op. cit.) Cf. also Williams, 1953, pp. 5–6; Wolterstorff, 1960, pp. 95–7 of reprint.
- ⁵⁹ Husserl, 1913/22, vol. II/1, p. 439, Eng. trans., vol. II, p. 606.
- ⁶⁰ *Ibid.*, p. 155, Eng. trans., vol. I, pp. 376f.
- ⁶¹ *Loc. cit.*
- ⁶² 1884, § 54, p. 66.
- ⁶³ On the manifold uses of colour-names see Künne, 1979, cb. 1, § 1.
- ⁶⁴ Seen from this point of view the language in which Stout’s theory is defended by J. R. Jones appears suspicious: the ‘red color spread over the surface’ (1951, p. 556), ‘patch of colour’, ‘extent of red’ (1949, pp. 159, 162).
- ⁶⁵ That Stout’s arguments for this conception (op. cit.) are not conclusive has been shown by G. E. Moore, 1923, pp. 25–30 of reprint.
- ⁶⁶ Thus my conclusion is in agreement with that of David Armstrong: “Every consideration of economy seems to plead for the elimination of the Particularist properties. They are a useless intermediary between ordinary particulars and their universal properties” (*Nominalism and Realism*, p. 86). Armstrong believes however that he can also demonstrate the ‘incoherence of particularism’. His argument suffers from the fact that he does not clearly separate the following three assumptions:
 (A) There exist individualised properties (‘Particularist properties’, ‘Stoutian particulars’, ‘Husserlian individual moments’).
 (B) The properties of concrete objects are not abstract entities (‘universals’), but individual moments; abstract entities which would also be properties (of the first level) do not exist (op. cit., pp. 79, 138).
 (C) Concrete objects are bundles of individualised properties (pp. 81, 86).
 Armstrong’s official characterisation of particularism is (B). Yet he asserts also that ‘Particularism could ... be combined with Realism’ (p. 85) and Realism is for him the doctrine that there exist abstract objects, in particular properties (of the first level) (p. 139 *et passim*). Such a realism could not however be combined with a particularism in the sense of (B). Armstrong admits that he knows of no philosopher who defends such a combination (p. 85). It may be that it is shyness of contradiction that has held philosophers back from it. Husserl, who seems to be unknown to Armstrong, combines Realism with (A). Aristotle would, with Armstrong, deny (C), in my opinion correctly.
- ⁶⁷ The most important works of Strawson for our purposes are his 1953/4, 1957, and 1959, Part II.
- ⁶⁸ 1959, Part I. Cf. on this Künne, 1975 a.
- ⁶⁹ 1959, pp. 168–70. Stout, too, in perfect correspondence, calls them ‘abstract particulars’ (where ‘abstract’ is used in the Husserlian sense). Cf. Stout, 1921, p. 178.
- ⁷⁰ Strawson, 1959, p. 227.
- ⁷¹ Not every unambiguous reply to the question ‘which?’ is a *definitive* answer. The hearer can e.g. always go on sensibly to ask: ‘and who is this *F* who was at place *P* at time *T*?’ On the concept of ‘definitive answer’ see Künne, 1979, ch. 4, § 7.
- ⁷² 1953/54, p. 49.

- ⁷³ 1953/54, p. 49.
- ⁷⁴ Husserl 1913/22, vol. II/1, pp. 108ff, Eng. trans., Vol. 1 pp 339ff.
- ⁷⁵ Cf. Kunne, 1979, ch. I, § 2 on μέθεξις.
- ⁷⁶ On the opposition 'particular vs. general' Strawson remarks: "In the case of propositions ... we may feel a very strong reluctance to classify in this way at all." (1953/54, p. 51).
- ⁷⁷ Cf. e.g. Husserl, 1913/22, Vol. II/1, p. 99, Eng. trans., Vol. 1, p. 329.
- ⁷⁸ Husserl, 1913/22, Vol. II/1, pp. 100f. Here Husserl applies his conception of 'individualised properties' criticised in the last section. Nevertheless the thesis that propositions are properties of acts could still be to the point even if, as we believe, this conception is itself unsupported.
- ⁷⁹ 1948, § 64 d; cf. 262 of Eng. trans.
- ⁸⁰ Husserl uses the term 'Satz', but it is clear from the context that this is to be understood as in 'der Satz des Pythagoras' (Pythagoras' theorem) i.e. to mean the same thing as 'proposition'.
- ⁸¹ In this (very peculiar) sense every entity is general: even the Eiffel Tower can after all be intended in infinitely many acts. Thus Husserl's usage here implies that the attribute of generality loses all its sharpness.
- ⁸² In his 1953/54, p. 50, Strawson himself admits of this.
- ⁸³ 1953/54, p. 50.
- ⁸⁴ See our discussion of the identity of the literary work in Kunne, 1979, Ch. 6, § 2.
- ⁸⁵ That certain types (e.g. words) are the referents of sense-determinate singular terms has been claimed already above. Cf. also Strawson, 1953/54, p. 51, n. 1.
- ⁸⁶ Cf. Strawson, 1952, p. 231.
- ⁸⁷ Cf. Dummett, 1973, pp. 143-44.
- ⁸⁸ In opposition to Strawson, 1953/54, p. 51, and 1957, pp. 443-4.
- ⁸⁹ Husserl, 1948, § 65, cf. p. 267 of Eng. trans.
- ⁹⁰ It is the shadow of Quine: see on this his 1960, p. 233, n. 2.
- ⁹¹ The assumption is defended in chs. 5 and 6 of my 1979. In the absence of this defence the result of the present essay must remain uncertain.
- ⁹² Or (see Kunne, op. cit., ch 6) under the heading 'extensionality vs. non-extensionality'.
- ⁹³ Cf. Plato, *Laches* 192 AB; *Theaetetus*, 147 C.
- ⁹⁴ Cf. *Theaetetus*, 147 Cff. (and on this Wittgenstein, 1958, p. 27). *Meno*, 75 E ff.
- ⁹⁵ Wittgenstein, 1958, pp. 19-20, cf. pp. 26f, and 1967, pp. 75f, 120f.
- ⁹⁶ 1953, § 65.

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Manifolds, Concepts and Moment Abstracta

§ 1 Introduction

In this paper we present a mathematical contribution to the resolution of certain metaphysical questions relating to the traditional notions of universals and abstraction. The main purpose of the paper is to introduce a theory of certain mathematical structures, called *manifolds*. We prove that one type of manifold has properties which have been historically ascribed to the (extensions of) least general universals. Within the Aristotelian and Thomistic hylemorphic ontology (a prototypical form of realist metaphysics) these universals were claimed to be hypostatizations of those forms which are the direct results of abstraction from sensible particulars, and were called *automon eide* or *infima species*. *Automon eide* were conceived within this tradition as those universals (secondary substances, or *eide*) which comprise least specific differences, and below which (in the hierarchy of generalisation) only individual (Aristotle's *primary*, or Aquinas' *composite*) substances (but not less general *eide*) fall. Considered ontologically as objects (rather than epistemologically as concepts), they may be referred to provisionally as *universal individuals of higher order*. Universal individuals of higher order are the individuals of the realm of *eide*.

This paper is the only contribution to the present volume conceived within the framework of the transcendental phenomenology inaugurated by Husserl in the *Ideas* of 1913, rather than within the realist framework of the earlier Husserl, Ingarden, *et al.* Within the transcendental tradition (which is heavily influenced by Kant's 'Copernican revolution' in metaphysics), metaphysics is identified with epistemology rather than with ontology. But in spite of the major differences in overall directions and concerns between the two traditions of Husserlian pheno-

menology, certain issues (including most of those confronted here) are common to both. From an *epistemological* point of view, we conceive manifolds (as defined below) as extensions of linguistically formulated concepts. We conceive concepts as intensions of unions of isomorphism types, and define a linguistically formulated concept (which we refer to as a 'predicative concept') as an intension of a manifold. By distinguishing different types of manifolds and relations between manifolds, we will therefore make possible analogous distinctions concerning predicative concepts. We shall define a linguistically formulated or predicative concept as *distinct* if and only if (iff) it is an intension of the type of manifold to be identified below as the extension of a universal individual of higher order (*automon eidos*); any concept which is an intension of any other type of manifold we refer to as *indistinct* (or, synonymously, *vague*).

We introduce the generic term 'conceptualisation' to denote processes of attention whereby universal objects of higher order are constituted as themes, and we will formulate definitions of several types of conceptualisation discussed by Husserl and others. Definitions will be formulated in terms of types of manifolds and relations between manifolds. Because of the epistemological orientation of the paper, it is concerned with the metamathematical analysis of manifolds largely as a means to achieving such formulations concerning the constitution of perceivable objects and universals as themes of attention.

The particular context within which the paper is conceived is provided by the perspective and set of problems sketched by Aron Gurwitsch toward the end of his career. Gurwitsch was originally a student of Carl Stumpf interested in Gestalt theory and the psychology of perception. At Stumpf's suggestion he became a student of Husserl's earlier works during the 1920's. Throughout his life Gurwitsch worked as a transcendental phenomenologist who expressed a preference for the first (i.e. 'realist') edition of Husserl's *Logical Investigations*, largely because the second edition was revised to reflect Husserl's 1913 introduction of the concept of *transcendental ego* and his use of it in defining attention of various types (see Gurwitsch, 1966, chs. 10, 11). In spite of his rejection of Husserl's egological formulations, Gurwitsch remained convinced that the programme of transcendental phenomenology sketched in the *Ideas* of 1913 (but to be developed in terms of a non-egological account of attention) embodied the most promising formulation of Western metaphysics to date. He also remained convinced that the

realisation of that programme demanded theoretical analysis and development of a concept introduced by Husserl in 1938, the concept of the lifeworld (*Lebenswelt*). The following quotes are selected to sketch Gurwitsch's position, and to introduce certain key notions of his non-ego-logical transcendental phenomenology in their native setting:

In recent decades the theory of science has not received sufficient attention in phenomenological literature. Thus the impression could arise that phenomenology did not have much to say in that field of research, had withdrawn from it altogether to leave it to those contemporary philosophical trends, such as logical positivism, which call themselves 'scientific'. Since the necessary preparatory work was done in Husserl's later writings, the time seems to have come for phenomenology to reclaim possession of the field from which it had its departure in Husserl's earliest writings (1974, p. 31–2).

The phenomenological theory of the sciences which Gurwitsch envisioned would be built around two fundamental themes. The first of these is the life-world:

The first presupposition of the sciences proves to be the life-world itself, our paramount and even sole reality. Whatever unity obtains among the sciences derives from their common rootedness in the life-world, from which all of them originate and to which most of them explicitly relate as their theme (ibid., p. 139).

The second fundamental theme is consciousness or, more specifically, the processes of attention involved in the perceiving and theorising which, according to the phenomenological account, hypostatise the entities studied within the cultural, formal, and especially natural sciences:

... the life-world proves to underlie and to be presupposed by the elaboration of 'objective' nature. Still, we have not yet reached the ultimate, but only the penultimate, presupposition and foundation. *The life-world, in its turn, refers to and, in that sense, presupposes mental life, acts of consciousness, especially perceptual consciousness through which it is experienced and presents itself as that which it is, that is, as that which we accept.* It is not until we arrive at consciousness as the universal medium of access (in the sense of Descartes' *Second Meditation*) to whatever exists and is valid, including the lifeworld, that our search for foundations reaches its final destination. As far as the processes of conceptualisation, idealisation, and formalisation are concerned, they now appear in their proper place as acts of consciousness of a higher order insofar as they presuppose the more elementary and more fundamental acts through which the life-world is given or, in Husserl's parlance, they are built on prepredicative experi-

ence. This is another expression of our previous conclusion that the universe of physics – objective nature as conceived by physicists – is a product of mental life constructed on the basis of the prepredicative experience of the life-world (*ibid.*, p. 58).

It is the failure to recognise the hypostatized status of the results of conceptualisation which Husserl criticised as responsible for the failure of Western metaphysics which he characterised in 1938 as the crisis of European science:

Failure to refer the accomplished products and results to the mental operations from which they derive and whose correlates they are makes one the captive of those products and results, that is, the captive of one's own creations, and that is a further aspect of traditionality. Thus, as Husserl expressed it, a cloak or tissue of ideas (*Ideenkleid*), of mathematical ideas and symbols, is cast on the life-world to conceal it to the point of being substituted for it. What in truth is a method and the result of that method come to be taken for reality. Thus we arrive at the conception of nature . . . as possessing a mathematical structure or being a mathematical manifold (*ibid.*, p. 45).

In view of such claims on the part of (non-egological) transcendental phenomenology of science, it is not surprising that the problem of conceptualisation emerges as a central issue:

. . . the first task of a phenomenological theory of the sciences is to develop a phenomenological theory of conceptualisation, that is, a phenomenological account of the transition from type to concept and eidos.

Conceptualisation is possible along two different lines of direction which Husserl has distinguished from one another under the headings 'generalisation' and 'formalisation'. In the present context, we must confine ourselves to pointing out that the theory of conceptualisation, generalisation, formalisation, and algebraisation is one of the most urgent tasks with which phenomenological research finds itself confronted at the present stage of its development . . . the problem is far from being exhausted. Not only is there ample room for further investigation, but some of Husserl's results – we submit – require revisions and modifications (*ibid.*, p. 143).

The late Husserlian notion of the life-world is an intuitive (and effectively anthropological or sociological) notion. It was held by Gurwitsch to be fundamental to metaphysics in general and to the phenomenology of conceptualisation in particular. While the term remains at present ill-defined (see Carr, 1974, Ch. 8), it may be understood as denoting the world (and also, ambiguously, culturally relative worlds) of common sense ex-

perience i.e. the milieu(s) studied in one form or another by the various sciences of culture. Within the Husserlian tradition the work of Alfred Schutz was devoted exclusively to the task of defining (i.e. removing the vagueness of) this concept by formulating invariantly shared properties of each possible world of common sense experience. While no further mention of the lifeworld and related issues will be made in this paper, we wish to mention here that we do not conceive of the life-world as the class of *actual wholes which are perceived* (as defined in § 5 below). The definitions which we shall offer in terms of manifold theory will explicitly cover certain types of conceptualisation (e.g. *formalisation* and *generalisation*) and certain structures involved in perception (e.g. *wholes as perceived, possible* and *actual wholes which are perceived*), and they will implicitly cover the notion of *lifeworld*. We omit discussion of themes like the influence of historicity upon the material content of the life-world, and of spatial, temporal, and causal relations within the life-world. However, we consider such questions both important and approachable (elsewhere) in terms of the late Husserlian notion of *lifeworld* and the results reported here.

The theory of manifolds is of particular interest in the context of the ontology of wholes and parts as developed by Husserl, and by Mulligan, Smith, and Peter Simons elsewhere in this volume. In this paper an attempt is made to formulate a connection between manifold theory and whole-part notions by interpreting the language of wholes and parts in manifold theory. One such interpretation is specified (§ 5), and the theorems of manifold theory are translated into claims concerning whole-part relations. The purpose of this translation is to present a set of claims in order that their truth may be evaluated. These claims appear conformable to intuitions expressed in Thomistic ontology and realist phenomenology concerning wholes and parts. Moreover, one claim in particular (theorem 20) expresses a fundamental Aristotelian, Thomistic, and Scholastic tenet. The evaluation of the validity of the remaining theorems is left to those engaged in eidetic phenomenology of whole-part relations, and it is hoped that this interpretation may be suggestive for such work.

We conclude this introduction with a brief statement of the preliminary assumptions, notation and terminology we adopt in our underlying set theory, logic, and metamathematics. Further metamathematical notions, including the concepts of *second-order equivalence* and *isomorphism* are defined in § 2. §§ 3–4 contain our theory of manifolds, in-

spired by Husserl (1891). § 5 develops an interpretation of whole-part notions in manifold theory so that the various theorems state properties and relations of wholes and parts of various kinds. In § 6 we consider some limitations and implications of the results presented in §§ 3–4 for the problem of conceptualisation.

The underlying set theory of the metalanguage used in this discussion is a class-set theory such as those of von Neumann, Bernays, Gödel, and A. P. Morse. We adopt set-theoretic axioms and basic definitions as in Monk (1969). The primitive (undefined) terms are 'class' and 'member' or 'element'. All other concepts of our class-set theory can be defined in terms of these primitives. E.g.: A class is a *set* iff it is a member of some other class; otherwise it is a *proper class*. Braces will be used to denote finite sets whose elements are listed within the braces, and ' \emptyset ' will denote the empty set. Also, the binary operations of *intersection* (\cap) and *union* (\cup) of classes can be defined as usual. Finally, A is a *subclass* of B ($A \subseteq B$) iff each member of A is a member of B , and A is a *proper subclass* of B ($A \subset B$) iff $A \subseteq B$ and not $B \subseteq A$. Manifold theory as we envision it in our ontological and epistemological interpretations cannot be captured within class-set theory (see § 5 *infra*). However, in this paper we will limit our discussion to those aspects of manifold theory which can be expressed in class-set theory, and which may be thought of as comprising the theory of *manifolds in extension*.

An *n-ary relation-in-extension* (or simply-*n-ary relation*) is a class of ordered *n*-tuples. An *equivalence relation* is a binary relation R satisfying the following conditions, for all elements x , y , and z to which the relation applies:

- (1) reflexivity: $\langle x, x \rangle \in R$
- (2) symmetry: if $\langle x, y \rangle \in R$, then $\langle y, x \rangle \in R$
- (3) transitivity: if $\langle x, y \rangle \in R$ and $\langle y, z \rangle \in R$, then $\langle x, z \rangle \in R$

For example, the relation R' such that $\langle x, y \rangle \in R'$ iff x and y are sets with the same number of members is an equivalence relation. An equivalence relation can be thought of as a relation of similarity between objects. Equivalence relations can thus be used to classify things by means of classes of similars. For any equivalence relation R and any element x , the *R-equivalence class* of x (also called the equivalence class of x under R) is the class of all y such that $\langle x, y \rangle \in R$. I.e., the *R-equivalence class* of x is the class of all things similar to x according to R .

The object languages used in this paper are those of *monadic second-order logics*, possessing all features of classical, first-order logic with equality (defined as a logical constant of the languages), but including also quantification over monadic predicate variables. We define an axiom set Σ as a consistent set of sentences formulated in one of these monadic second-order languages. More than one object language is used because the notion of the completeness of an axiom set Σ is defined in terms of the set of predicate constant symbols used in the sentences of Σ . The symbols which distinguish one object language from another are its predicate constants; the different object languages are thus identical, except that they differ from each other in the number and type (monadic, dyadic, etc.) of their predicate constants.

A *model* is a $(k+1)$ -tuple for some ordinal k , consisting of a non-empty set called the *universe of the model*, followed by k distinct relations on that universe. In this paper we shall further assume for each model that its universe has at least 2 elements and that for each positive integer n , the intersection of any finite number of its n -ary relations is non-empty. We assume a semantics based on Tarski (1956) for the object languages, and the notion of a sentence of an object language being true in a model iff it is satisfied by an assignment to that model (see Enderton, 1972, pp. 81–82). M is a *model of an axiom set* Σ iff each sentence of Σ is true in M . E.g., let $M_3 = \langle \{1,2,3\}, R \rangle$ where R is the usual \leq ordering on this 3-element set. Where the dyadic predicate constant ' P ' denotes \bar{K} , and the symbol ' \approx ' denotes equality defined as a logical constant of the object languages, M_3 is a model of the axiom set Σ_2 consisting of the following two sentences:

1. $\forall u \exists v \neg (u \approx v)$
2. $\exists u \forall v Pvu$

On the other hand, let N be the set of all positive integers and R' be the usual \leq ordering on N . Then $\langle N, R' \rangle$ is not a model of Σ_2 because sentence 2., asserting that there exists a greatest element, is not true in $\langle N, R' \rangle$. A model M is *compatible* with the language \mathfrak{L} iff for each positive integer n , M has exactly as many n -ary relations as \mathfrak{L} has n -adic predicate constants. We offer only the following comments concerning the syntactics of the object languages:

A *deduction system* for any object language \mathfrak{L} is a system of axioms and/or inference (transformation) rules which define derivations. We

define a *theorem of an axiom set Σ formulated within \mathfrak{L}* as any sentence of \mathfrak{L} which results from a derivation from Σ . Two properties defined on deduction systems are:

1. *Correctness*: Any deduction system is *correct* iff for all object language sentences ϕ , if ϕ is a theorem of Σ then ϕ is true in each model of Σ .
2. *Completeness*: Any deduction system is *complete* iff for all object language sentences ϕ , if ϕ is true in each model of Σ , then ϕ is a theorem of Σ .

We assume any *correct* deduction system for monadic second-order logic with identity that extends a *correct and complete deduction system* for first-order logic with identity for our object languages. Further, for any axiom set Σ consisting of sentences formulated in \mathfrak{L} , Σ is *complete* iff for each sentence ϕ of \mathfrak{L} , either ϕ or $\neg \phi$ is a theorem of Σ .

These assumptions are common conventions in contemporary logic. While space will not permit their further elaboration here, they may be found more fully described in many currently available textbooks (e.g. Enderton, 1972).

§ 2 Isomorphism Types

This section consists mainly of a brief list of definitions of the basic metamathematical concepts needed for the present work. The section concludes with three theorems establishing that isomorphism is an equivalence relation, and that isomorphism types and second-order equivalence types are proper classes. These facts (along with theorem 4) are included here in order to establish the relationship of the universe of discourse of manifold theory to that of class-set theory; they are neither proven nor used in this work.

Two models M and M' which are compatible with the same languages are said to be *isomorphic* iff there is a one-to-one function f whose domain is the universe of M , whose range is the universe of M' , and which preserves the relations of M in the following sense: the function f *preserves the relations of M* (maps M isomorphically to M') iff for each n -tuple $\langle a_1, \dots, a_n \rangle$ of elements of M and for each n -ary relation

R of M , $\langle a_1, \dots, a_n \rangle \in R$ iff $\langle f(a_1), \dots, f(a_n) \rangle \in R'$, where R' is the relation in M' corresponding to R . Roughly speaking, M and M' are isomorphic just in case there is a 1-1 correspondence between their universes and between their relations such that corresponding elements are in the corresponding relations.

The relation(-in-extension) of isomorphism is the class of all ordered pairs of isomorphic models. I.e. $\langle M, M' \rangle \in \text{isomorphism}$ iff M and M' are isomorphic. An axiom set is *categorical* iff any two models of it are isomorphic.

Theorem 1: Isomorphism is an equivalence relation.

Mathematicians refer to equivalence classes under the isomorphism relation as 'isomorphism types'. An *isomorphism type* is the class of all models isomorphic to a particular model. Isomorphism types are philosophically interesting for a number of reasons. Every manifold is a union of isomorphism types (theorem 13, *infra*), and ordinals can be defined as isomorphism types of a certain sort (*viz.* well-ordering types; see Wilder, 1952, ch. 5). Since Husserl and Gurwitsch identified formalisation as the type of conceptualisation which is involved in the hypostatisation of the finite and first transfinite ordinals, we have elected to define this type of conceptualisation in terms of isomorphism types. In view of the use we intend to make of isomorphism types, their class-theoretic status is worth stating as a theorem:

Theorem 2: Each isomorphism type is a proper class.

Theorem 2 can be proven using the set-theoretic axioms of substitution (also called 'replacement') and regularity (Monk, 1969, p. 180).

Two models M and M' are *monadic second-order equivalent* (in symbols $M \equiv_{m_2} M'$) iff exactly the same sentences are true in each of them; i.e. $M \equiv_{m_2} M'$ iff each sentence true in either one of these two models is also true in the other. M and M' are *elementarily equivalent* (in symbols $M \equiv M'$) iff exactly the same *elementary sentences* (sentences containing no predicate variables) are true in each of these models. The \equiv_{m_2} -type of a model M is the class of models which are monadic second-order equivalent to M , and the \equiv -type of a model M is the class of models which are elementarily equivalent to M .

Theorem 3: Each monadic second-order equivalence type (\equiv_{m_2} -type) is a proper class.

Theorem 3 can be proven from theorem 2 by observing that each \equiv_{m_2} -type contains an isomorphism type as a subclass. We include the statements of theorems 2 and 3 in order to make it clear that any theory of isomorphism types, \equiv_{m_2} -types, and manifolds must make statements attributing properties to and imposing relations on proper classes. We now develop some fundamental notions of one such theory.

§ 3 Manifolds

This section deals primarily with one (extensional) version of the theory of manifolds, a theory similar to (and inspired by) the theory of manifolds developed by Edmund Husserl in *Philosophie der Arithmetik* and subsequent works. The following quotes, taken from the (amended) English translation of *Formale und transzendente Logik*, sketch the notions of *manifold* and *definite manifold* developed first in *Philosophie der Arithmetik*.

The first passage introduces Husserl's notion, and shows clearly that he had a metamathematical context in mind when speaking of manifolds; the relationship between what he called 'formal apophantics' (the study of 'judgment-forms, . . . proof forms, . . . *judgment-systems in their entirety*') and ('*auf der gegenständlichen Seite*') 'formal ontology' (the study of 'any objects whatever, any set and any set-relationship whatever; any combinations, orders, magnitudes, or the like, . . . of objectual totalities (manifolds)') is analogous (to say the least) to the metamathematical relationship between logic and axiomatics on the one hand, and model theory on the other.

. . . a beginning was found here for a theory of deductive systems or, in other words, a logical discipline relating to the deductive sciences as such and considered as theoretical *wholes*. As the earlier level of logic had taken for its theme the pure forms of all meaning formulations that, as a matter of *a priori* possibility, can occur within a science: judgment-forms (and the forms of their elements), argument-forms, proof-forms – correlatively (on the objectual side): any objects whatever, any set and any set-relationship whatever; any combinations, orders, magnitudes, or the like, with the pertinent formal essential relationships and connections. So now *judgment-systems in their entirety* become the theme – sys-

tems each of which makes up the unity of a possible deductive theory, a (possible) 'theory in the strict sense'. As the concept of an objectual totality (a concept always understood in formal generality), there appears here that which mathematics, without any explicative determination of its sense, has in mind under the name '*Mannigfaltigkeit* (manifold)'. It is the form-concept of the object-realm of a deductive science, this being thought of as a systematic or total unity of theory (§ 28).

Husserl's references to the originator of differential geometry as the source of the concept of *manifold* leave no doubt concerning the meta-mathematical nature of the notion:

The great advance of modern mathematics, particularly as developed by Riemann and his successors, consists not merely in its having made clear to itself the possibility of going back in this manner to the form of a deductive system . . . but rather in its having also gone on to view *such system-forms themselves as mathematical objects* . . . (§ 30).

Where Husserl appears to have been working with an inadequate distinction between the manifold and a model of an axiom set, our definitions will respect this distinction, and may therefore be viewed as a slight but perhaps useful variation on Husserl's work. We are nevertheless able to establish enough of the properties attributed by Husserl to manifolds that we believe the definitions proposed here clarify his own ideas. To see this relationship between our results and those of Husserl, it is useful to consider some of his comments regarding the type of manifold which he characterised as *definite*:

The tendency towards a preeminent version of the mathematical concept of the manifold (and therefore toward one particular aim in the theory of manifolds) was determined by the Euclidean ideal. I attempted to give that version concrete formulation in the *concept of the definite manifold*.

The hidden origin of this concept, which, it seems to me, has continually guided mathematics from within, is as follows. If we imagine the *Euclidean ideal* as realised, then the whole infinite system of space-geometry would be derivable from an irreducible finite system of axioms by purely syllogistic deduction (that is to say, according to the principles of the lower level of logic); and thus the *a priori* essence of space would be *capable of becoming completely disclosed in a theory*. The transition to form thus yields the form-idea of any manifold whatever that, conceived as subject to an axiom-system with the *form* derived from the Euclidean axiom-system by formalisation, could be *completely explained nomologically*, and indeed in a deductive theory that would be (as I used to express it in my Göttingen lectures) 'equiform' with geometry. If a manifold of indeterminate generality is conceived from the start as defined by such a system of forms

of axioms – if it is conceived as determined exclusively thereby – then the wholly determinate system of the forms belonging to the theorems and component theories, and ultimately the whole *science-form* necessarily valid for such a manifold, can be derived by pure deduction. Naturally all the concretely exhibited material manifolds subject to axiom-systems that, on being formalised, turn out to be equiform, have the same deductive science-form in common; they are equiform precisely in relation to this deductive science-form (§ 31).

It may be noted that the term ‘equiform’ used by Husserl is the Latin translation of the Greek ‘isomorphic’, and that the notions of isomorphism and formalisation play fundamental roles in Husserl’s conception of the definite manifold. However, in view of the references made by Husserl to Hilbert in these contexts, and of Husserl’s suggestions concerning the equivalence of the property of completeness of an axiom set and the definiteness of its manifold, we have elected to use a more general notion than isomorphism in defining the property of Husserl-definiteness:

In proceeding from such considerations of the peculiar nature of a nomological object-realm to formalisation, there was yielded that which is pre-eminently distinctive of a *manifold-form in the pregnant sense*, i. e. in the sense of a form that is nomologically explicative. Such a manifold-form is defined *not by just any formal axiom-system* but by a ‘complete’ one. Reduced to the precise form of the concept of the definite manifold, this implies:

That the axiom-system formally defining such a manifold is distinguished by this, that every proposition (proposition-form, of course) that can be constructed, in accordance with the grammar of pure logic, out of the concepts (concept-forms) occurring in that system, is either ‘true’, i. e. is an analytic (purely deductive) consequence of the axioms, or ‘false’, i. e. is an analytic contradiction: *tertium non datur* . . .

Throughout the present exposition I have used the expression ‘complete system of axioms’, which was not mine originally but derives from Hilbert . . . The analyses given above should make it clear that the inmost motives that guided him mathematically were, even though inexplicitly, tending essentially in the same direction as those that determined the concept of the definite manifold (§ 31).

In the development of the definitions of the present section, we have chosen to take as fundamental Husserl’s claim concerning the definiteness of the manifold of any complete axiom set, and will therefore define definite manifolds as $=_{m_2}$ -types, rather than as isomorphism types. However, it will be found that some Husserl-definite manifolds (including the type which are most philosophically significant; see theorem 22, *infra*) are isomorphism types.

In this section we develop definitions of *manifold*, *Husserl-definite manifold*, and *formal manifold*, as well as theorems stating several properties of these manifolds. Theorem 4 states that each manifold is a proper class. Theorems 5 through 10 deal with the fundamental properties which Husserl suggested hold for definite manifolds, most notably that a manifold of an axiom set is definite iff the axiom set is complete. The “if” direction of this property is established in theorem 6. Theorem 8, however, provides a counter-example showing that the “only if” direction does not hold. Theorem 9 is a weaker version of the “only if” condition which does hold, and theorem 10 establishes the full strength of Husserl’s property for first-order logic.

The *manifold of an axiom set* Σ (denoted by ‘ $\mathfrak{M}\Sigma$ ’) is the class of all models of Σ which are compatible with the language whose only predicate constants are equality and those appearing in at least one sentence of Σ .

Theorem 4: Each manifold is a proper class.

Theorem 4 can be proved from theorem 2 by observing that each \equiv_{m_2} -type contains an isomorphism type as a subclass.

For any manifold \mathfrak{M} , \mathfrak{M} is *Husserl-definite* iff \mathfrak{M} is an $=_{m_2}$ -type. In the following theorems we prove several of the fundamental properties claimed by Husserl (1891) for Husserl definite manifolds.

Theorems 5: Each manifold which is an isomorphism type is Husserl-definite.

Proof: Let $\mathfrak{M}\Sigma$ be any manifold which is an isomorphism type. We shall show that $\mathfrak{M}\Sigma$ is an \equiv_{m_2} -type by showing that (1) for each model M in $\mathfrak{M}\Sigma$, and each model M' , if $M' \equiv_{m_2} M$, then $M' \in \mathfrak{M}\Sigma$; and (2) if $M \in \mathfrak{M}\Sigma$ and $M' \in \mathfrak{M}\Sigma$, then $M \equiv_{m_2} M'$. The hypotheses of (1) imply that every sentence of Σ is true in M and that M' has the same true sentences as M . Thus M' is a model of Σ and the conclusion of (1) follows. The hypotheses of (2) along with the assumption that $\mathfrak{M}\Sigma$ is an isomorphism type imply that M and M' are isomorphic. Thus $M \equiv_{m_2} M'$. Consequently $\mathfrak{M}\Sigma$ is an $=_{m_2}$ -type, and hence a Husserl-definite manifold.

We define any manifold which is an isomorphism type as a *formal manifold*. Thus, some Husserl-definite manifolds are formal manifolds, and every formal manifold is Husserl-definite by theorem 5.

Theorem 6: For each complete axiom set Σ , the manifold $\mathfrak{M}\Sigma$ is Husserl-definite.

Proof: Let Σ be any complete axiom set. Again, we show that $\mathfrak{M}\Sigma$ is an \equiv_{m_2} -type by proving that (1) for each model $M \in \mathfrak{M}\Sigma$, if $M' \equiv_{m_2} M$ then $M' \in \mathfrak{M}\Sigma$; and (2) if $M \in \mathfrak{M}\Sigma$ and $M' \in \mathfrak{M}\Sigma$, then $M \equiv_{m_2} M'$. As in the preceding proof, (1) is clear since its hypotheses imply that every sentence of Σ is true in M and that M' has the same true sentences as M . To prove (2), observe that the hypotheses imply that each sentence in Σ is true in M as well as in M' . Since Σ is complete, for each sentence ϕ of the language of Σ , either ϕ or $\neg \phi$ is a theorem of Σ . Thus by the correctness property, either ϕ or $\neg \phi$ is true in every model of Σ . Hence M and M' have the same true sentences, and thus $M \equiv_{m_2} M'$.

Lemma 7: For any axiom set Σ , if Σ is categorical, then $\mathfrak{M}\Sigma$ is a Husserl-definite, formal manifold.

Proof: Let Σ be any categorical axiom set. Since Σ is categorical, $\mathfrak{M}\Sigma$ is a manifold and an isomorphism type. The result now follows by theorem 5.

Husserl claimed the converse of theorem 6 (see preceding quote), and seemed to suggest theorem 6 as properties of definite manifolds. But using Gödel's incompleteness theorem of 1931, we show that the converse of theorem 6 does *not* hold:

Theorem 8: There exists a (categorical) set Σ of monadic second-order sentences such that the manifold $\mathfrak{M}\Sigma$ is Husserl-definite, but Σ is not complete.

Proof: Let Σ_1 be the set of the following seven axioms for the non-negative integers with ' $<$ ' and ' S ' respectively de-

noting the *less than relation* and the *successor operation*.

- Axiom 1:** $\forall u \forall v \{ \neg (u \approx v) \rightarrow [(u < v) \vee (v < u)] \}$
- Axiom 2:** $\forall u \forall v [(u < v) \rightarrow \neg (u \approx v)]$
- Axiom 3:** $\forall u \forall v \forall w \{ [(u < v) \wedge (v < w)] \rightarrow (u < w) \}$
- Axiom 4:** $\forall V \{ \exists u \forall u \rightarrow \exists v \{ Vv \wedge \forall w \{ Vw \rightarrow [(v < w) \vee (v \approx w)] \} \} \}$
- Axiom 5:** $\neg \exists u \forall v [(v < u) \vee (v \approx u)]$
- Axiom 6:** $\forall u \{ \exists v (v < u) \rightarrow \exists u_1 \{ (u_1 < u) \wedge \neg \exists u_2 [(u_1 < u_2) \wedge (u_2 < u)] \} \}$
- Axiom 7:** $\forall u \forall v \{ (Su \approx v) \leftrightarrow \{ (u < v) \wedge \neg \exists w [(u < w) \wedge (w < v)] \} \}$

Let Σ_2 be a set of axioms for addition and multiplication of natural numbers, such as axioms M1, M2, E1, and E2 in Enderton (1972, p. 194). And let $\Sigma = \Sigma_1 \cup \Sigma_2$. Σ is categorical, and hence $\mathfrak{M}\Sigma$ is Husserl-definite. But Σ is not complete, by Gödel's incompleteness theorem (1931). In effect, since $\mathfrak{M}\Sigma$ is Husserl-definite, there is a complete set Σ' of sentences such that $\mathfrak{M}\Sigma = \mathfrak{M}\Sigma'$. (E.g. let Σ' be the set of all sentences true in some particular model of Σ). Moreover, $\Sigma \subseteq \Sigma'$, but their *equality* would be equivalent to a completeness theorem, which does not hold in monadic second-order logic.

Nevertheless, we are able to provide the following weaker relationship between the definiteness of a manifold and the completeness of some axiom set for it:

Theorem 9: For all manifolds \mathfrak{M} , if \mathfrak{M} is Husserl-definite, then there is some perhaps infinite axiom set Σ such that \mathfrak{M} is the manifold of Σ and Σ is complete.

Proof: Assume \mathfrak{M} is a Husserl-definite manifold. For any model $M \in \mathfrak{M}$, let Σ be the set of all sentences which are true in M . Then for each sentence φ of the language compatible with M , either $\varphi \in \Sigma$ or $\neg \varphi \in \Sigma$. Thus Σ is complete. To show that $\mathfrak{M} = \mathfrak{M}\Sigma$, observe that \mathfrak{M} is the

set of all models which are monadic second-order equivalent to M , and hence $\overline{\mathfrak{M}}$ is the set of all models which have the same true sentences as M .

It should be pointed out that Husserl's comments indicate that he never considered the notion of an infinite axiom set, and would probably have found it philosophically unsatisfying. This development of manifold theory is based on monadic second-order object languages. Such languages are used here for two reasons: (1) Husserl seemed to be conceiving of 2nd-order entities in his concern with some objects of higher order, and (2) we intend to apply these results to a (noetic) foundation of mathematics which will be based on second-order axioms of well-orderings (see now Null and Simons, 1981). The proof of theorem 8 depends on the use of a second-order sentence (axiom 4). Moreover, if we restrict to first-order languages, it can be proven that an axiom set is complete iff its manifold is definite.

A *first-order Husserl-definite manifold* is a manifold which is an equivalence type under the relation of elementary equivalence (\equiv). Also, a *first-order axiom set* is an axiom set containing no sentence in which a predicate variable occurs, and a *first-order complete axiom set* is a first-order axiom set Σ such that for each first-order (elementary) sentence ϕ of the language of Σ , either ϕ or $\neg \phi$ is a theorem of Σ .

Theorem 10: For each first-order axiom set Σ , $\overline{\mathfrak{M}}\Sigma$ is a first-order, Husserl-definite manifold iff Σ is first-order complete.

Proof: Let Σ be any first-order axiom set. First, assume that $\overline{\mathfrak{M}}\Sigma$ is a first-order, Husserl-definite manifold containing the model M . I.e. all models of Σ are elementarily equivalent to M . Let ϕ be any first-order sentence formulated in the language of Σ , and clearly either ϕ is true in M or $\neg \phi$ is true in M . Hence either ϕ is true in each model of Σ or $\neg \phi$ is true in each model of Σ . Now, by the completeness theorem of first-order logic, either ϕ or $\neg \phi$ is a theorem of Σ . Thus Σ is first-order complete. Secondly, we must show that if Σ is first-order complete, then $\overline{\mathfrak{M}}\Sigma$ is a first-order, Husserl-definite manifold. This can be proven by the argument used for theorem 6, with ' $\equiv_{\mathfrak{M}}$ ' replaced by ' \equiv ' and

terms like 'sentence' and 'complete' referring to their first-order cases.

We claim that theorems 5 through 10 show that we have formulated a close approximation to the notion of *definite manifold* which Husserl had in mind, and that the disparity between our description and his shown by theorems 8 through 10 is a result of the correction of his assumption of completeness of second-order logic. Some further disparities may be expected as a result of our distinction between *models* and *manifolds* of axiom sets. A further property (which we shall refer to as the *automon eidos* property) claimed by Husserl for definite manifolds will be proven (along with certain other properties) in the next section.

§ 4 Further Aspects of Manifold Theory

This section begins with a lemma establishing a technical fact about equivalence classes which is to be used only in proving theorems 12 and 13. Theorems 12 through 24 state important relationships among manifolds, Husserl-definite manifolds, isomorphism types, and certain types of models. They climax with theorems 18–24, which indicate that one type of Husserl-definite manifold is a mathematical structure which exhibits properties historically ascribed (within the Aristotelian and Thomistic traditions) to least general universals (i.e. to *automon eide*).

Lemma 11: Let \mathfrak{M} be any class and E be any equivalence relation defined on \mathfrak{M} . For each $M \in \mathfrak{M}$, let E_M be the equivalence class under E which contains M as a member. If for each $M \in \mathfrak{M}$, E_M is a subclass of \mathfrak{M} , then \mathfrak{M} equals an E -equivalence class or a union of at least two disjoint E -equivalence classes.

Proof: Let U be the union of all E_M such that $M \in \mathfrak{M}$. Clearly, each $M \in \mathfrak{M}$ is a member of its E_M , and thus $\mathfrak{M} \subseteq U$. By hypothesis each $E_M \subseteq \mathfrak{M}$, and thus the union U of all those E_M 's is a subclass of \mathfrak{M} . Hence $U = \mathfrak{M}$. If all those E -equivalence classes are equal, $\mathfrak{M} =$ that E -equivalence class. Otherwise U (and hence \mathfrak{M}) can be written as a union of *disjoint* equivalence classes (because two equivalence classes are either equal or disjoint).

The next two theorems are proven by applying Lemma 11, using \equiv_{m^2} -equivalence and isomorphism, respectively, as the equivalence relation E .

Theorem 12: Each manifold is either Husserl-definite or is a union of at least two disjoint Husserl-definite manifolds.

Proof: Given any manifold $\mathfrak{M}\Sigma$ and any $M \in \mathfrak{M}\Sigma$, let \mathfrak{M}_M be the Husserl-definite manifold containing M , and we shall show that $\mathfrak{M}_M \subseteq \mathfrak{M}\Sigma$. Let M' be any member of \mathfrak{M}_M . Then $M \equiv_{m^2} M'$, and hence M and M' have the same true sentences. Thus, since M is a model of Σ , M' is also. I.e. $M' \in \mathfrak{M}\Sigma$, establishing that each \mathfrak{M}_M is a subclass of $\mathfrak{M}\Sigma$. The conclusion now follows by lemma 11.

Theorem 13: Each manifold is either an isomorphism type or a union of (disjoint) isomorphism types.

Proof: Let M be any member of the manifold $\mathfrak{M}\Sigma$. Let I_M be the isomorphism type containing M as an element. We shall show that $I_M \subseteq \mathfrak{M}\Sigma$. Let M' be any member of I_M . Then M and M' are isomorphic, and thus they have the same true sentences. Hence, since M is a model of Σ , M' is also, and $M' \in \mathfrak{M}\Sigma$. Thus $I_M \subseteq \mathfrak{M}\Sigma$ and the conclusion follows by lemma 11.

Theorem 14: (a) There exists an isomorphism type which is not a formal manifold; (b) there exists a manifold which is neither a formal manifold nor a union of formal manifolds; (c) there exists a Husserl-definite manifold which is neither a formal manifold nor a union of formal manifolds.

Proof: Let $\Sigma_{w.o.}$ consist of axioms 1–4 in the proof of theorem 8. Then $\mathfrak{M}\Sigma_{w.o.}$ (the manifold of well-ordered models) has an isomorphism type for each ordinal number, and each of those isomorphism types is a subset of $\mathfrak{M}\Sigma_{w.o.}$. In particular, $\mathfrak{M}\Sigma_{w.o.}$ contains more than 2^{\aleph_0} isomorphism types. However, there are only \aleph_0 many sen-

tences in the language of $\Sigma_{w.o.}$ and thus at most 2^{\aleph_0} many sets of sentences. Thus at most 2^{\aleph_0} many of the isomorphism types contained in $\overline{\mathfrak{M}}\Sigma_{w.o.}$ can be manifolds of the form $\mathfrak{M}\Sigma$ for some axiom set Σ in the same language as $\Sigma_{w.o.}$. I.e. at most 2^{\aleph_0} many isomorphism types of $\mathfrak{M}\Sigma_{w.o.}$ are formal manifolds. Thus the manifold $\mathfrak{M}\Sigma_{w.o.}$ contains more isomorphism types than formal manifolds, which means that some isomorphism type I of $\mathfrak{M}\Sigma_{w.o.}$ is not a formal manifold, establishing (a). By theorem 13, $\mathfrak{M}\Sigma_{w.o.}$ is the union of its isomorphism types. Since distinct isomorphism types are disjoint, the members of I are absent from the union of the formal manifolds of $\mathfrak{M}\Sigma_{w.o.}$. Thus $\mathfrak{M}\Sigma_{w.o.}$ is not a union of formal manifolds of $\mathfrak{M}\Sigma_{w.o.}$. Thus $\overline{\mathfrak{M}}\Sigma_{w.o.}$ is not a union of formal manifolds, completing the proof of (b). To prove (c), let M be any model in I and let \mathfrak{M} be the Husserl-definite manifold containing M . By a proof similar to that of theorem 13, we can show that $I \subseteq \mathfrak{M}$. Since I is not a manifold, $\mathfrak{M} \neq I$. Thus $I \subset \mathfrak{M}$ and by theorem 13, \mathfrak{M} is a union of its isomorphism types. Again, these isomorphism types are disjoint, and hence the members of I are absent from the union of the formal manifolds of \mathfrak{M} , establishing (c).

Theorem 15: Each isomorphism type is a subclass of some Husserl-definite manifold, but is not necessarily itself a manifold.

Proof: Let I be any isomorphism type, let M be any model in I , and let \mathfrak{M} be the Husserl-definite manifold containing M . We shall show that $I \subseteq \mathfrak{M}$. Let M' be any member of I . Then M and M' are isomorphic, and hence have the same true sentences. I.e. $M \equiv_{\text{true}} M'$, and thus $M' \in \mathfrak{M}$, establishing that $I \subseteq \mathfrak{M}$. Moreover, by theorem 14 (a), I is not necessarily a manifold.

An *expansion* of a model M is a model M' having the same universe as M such that the set of relations on M is a proper subset of the set of relations on M' , and a *reduct* of a model M' is a model M such that M' is an

expansion of M . Let \mathfrak{M}_1 and \mathfrak{M}_2 be any manifolds. First, \mathfrak{M}_1 is a *generalisation of \mathfrak{M}_2 by weakening axioms* iff $\mathfrak{M}_2 \subset \mathfrak{M}_1$. In such a case, Σ_1 is *weaker than Σ_2* in the sense that each sentence of Σ_1 is a consequence of Σ_2 , while some sentence of Σ_2 is not a consequence of Σ_1 . Secondly, \mathfrak{M}_1 is a *generalisation of \mathfrak{M}_2 by removals* iff there exist two sets of sentences Σ_1 and Σ_2 such that $\mathfrak{M}_1 = \mathfrak{M}\Sigma_1$, $\mathfrak{M}_2 = \mathfrak{M}\Sigma_2$, $\Sigma_1 \subset \Sigma_2$, and each model in \mathfrak{M}_2 is an expansion of some model in \mathfrak{M}_1 . This style of generalising \mathfrak{M}_2 involves removing axioms from an axiom set Σ_2 for \mathfrak{M}_2 and removing relations from each model in \mathfrak{M}_2 along with corresponding predicates from the language of Σ_2 . Last, we define \mathfrak{M}_1 as a *generalisation of \mathfrak{M}_2* iff it is one or the other kind of generalisation defined above, or there exists a manifold \mathfrak{M}_3 such that \mathfrak{M}_3 is a generalisation of \mathfrak{M}_2 by removals, and \mathfrak{M}_1 is a generalisation of \mathfrak{M}_2 by weakening axioms.

Theorem 16: For each isomorphism type I there exists a Husserl-definite manifold \mathfrak{M} such that I is a subclass of \mathfrak{M} and of every generalisation of \mathfrak{M} by weakening axioms.

Proof: Consider any isomorphism type I . By theorem 15, I is a subclass of some Husserl-definite manifold \mathfrak{M} . For each generalisation by weakening axioms \mathfrak{M}' of \mathfrak{M} , we have $\mathfrak{M} \subset \mathfrak{M}'$, and hence $I \subseteq \mathfrak{M}'$. Thus I is a subclass of \mathfrak{M} and of each generalisation of \mathfrak{M} by weakening axioms.

Theorem 17: Each Husserl-definite manifold can be generalised by weakening axioms to a manifold (which by the next theorem is not Husserl-definite).

Proof: Let $\mathfrak{M}\Sigma$ be any Husserl-definite manifold. By vacuous quantification, for each model M , every axiom in the null axiom set is true in M . Thus each model is a model of the null axiom set Σ_\emptyset . Hence $\mathfrak{M}\Sigma \subseteq \mathfrak{M}\Sigma_\emptyset$. Moreover, for each language, there exist models M and M' compatible with that language such that $M \neq_{m_2} M'$. Thus $\mathfrak{M}\Sigma_\emptyset$ is not an $=_{m_2}$ -type, and hence $\mathfrak{M}\Sigma \subset \mathfrak{M}\Sigma_\emptyset$.

In view of the proof tactic for theorem 17, it is worth pointing out that there are also manifolds $\mathfrak{M}\Sigma'$ such that $\mathfrak{M}\Sigma'$ is a generalisation by weak-

ening axioms of $\mathfrak{M}\Sigma$ and $\mathfrak{M}\Sigma_{\emptyset}$ is a generalisation by weakening axioms of $\mathfrak{M}\Sigma'$.

Theorem 18: A manifold is Husserl-definite iff there exists no manifold of which it is a generalisation by weakening axioms.

Proof: First, assume that \mathfrak{M} is a Husserl-definite manifold, and for a *reductio ad absurdum* proof, assume that \mathfrak{M} is a generalisation by weakening axioms of the manifold $\mathfrak{M}\Sigma$. Thus $\mathfrak{M}\Sigma \subset \mathfrak{M}$. Since each model M in $\mathfrak{M}\Sigma$ is a member of \mathfrak{M} and \mathfrak{M} is a \equiv_{m_2} -type, M is \equiv_{m_2} to every model in \mathfrak{M} . Thus each model in \mathfrak{M} has the same true sentences as M , which is a model of Σ . Hence each model in \mathfrak{M} is a model in $\mathfrak{M}\Sigma$. I.e. $\mathfrak{M} \subseteq \mathfrak{M}\Sigma$, contradicting the *reductio* assumption that $\mathfrak{M}\Sigma \subset \mathfrak{M}$. Secondly, assume \mathfrak{M} is a manifold which is not the generalisation by weakening axioms of any manifold. For a *reductio ad absurdum* proof, assume \mathfrak{M} is not a Husserl-definite manifold. Then by theorem 12, \mathfrak{M} is a union of at least two Husserl-definite manifolds. If \mathfrak{M}' is one of them then $\mathfrak{M}' \subset \mathfrak{M}$, and thus \mathfrak{M} is a generalisation of \mathfrak{M}' by weakening axioms, contradicting the assumption.

Define a *fully expanded model* as a model M such that for each positive integer n , the set S_n of all n -ary relations in M is an *ultrafilter*. Such an S_n is an *ultrafilter* iff:

- (i) $S_n \neq \emptyset$
- (ii) $\emptyset \notin S_n$
- (iii) Whenever $R \in S_n$ and $R' \in S_n$; $R \cap R' \in S_n$
- (iv) Whenever $R \in S_n$, $R \subseteq R'$ and R' is an n -ary relation on U then $R' \in S_n$
- (v) Whenever R is an n -ary relation on U then either R or its complement is a member of S_n (where the complement of $R = \{(a_1, \dots, a_n) \mid \text{each } a_i \in U \text{ and } (a_1, \dots, a_n) \notin R\}$).

Lemma 19: (a) No fully expanded model has an expansion.
 (b) Every model is fully expanded or has an expansion which is.

Proof:

(a) Let M be any fully expanded model. Suppose we attempt to expand M to M' by including the additional n -ary relation R . If R is not a relation of M , then the complement of R is a relation of M . Thus M' has two relations, R and its complement, whose intersection is empty. Hence, M' is not a model by our definition. If R is a relation of M , then the relation R appears twice in M' . Such an M' does not fit our definition of a model, because not all of its relations are distinct.

(b) Let M be any model which is not fully expanded. Let M_2 be a model defined as follows: (1) M and M_2 have the same universe; (2) for each n , M_2 contains the intersection of any finite number of n -ary relations of M , and (3) M_2 contains each relation which is defined on its universe and which includes at least one of the above intersections as a subset. Then M_2 is M or an expansion of M . Moreover, for each n , the set of all n -ary relations of M_2 is either empty or satisfies properties (i) – (iv) of the definition of an ultrafilter and hence is a ‘filter’. By the ultrafilter theorem (Kopperman, 1972, p. 76) each filter is a subset of some ultrafilter. Now let M_3 be a model with the same universe as M_2 obtained as follows: For each m , select an ultrafilter F_n containing all n -ary relations of M_2 , and let M_3 contain all relations in F_n . Then M_3 is a full expansion which is an expansion of M_2 and hence of M .

A *manifold of full expansions* is a manifold, each model of which is a fully expanded model. We now state the *automon eidos* property claimed by Husserl (1970, p. 473, lines 8–23) to hold for definite manifolds:

Theorem: 20: A manifold is a Husserl-definite manifold of full expansions iff there exists no manifold of which it is a generalisation.

Proof: First, let \mathfrak{M} be any Husserl-definite manifold of full expansions. By lemma 19, no model in \mathfrak{M} has an expansion. Thus there is no manifold \mathfrak{M}_2 of which \mathfrak{M} is a generalisation by removals, and by theorem 18, there is no manifold of which \mathfrak{M} is a generalisation.

Secondly, let $\mathfrak{M}\Sigma$ be any manifold which is not a generalisation of any other. By theorem 18, $\mathfrak{M}\Sigma$ is a Husserl-definite manifold. For a *reductio ad absurdum* proof, assume that some model M in $\mathfrak{M}\Sigma$ is not fully expanded. By lemma 19, there exists an expansion M_2 of M . Let Σ_2 be the set of all sentences true in M_2 , and let Σ_1 be the set of all sentences true in M . Since M_2 is an expansion of M , $\Sigma_1 \subset \Sigma_2$. Moreover, since $M \in \mathfrak{M}\Sigma$, all sentences of Σ are true in M . I.e. $\Sigma \subseteq \Sigma_1$, and hence $\Sigma \subset \Sigma_2$. In order to show that $\mathfrak{M}\Sigma$ is a generalisation of $\mathfrak{M}\Sigma_2$, it remains only to establish that each model in $\mathfrak{M}\Sigma_2$ is an expansion of some model in $\mathfrak{M}\Sigma$. Let M_3 be any model in $\mathfrak{M}\Sigma_2$. Then each sentence in Σ_2 , and hence each sentence in Σ , is true in M_3 . Let M_4 be the reduct of M_3 whose relations are all and only the denotations in M_3 of the predicates of Σ . It follows that $M_4 \in \mathfrak{M}\Sigma$. Hence $\mathfrak{M}\Sigma$ is a generalisation of $\mathfrak{M}\Sigma_2$, contradicting the assumption.

Theorem 21: Each manifold which contains a countable model is a Husserl-definite manifold of full expansions or is a generalisation of one.

Proof: Let $\mathfrak{M}\Sigma$ be any manifold which contains a countable model M . Let M_2 be a full expansion which is either M itself or any expansion of M , and let Σ_2 be the set of all sentences true in M_2 . Clearly $\Sigma \subseteq \Sigma_2$. We show that Σ_2 is categorical by considering cases depending on whether the universe of M is (1) finite or (2) denumerable. In case (1), for each pair of relations in M_2 , there are predicates P_1 and P_2 denoting those relations in the language of Σ_2 . There is a sentence of Σ_2 stating that there are exactly k members of the universe, and indicating exactly which n -tuples do and do not satisfy P_1 and which m -tuples do and do not satisfy P_2 . Any models in which all of these kinds of sentences are true must be isomorphic to M_2 . Thus Σ_2 is categorical.

In case (2), we consider subcases depending on whether there is a relation R in M_2 which is a strict well-ordering of the universe U of M (i.e. such that $\langle U, R \rangle$

is isomorphic to $\langle N, < \rangle$ where N is the set of natural numbers). If there is such an R in M_2 , let P denote R , and for each natural number n , there is a formula uniquely describing the n^{th} element in the ordering R . For example, the following formula uniquely describes the third element (in that only the assignment of the third element to u satisfies the formula in M_2):

$$\exists v \langle \neg \exists w Pvw \wedge \exists v_2 \{ Pvv_2 \wedge Pv_2u \wedge \forall w [(Pvw \wedge Pwu) \rightarrow (w \approx v_2)] \} \rangle$$

Thus each element of U is uniquely described by a formula in the language of Σ_2 . In the second subcase, there is no such R in M_2 . Since the set of binary relations of M_2 is an ultrafilter, there is a relation R of M_2 whose complement $R' = \{ \langle x, y \rangle \mid x \in U, y \in U, \text{ and } \langle x, y, \rangle \notin R \}$ is a strict well-ordering of U . Let P denote R . A formula uniquely describing the n^{th} element of R' can be obtained from the one used in the previous subcase by replacing all occurrences of ' P ' with ' $\neg P$ '. In either subcase, each element of U is uniquely described by a formula in the language of Σ_2 . Σ_2 contains sentences which specify, using such formulas, any n -tuple of members of the universe of M_2 (for any n) and indicate whether or not that n -tuple is a member of any particular n -ary relation of M_2 . Σ_2 also contains sentences (like axioms 1–6 in the proof of theorem 8) which state that P (or the complement of P) is a well-ordering isomorphic to $\langle N, < \rangle$. Hence, every model of Σ_2 is isomorphic to M_2 .

Thus in both cases, Σ_2 is categorical. Hence, by lemma 7, $\mathfrak{M}\Sigma_2$ is Husserl-definite. Also, each model in $\mathfrak{M}\Sigma_2$, being isomorphic to M_2 , is fully expanded. Since $\Sigma \subseteq \Sigma_2$, $\mathfrak{M}\Sigma$ is either equal to or a generalisation of $\mathfrak{M}\Sigma_2$, which is a Husserl-definite manifold of full expansions.

Theorem 22: Each Husserl-definite manifold of full expansions is a formal manifold.

Proof:

Let \mathfrak{M} be any Husserl definite manifold of full expansions. Let M and M_2 be any models in $\overline{\mathfrak{M}}$, and we must show that they are isomorphic. Let Σ be the set of all sentences true in M . Then $\mathfrak{M}\Sigma \subseteq \mathfrak{M}$ and by theorem 18, $\mathfrak{M}\Sigma = \mathfrak{M}$. We show, by two cases, that each member of the universe U of M is uniquely described by a formula in the language of Σ . For the first case we assume that *no* unary relation of M is a singleton. Since the set of unary relations of M is an ultrafilter, the complement of each singleton $\{x\}$, where $x \in U$, is a relation of M denoted by P_x in Σ . Then $\neg P_x u$ is a formula uniquely describing x . In the second case there is a singleton unary relation $\{y\}$ in M . For each $x \neq y$, define P_x exactly as in the first case, and define P_y as the predicate denoting $\{y\}$. Clearly, each member of U is uniquely described by a formula in the language of Σ in this case as well as in the first case. Let f be the function mapping each member of U to the member of the universe U_2 of M_2 which satisfies the same formula. Since every model in \mathfrak{M} is fully expanded, each member of U_2 must satisfy one of these formulas. Thus the function f maps onto the universe of M_2 . Moreover, for each n , each n -tuple is uniquely described by a conjunction of the above kinds of formulas. Thus, for each n -tuple and each n -ary relation, there is a sentence of Σ stating whether or not that n -tuple satisfies that relation. These sentences of Σ assure that f is an isomorphism. Hence M is isomorphic to M_2 .

Theorem 23: There exists a Husserl-definite manifold containing only one isomorphism type, and any such manifold is coextensive with that isomorphism type.

Proof: First, by theorem 8, there exists a categorical set Σ of sentences such that $\mathfrak{M}\Sigma$ is a Husserl-definite manifold. Since Σ is categorical, $\mathfrak{M}\Sigma$ is an isomorphism type. Since isomorphism types are either equal or disjoint, $\mathfrak{M}\Sigma$ contains only the one isomorphism type, namely $\mathfrak{M}\Sigma$ itself. Secondly, let \mathfrak{M} be any Husserl-de-

finite manifold which contains only one isomorphism type I . By theorem 13, $\mathfrak{M} = I$.

Theorem 24: There exists a Husserl-definite manifold containing at least two isomorphism types, and the isomorphism types contained as subclasses of any such manifold are not themselves manifolds.

Proof: First, by theorem 14 (c), there exists a Husserl-definite manifold \mathfrak{M} which is not a formal manifold. Thus \mathfrak{M} is not an isomorphism type, and by theorem 13, \mathfrak{M} is a union of (at least two) disjoint isomorphism types. Secondly, assume \mathfrak{M} is such a Husserl-definite manifold, and for a *reductio ad absurdum* proof, assume that \mathfrak{M}_1 is an isomorphism type contained as a subclass within \mathfrak{M} , and that \mathfrak{M}_1 is a manifold. Then \mathfrak{M} is a generalisation of \mathfrak{M}_1 by weakening axioms, contradicting theorem 18. Thus each isomorphism type contained as a subclass of \mathfrak{M} is not a manifold.

§ 5 Interpreting Whole-Part Notions in Manifold Theory

Abstraction gets to work on a basis of primary intuitions, and with it a new categorial act-character emerges, in which a new style of objectivity becomes apparent, an objectivity which can *only* become apparent – whether given as ‘real’ or as ‘merely imagined’ – in just such a founded act. Naturally I do not here mean abstraction merely in the sense of setting-in-relief of some dependent moment in a sensible object, but ideational abstraction, where no such dependent moment, but its Idea, its Universal, is brought to consciousness, and achieves *actual givenness*. We must presuppose such an act in order that the very sort, to which the manifold single moments ‘of one and the same sort’ stand opposed, may *itself* come before us, and may come before us *as one and the same* (Husserl, LU VI, § 52).

The intuitive concepts of *whole* and *part* are paradigm cases of the type of concept which, following Husserl, we can characterise as *vague* or

(synonymously) *indistinct*. Within the context of this paper we depart in letter (but not in spirit) from Husserl's definition of 'distinctness'. We define a concept as *formally distinct* iff for each two particulars which are (correctly) intended as instances of it, those two particulars are isomorphic. To anticipate our epistemological application of notions developed in § 4, this means that a concept is *completely distinct* if it is the intension of a Husserl-definite manifold of full expansions. Any intuitive concept can be shown to be formally vague or indistinct by showing that there is some sentence true of one but false of some other instance of it. That the intuitive concept of *part* is formally vague in this sense can be shown by a consideration of the Stumpf-Husserl distinction between two distinct types of parts; dependent parts (moments) and independent parts (pieces).

It should be remembered that this distinction was developed originally not within ontology, but within the psychology of perception. Stumpf (1873, pp. 109 ff.) and, following him, Husserl characterised a part as *dependent* iff it cannot be perceived separately from the perceptual whole of which it is a part, and as *independent* iff it can be so perceived. Perceptual examples of countable wholes might be the perception of a minor chord or of a swarm of bees: Each note in the chord and each bee in the swarm is an independent part, while the 'flatted third' sound distinctive of this minor but not of that major chord, or the 'swarm' character distinctive of this group of bees but not that group of birds, are dependent parts of their respective wholes. In his 1929 article on Gestalt theory and the phenomenology of (perceptual) thematics, Gurwitsch (1966, pp. 263–5) reformulated the definition of 'independent part', but maintained the distinction between the two different types of parts. Their mutual insistence on this distinction illustrates the opinion of Stumpf, Husserl, and Gurwitsch that the intuitive concept of *part* is vague in the sense defined above.

Husserl (1970a, Investigation V, § 17) and Gurwitsch (1966, pp. 131–4, 143, 186, 332–49, esp. 340) insist on a similar distinction between two types of whole; a *whole which is perceived* is distinguished from the many instances of the whole which is perceived, each of which is the *whole as perceived*. A single whole which is perceived may be perceived in a variety of ways, i.e. under a variety of aspects or as instancing a variety of types of things. For example, the collection of bees may be perceived as a swarm, as a threat, as an indication of a source of honey, or under any combination of these aspects, and so on. This example pro-

vides us with one *whole which is perceived*, but with six *wholes as perceived*. This distinction between two types of perceptual wholes (*wholes which are perceived* vs. *wholes as perceived*) illustrates the formal vagueness of the unqualified concept of *whole*.

We will maintain the distinction between the *whole which is perceived* and the *whole as perceived* in this paper, and will follow Husserl and Gurwitsch in characterising the latter as a dependent part of the former:

Husserl has quite correctly observed that what ‘genuinely appears’ forms, in the full thing-sense, a *dependent* part which can only possess sense unity and sense independence in a whole necessarily containing empty and indeterminate [i.e. *openly possible*] components (Gurwitsch, 1966, p. 186; Cf. Husserl, 1931, p. 355, lines 7–13, which are inadequately translated; and Husserl, 1973, pp. 96–9).

Since the expression ‘*whole as perceived*’ denotes a dependent part of the *whole which is perceived*, we will use it synonymously with the expression ‘thing-tied moment of the whole which is perceived’. In contrast, we define a *moment abstract* as an intension of a class of concrete moments belonging to different wholes which are perceived. In distinguishing thing-tied or concrete moments from moment-abstracta we adopt a distinction proposed by Guido K ung (1967, pp. 172 ff) which departs from Husserl’s terminology (1970a, pp. 426–32). While dependent parts cannot be perceived independently of their wholes, we claim that they can be so *conceived* (and that the theme of such a *conceiving* is a moment-abstractum).

The thing-tied moments of different wholes which are perceived may establish similarities amongst those wholes. For example, all swarms are similar because they are all wholes which are perceived as having thing-tied ‘swarm’ moments; all minor chords are similar because they are all wholes which are perceived as having thing-tied ‘flatted third’ moments. The shared character of all ‘swarm’ (or ‘flatted third’) moments, i.e. that which all such thing-tied moments have in common, we call a ‘swarm’ (or ‘flatted third’) abstractum. We will interpret wholes which are perceived, wholes as perceived (i.e. thing-tied moments of wholes which are perceived), and moment-abstracta (which will also be referred to as ‘perceptual types’ in § 6) as different structures definable in terms of our manifold theory in this section. Before stating this interpretation, we provide the following comments in order to clarify our motivation.

No whole as perceived is a mere class. The swarm and chord cited as examples might at first consideration appear to be classes, but they are

more than mere collections of elements (independent parts, or pieces). They involve not only the founding elements but also their particular *swarm* and *minor tonality* characters. These qualitative characters are relational characteristics of the founding independent parts. The absence of such qualitative and relational co-determinations of the founding independent parts characterises classes which are not wholes as perceived. It is this fact which has motivated our choice of models (rather than sets) to serve for wholes as perceived in the manifold-theoretic interpretation of the present section. Further, it is the relation of wholes as perceived (i.e. of concrete moments) to wholes which are perceived which has motivated us to restrict our definition of 'model' to full expansions and reducts of full expansions. Any reduct is related to each of its full expansions in the same way that a whole as perceived (i.e. a thing-tied moment) is related to each of the possible wholes which are perceivable via it.

Because our primary concern is with abstract, rather than with thing-tied moments, we present no theorems concerning relations amongst thing-tied moments and wholes which are perceived. In this paper we conceive of thing-tied moments (wholes as perceived) as Gurwitsch's *themes of perceptual attention*. He has identified this structure as the nucleus of the perceptual noema, and has characterised the whole which is perceived as a Gestalt contexture of such structures. We accordingly consider the thing-tied moments of a single whole which is perceived to be mutually codetermining and to be unified by Gestalt coherence, and consider the whole which is perceived to be a Gestalt contexture of all its thing-tied moments. Successive thematisations M_1 , M_2 , $M_3 \dots$ of the *same* whole which is perceived M comprise successive *explications* of the whole M . Each explication of M reduces the degree of *material vagueness* expressed as the open possibilities of the inner horizon of the nucleus of the perceptual noema which presents M to attention. When the explication involves *articulating thematisation* (Gurwitsch, 1974, p. 261; Cf. Husserl, 1973, § 50), this specification of the inner horizon is accomplished via the constitution of an individual *Sachverhalt* $\langle M, M_i \rangle$ in which the whole M is involved as exhibiting one of its thing-tied moments M_i . The *telos* of such a process of thematising (or, more specifically, of *explicating immediate concrete moments* of) a single whole which is perceived is the elimination of all open possibilities, and the determination of the entire contexture M of all thing-tied moments M_i of the whole. It is this aspect of Gurwitsch's analysis which

motivates our choice of full expansions to serve as possible wholes which are perceived, of some full expansions to serve as actual wholes which are perceived, and of the following manifold theoretic interpretations of the language of whole-part theory.

M is a whole iff *M* is a full expansion or an f.m.r. reduct. *M* is an f.m.r. reduct iff *M* is a model with finitely many relations. *M* is a whole which is perceived iff *M* is a full expansion, and *M* is a whole as perceived (also, a concrete moment and a thing-tied moment) iff *M* is an f.m.r. reduct. *M* is a piece of the whole *M'* iff *M* is a whole which is perceived, and the universe of *M* is a proper subset of the universe of *M'*. The notions *M* is the whole *M'* as perceived, *M* is an immediate concrete moment of the whole *M'*, and *M* is a thing-tied moment of the whole *M'* are equivalent; any *M* and *M'* have this relation iff *M* is an f.m.r. reduct of *M'*. *M* is an immediate part of *M'* iff *M* is a piece or an immediate concrete moment of *M'*.

For any f.m.r. reducts *M* and *M'* and their respective classes \mathfrak{M}_M and $\mathfrak{M}_{M'}$ of full expansions, *M* is founded on *M'* iff $\mathfrak{M}_M \subset \mathfrak{M}_{M'}$, and *M* and *M'* are mutually founding iff neither is founded on the other, but there is some f.m.r. reduct *M₁* which is founded on both *M* and *M'*. For each two concrete moments, there is a founding-founded relation between them iff one is founded on the other, or they are mutually founding. *M* and *M'* are relatively dependent iff there is a founding-founded relation between them. *M* is an immediate dependent part of the whole *M₁* iff *M* is a part of *M₁*, and for each immediate concrete moment *M'* of *M₁* other than *M*, *M* and *M'* are relatively dependent. It can be shown that any two distinct immediate concrete moments of the same whole *M* are relatively dependent, and are thus immediate dependent parts of *M*.

If the whole as perceived *M* is veridical (non-illusory), the actual whole which is perceived from the point of view of *M* is exactly one member of \mathfrak{M}_M , and if *M* is non-veridical (presents an illusion to perceptual attention), then the actual whole which is perceived (if there is one) is not a member of \mathfrak{M}_M . In this paper we refer on occasion to possible wholes which are perceived from the point of view of *M* (i.e. to members of \mathfrak{M}_M) as 'perceivable wholes', and to *M* as a 'perceived whole', and define the notion '*M* is a whole' as '*M* is a perceived or a perceivable whole'. (We restrict the denotation of 'whole' in this paper to perceived and perceivable wholes; however *eide* (as defined in the next paragraph) may justifiably be conceived as wholes which are neither perceived nor perceivable.)

We refer ontologically to intensions of unions of isomorphism types as '*eide*' and 'universal objects of higher order'. In cases where for some cardinal K , each isomorphism type which is a subclass of the union has some f.m.r. model with a universe of cardinality K as a member, we refer to its *eide* as 'moment-abstracta' and (in § 6) 'perceptual types'. Where the union of isomorphism types is a Husserl-definite manifold of full expansions, we refer to its *eide* as '*automon eide*', and 'universal individuals of higher order'. We define '*an intension $\pi\mathfrak{M}$ of some union \mathfrak{M} of isomorphism types*' as a hyperultraproduct $\pi\mathfrak{M}$ of \mathfrak{M} (by some ultrafilter on \mathfrak{M}). This hyperultraproduct turns out to be an entity beyond the class-set theory used in §§ 2–4. Our definition is based on the standard notion of an ultraproduct, as in Kopperman (1972, pp. 74–7), which is a complicated structure built up from a given *set* of models. But the hyperultraproducts we are considering as intensions are built up in an analogous manner from a *proper class* of models.

In taking an ultraproduct of a proper class, the class is treated as a member of 'collections' which are members of the universe of the resulting *hyperultraproduct*. Thus these collections and the hyperultraproducts themselves are ontologically outside of standard class-set theory, and can be thought of as higher order entities in a realm beyond our class-set theory. In view of our manifold-theoretic interpretation of perceivable wholes as full expansions of some model, of perceived wholes (or concrete moments of perceivable wholes) as reducts of full expansions, and of *eide* as hyperultraproducts of proper classes of models, no perceived or perceivable whole is of an order as high as any universal object of higher order. It does not follow from this fact, however, that all perceived and/or perceivable wholes are of the same order. In stating the following interpretation, we use 'and/or' for weak, and 'or' for strong disjunction:

Whole-part notions

1. M is a thing-tied moment of M' (M' is a whole which is perceived as M).
2. M' is a possible whole which is perceived.

Manifold-theoretic interpretation:

1. M is a model with finitely many relations (an *f.m.r.* model) which is a reduct of the full expansion M' .
2. M' is a full expansion.

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| <p>3. M is a whole.</p> <p>4. to explicate the whole M which is perceived.</p> <p>5. M is described by Σ.</p> <p>6. $\pi\mathfrak{M}$ is an <i>eidōs</i> instanced by M.</p> <p>7. $\pi\mathfrak{M}$ is a predicative <i>eidōs</i> instanced by M.</p> <p>8. $\pi\mathfrak{M}$ is an <i>eidōs</i> instanced by everything described by Σ.</p> <p>9. $\pi\mathfrak{M}$ is an <i>eidōs</i> which is a moment-abstractum instanced by the thing-tied moment M.</p> <p>10. $\pi\mathfrak{M}$ is a predicative <i>eidōs</i> which is an abstract predicative moment-abstractum instanced by the thing-tied moment M.</p> <p>11. $\pi\mathfrak{M}$ is a formal <i>eidōs</i> instanced by M.</p> <p>12. $\pi\mathfrak{M}$ is a predicative formal <i>eidōs</i> instanced by M.</p> <p>13. $\pi\mathfrak{M}$ is a formal <i>eidōs</i> which is a moment-abstractum instanced by the thing-tied moment M.</p> | <p>3. M is a full expansion or an f.m.r. model</p> <p>4. to expand some f.m.r. model M which is a reduct of the full expansion M'.</p> <p>5. M is a model of Σ.</p> <p>6. $\pi\mathfrak{M}$ is a hyperultraproduct (hup.) of a union \mathfrak{M} of isomorphism types such that $M \in \mathfrak{M}$.</p> <p>7. $\pi\mathfrak{M}$ is a hup. of some manifold \mathfrak{M} where $M \in \mathfrak{M}$ and M is a model.</p> <p>8. $\pi\mathfrak{M}$ is a hup. of $\mathfrak{M}\Sigma$.</p> <p>9. $\pi\mathfrak{M}$ is a hup. of a union \mathfrak{M} of isomorphism types such that $M \in \mathfrak{M}$, and M each $M' \in \mathfrak{M}$ is an f.m.r. model of the same cardinality.</p> <p>10. $\pi\mathfrak{M}$ is a hup. of some manifold \mathfrak{M} where $M \in \mathfrak{M}$ and each $M' \in \mathfrak{M}$ is an f.m.r. model of the same cardinality.</p> <p>11. $\pi\mathfrak{M}$ is a hup. of the isomorphism type of M.</p> <p>12. $\pi\mathfrak{M}$ is a hup. of some manifold $\mathfrak{M}\Sigma$ such that Σ is categorical, and M is a model of Σ.</p> <p>13. $\pi\mathfrak{M}$ is a hup. of the isomorphism type of M such that M is an f.m.r. model.</p> |
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14. $\pi\mathfrak{M}$ is a Husserl-definite *eidos* instanced by M .
15. $\pi\mathfrak{M}$ is an *automon eidos* instanced by the whole M which is perceived.
16. The *eide* $\pi\mathfrak{M}_1$ and $\pi\mathfrak{M}_2$ are coextensive.
- Note: $(\pi\mathfrak{M}_1 = \pi\mathfrak{M}_2)$ implies $(\mathfrak{M}_1 = \mathfrak{M}_2)$, but it is not the case that $(\mathfrak{M}_1 = \mathfrak{M}_2)$ implies $(\pi\mathfrak{M}_1 = \pi\mathfrak{M}_2)$.
17. $\pi\mathfrak{M}$ is a predicative *eidos* which is a least generalisation by weakening axioms of at least two predicative *eide* $\pi\mathfrak{M}_1, \pi\mathfrak{M}_2, \dots$
14. $\pi\mathfrak{M}$ is a hup. of a Husserl-definite manifold \mathfrak{M} such that $M \in \mathfrak{M}$.
15. $\pi\mathfrak{M}$ is a hup. of some class \mathfrak{M} such that $M \in \mathfrak{M}$ and \mathfrak{M} is a Husserl-definite manifold of full expansions.
16. $\pi\mathfrak{M}_1$ is a hup. of \mathfrak{M}_1 , $\pi\mathfrak{M}_2$ is a hup. of \mathfrak{M}_2 , and $\mathfrak{M}_1 = \mathfrak{M}_2$.
17. $\pi\mathfrak{M}$ is a hup. of the union of at least two manifolds $\mathfrak{M}_1, \mathfrak{M}_2, \dots$

Whole-part interpretations of the theorems of manifold theory:

Note: The following interpretations are either equivalent to or consequences of the corresponding theorems as stated in §§ 3–4.

- Theorem 1:** The binary relation of two wholes which instance a single formal *eidos* is an equivalence relation.
- Theorem 2:** No formal *eidos* is a whole.
- Theorem 3:** No Husserl-definite *eidos* is a whole.
- Theorem 4:** No predicative *eidos* is a whole.
- Theorem 5:** Each predicative *eidos* which is coextensive with a formal *eidos* is coextensive with a Husserl-definite *eidos*.
- Theorem 6:** For each axiom set Σ , if Σ is complete, then each predicative *eidos* $\pi\mathfrak{M}\Sigma$ is Husserl-definite.
- Lemma 7:** For each axiom set Σ , if Σ is categorical, then any predicative *eidos* $\pi\mathfrak{M}\Sigma$ is both Husserl definite and formal.
- Theorem 8:** There exists a categorical set Σ of monadic second-order sentences such that each predicative *eidos* $\pi\mathfrak{M}\Sigma$ is Husserl-definite, but Σ is not complete.

- Theorem 9:** For each predicative *eidōs* $\pi\mathfrak{M}$, if $\pi\mathfrak{M}$ is Husserl-definite, then there is some perhaps infinite axiom set Σ such that $\pi\mathfrak{M}$ is instanced by all and only wholes described by Σ and Σ is complete.
- Theorem 10:** For each first-order axiom set Σ , each $\pi\mathfrak{M}\Sigma$ is a first-order Husserl-definite *eidōs* instanced by all and only wholes described by Σ iff Σ is first-order complete.
- Theorem 12:** Each predicative *eidōs* is either Husserl-definite, or it is a least generalisation by weakening axioms of at least two extensionally disjoint Husserl-definite *eide*.
- Theorem 13:** Each predicative *eidōs* is either coextensive with a formal *eidōs*, or is a least generalisation by weakening axioms of extensionally disjoint formal *eide*.
- Theorem 14:** (a) There exists a formal *eidōs* which is not predicative; (b) there exists a predicative *eidōs* which is neither a predicative formal *eidōs* nor a least generalisation by weakening axioms of predicative formal *eide*; (c) there exists a Husserl-definite *eidōs* which is neither a predicative formal *eidōs* nor a least generalisation of predicative formal *eide*.
- Theorem 15:** Each formal *eidōs* I is an *eidōs* instanced by something which instances a Husserl-definite *eidōs*, but I is not necessarily predicative.
- Theorem 16:** For each formal *eidōs* πI , there exists a Husserl-definite *eidōs* $\pi\mathfrak{M}$ such that πI is instanced by a whole which instances $\pi\mathfrak{M}$ and every generalisation by weakening axioms of $\pi\mathfrak{M}$.
- Theorem 17:** Each Husserl-definite *eidōs* can be generalised by weakening axioms to a predicative *eidōs* (which by the next theorem is not Husserl-definite).
- Theorem 18:** Any predicative *eidōs* is Husserl-definite iff there exists no predicative *eidōs* of which it is a generalisation by weakening axioms.
- Lemma 19:** No whole which is perceived is a whole as perceived.
- Theorem 20:** Any predicative *eidōs* is an *automon eidōs* iff there exists no predicative *eidōs* of which it is a generalisation.
- Theorem 21:** Each predicative *eidōs* which is instanced by a countable whole is an *automon eidōs* or a generalisation of one.

Theorem 22: Each *automon eidos* is a formal predicative *eidos*.

Theorem 23: There exists a Husserl-definite predicative *eidos* $\pi\mathfrak{M}$ such that there exists a formal *eidos* πI instanced by all wholes which instance $\pi\mathfrak{M}$, and in all such cases πI and $\pi\mathfrak{M}$ are coextensive.

Theorem 24: There exists a Husserl-definite *eidos* $\pi\mathfrak{M}$ instanced by at least two wholes M and M' , where there are no coextensive formal *eide* instanced by both M and M' .

§ 6 Ontological and Epistemological Interpretations of Manifold Theory

In this section we discuss some implications of the theory of manifolds from the point of view of the ontological interpretation of § 5, and of the epistemological interpretation (i.e. as a theory of the transcendental constitution of perceptual wholes and universals) which we have had in mind but have not developed in detail.

The problem of carrying out an adequate phenomenology of conceptualisation involves the formulation of descriptions of processes of attention (thematic transitions) within which formalisation, generalisation, and other processes of attention involved in conceptualisation occur. While no such descriptions will be attempted here, certain comments regarding epistemological application of the results of §§ 2–4 and lines of approach to the problem of conceptualisation are appropriate. We begin these comments with a list of epistemological definitions which will enable us to establish the relation of an *Idea in the Kantian sense* to perception, and will conclude by delineating certain types of conceptualisation as problems for further research.

1. A concept is *formally distinct* iff it is an intension of an isomorphism type.
2. A concept is *materially distinct* iff it is an intension of a union of isomorphism types containing as members only full expansions.
3. A concept is completely *distinct* iff it is both formally and materially distinct.

Remark: Any intension of an isomorphism type is a formally distinct concept, and any intension of a Husserl-definite manifold of

full expansions is a formally and materially (thus completely) distinct concept, and the epistemological correlate of an *automon eidos*, by theorem 22.

4. A concept is *clear* iff it is instanced (in perception) by some thing-tied moment.
5. (M_f, M) is an individual Sachverhalt iff M_f is a whole which is perceived, and M is an immediate part of M_f .

Remark: The existence of an individual Sachverhalt (M_f, M) is a necessary and sufficient condition for the truth of a sentence (formulated in some language other than the object languages considered in this paper) ascribing the immediate part M to M_f as an articulated explicite.

6. A process of explicating an immediate dependent part of a whole M_f which is perceived is interpreted as a process of expanding M_1 to M_2 where both M_1 and M_2 are f.m.r. models which are reducts of the full expansion M_f .
7. Each explication of an immediate dependent part of the whole M_f which is perceived via a thing-tied moment M_1 results in M_f being perceived via a thing-tied moment M_2 (where M_2 is a reduct of M_f and M_1 is a reduct of M_2).

Remark: In each such case, the thing-tied moment M_2 is an *explicate* of the whole M_f which is perceived.

8. Immediate dependent part explication which involves *articulating thematisation* (see Gurwitsch, 1974, ch. 10; Cf. Husserl, 1973, §§ 22–32, 47–65) constitutes the individual Sachverhalt $\langle M_f, M_2 \rangle$, i.e. the situation of M_2 being articulated as a thing-tied moment of the whole M_f which is explicated.

Remark: In each such case, the thing-tied moment M_2 is an *articulated explicate* of the whole M_f which is perceived.

Theorem 25: If a concept $\pi\mathfrak{M}$ is completely distinct, then it is unclear.

Proof: If a concept $\pi\mathfrak{M}$ is completely distinct, then it is instanced only by models which are full expansions and isomorphic to each other. Thus $\pi\mathfrak{M}$ is instanced by no

reduct of any full expansion. By the definition of 'thing-tied moment', $\pi \mathfrak{M}$ is instanced by no thing-tied moment, and is therefore not clear. We note also that if a concept is clear, then it is materially indistinct (materially vague), though it may be formally distinct.

The material vagueness characteristic of perceptual evidence may be progressively diminished by successive explications, and the *telos* of this process (of explicating dependent parts of the same whole which is perceived) is the ideal of *adequate* perceptual (i.e. clear and completely distinct) evidence. Perceptual evidence which is *adequate* is therefore never in fact available, but is a limit approached by successive explications of immediate dependent parts of a single whole which is perceived, and is experienced (i.e. is originally constituted in transcendental consciousness) as the *telos* of such a sequence of explications (Cf. Husserl, 1970a, pp. 720, 731–2, 734–6, 745–8, 760–70). Because of this *limit* feature *vis-à-vis* perceived objects and moment abstracta, a completely distinct concept (i.e. an intension of a Husserl-definite manifold of full expansions) has the status of an *Idea in the Kantian sense* (Cf. Husserl, 1931, §§ 22, 74, 83, 149). The constitution of such a *Kantian Idea* in thematising an *automon eidos* (called by Husserl a *concretum*) is one type of conceptualisation, which we shall refer to as 'ideation'.

We propose that each proper class which is a union of isomorphism types is a unary relation capable of hypostatisation as an *eidos*, i.e. as a more or less general universal (object of higher order) via conceptualisation of one sort or another. Each such proper class of models can be viewed as the extension of an *eidos*. Within our ontological and epistemological interpretations, a hyperultraproduct of such an extension is identified as its intension, and the instantiation of an *eidos* by wholes of a given type is characterised as *instancing*. For any moment-abstractum (perceptual type) $\pi \mathfrak{M}_1$ and *eidos* $\pi \mathfrak{M}_2$, we define $\pi \mathfrak{M}_2$ as a *specification of $\pi \mathfrak{M}_1$ by explication of immediate dependent parts* iff \mathfrak{M}_1 is a generalisation of \mathfrak{M}_2 by removals (see p. 458, *supra*). Since the phenomenology of science is concerned with describing an enterprise essentially involving language, we are particularly interested in the cases where the proper classes involved in these processes of conceptualisation are manifolds.

Where the processes of conceptualisation are scientific, the proper classes of models to be hypostatized via abstraction and specified via ex-

plication must be expressed linguistically (predicatively). It appears plausible to consider manifolds to be the classes which become hypostatised and specified as universal objects of higher order (predicative *eide*) via theorising activities (i.e. via predicative processes of attention) of various sorts. Our results permit us to distinguish ontologically between those manifolds amenable to hypostatisation as individual universal objects of higher order, and those amenable to hypostatisation as non-individual universals. Theorem 20 indicates that all and only Husserl-definite manifolds of full expansions satisfy the conditions for hypostatisation as universal individuals (i.e. as *automon eide*). Manifolds of f.m.r. reducts can be hypostatised as moment-abstracta which are non-individual universals, i.e. as generalised *eide* which are capable of further specification.

Similarly, we can distinguish epistemologically between distinct and vague concepts on the basis of our manifold-theoretic results. Moment-abstracta are, considered as concepts, materially indistinct. We therefore associate clear concepts with moment-abstracta (perceptual types), considering them as epistemological correlates of generalised *eide*, and contrast them with completely distinct concepts (Kantian Ideas), which we identify as the epistemological correlates of *automon eide*. Since all moment-abstracta are instanced by thing-tied moments while no *automon eidos* is instanced by any concrete moment, no *automon eidos* is a moment-abstractum (perceptual type). However, each *automon eidos* is the *telos* of a sequence of progressively more specific, less general moment-abstracta (perceptual types). This means that ideation should be viewed as involving first the hypostatisation of a moment-abstractum, and secondly the specification of that hypostatised type via thematic transitions (e.g. Husserl's free variation in imagination) which are equivalent to an infinite sequence of progressive specifications which approach the constitution of a distinct concept and the thematisation of an *automon eidos* as an ideal limit of the process of specification. This transition (via specification) from moment-abstractum to *automon eidos* is, we submit, the 'transition from type to (completely distinct) concept and *eidos*' mentioned by Gurwitsch in the passage quoted on p. 442 above. The account of conceptualisation which Gurwitsch envisioned must therefore include not only a description of the processes of attention involved in the hypostatisation of moment-abstracta, but also descriptions of the processes of attention involved in the progressive specification of such hypostatised moment-abstracta. We suggest that such

accounts will have to involve descriptions not only of prepredicative and predicative explication in perception (i.e. specification via both *schlichte* and *kategoriale Anschauung*; Husserl, 1970a, pp. 773–815), but also of free variation in the imagination (Husserl, 1973, pp. 321–64 and 1977, pp. 53–78).

The problem of conceptualisation indicated by Gurwitsch as a pressing desideratum of non-egological transcendental phenomenology at the present stage of its development therefore involves more than just indicating the mathematical operation of hyperultraproduct as a promising formalisation of abstraction, and specifying which relations can, and which cannot be thematised via abstraction as universal individuals and/or objects of higher order. It requires also epistemological descriptions (to be developed in terms of the general theory of intentionality) of the processes of attention which are involved in such abstractive hypostatizations, as well as of other processes of attention (such as specification to less general moment-abstracta, and ideation to *automon eide*). In view of the distinctions we have developed in this paper, we suggest that the processes of attention involved in the constitution of generalised and *automon eide* (and the corresponding epistemological structures which we have called concepts of various sorts) should be described separately and in relation to each other. We further suggest that this epistemological task can be approached in terms of the notion of hyperultraproduct and manifold theory, and that it ultimately cannot avoid the concept of the lifeworld.

To this end, we propose considering (*ex hypothesi*) the theme of perceptual attention (i.e. the nucleus of the perceptual noema) to be an f.m.r. model of some axiom set Σ , the *order of existence* O of M to be the class of f.m.r. models which are cardinally equivalent to M and compatible with the language of Σ , the *thematic field* T of M to be $\mathfrak{M}\Sigma \cap O$, and the *principle of material relevance* which organizes T to be the moment abstractum πT . Some relevant processes of conceptualization would then be:

Ontological abstraction: Those thematic transitions by which attention can, given as an initial theme some whole M perceived as a member of some thematic field T , constitute as a subsequent theme the *eidos* (moment abstractum, or perceptual type) πT .

Specification by strengthening axioms: Those thematic transitions by which attention can, given as an initial theme some whole M perceived

as a member of some thematic field T , constitute as a subsequent theme the whole M perceived as a member of a thematic field T' such that $\mathfrak{M}\Sigma$ is a generalisation of $\mathfrak{M}\Sigma'$ by weakening axioms.

Specification by explication of immediate dependent parts: Those thematic transitions by which attention can, given as an initial theme some whole M perceived as a member of some thematic field T , constitute as a subsequent theme the whole M perceived as a member of some thematic field T' such that M is an immediate dependent part of M' and $\mathfrak{M}\Sigma$ is a generalisation of $\mathfrak{M}\Sigma'$ by removals.

Specification: Those thematic transitions by which attention can, given as an initial theme some whole M perceived as a member of some thematic field T , constitute as a subsequent theme the whole M' perceived as a member of a thematic field T' such that $\mathfrak{M}\Sigma$ is a generalisation of $\mathfrak{M}\Sigma'$, and either $M = M'$, or M is an immediate dependent part of M' .

Formalising abstraction: Any combination of ontological abstractions and/or specifications whereby a formal *eidos* is thematised.

Ideation: Any combination of ontological abstractions and/or specifications whereby an *autonom eidos* is thematised.

In conclusion, it should be noted that specification is the inverse of generalisation, and that we have indicated the need for an epistemological account of specification rather than of generalisation because ideation is a limit approached by progressive specification, rather than by generalisation. It should also be noted that we have made no attempt in this paper to formulate epistemological descriptions of the processes of attention defined here as involved in conceptualisation, but have merely categorised these processes in terms of the types of concepts they involve, and the types of universal objects which they constitute as themes of attention. While we have suggested the mathematical operation of ultraproduct as defined on unions of isomorphism types as the structure we consider interpretable as ontological abstraction, we have offered neither a mathematical definition, nor an epistemological interpretation (i.e. description in terms of the general theory of intentionality) of that operation. Such a definition, epistemological descriptions, and formal ontological specifications of the properties common to each life-world as experienced (i.e. to each cultural world), and consideration of the role of culture and consensus in determining the properties characteristic of

a particular cultural world then remain outstanding desiderata within the programme sketched by Gurwitsch. Hence the results which we have presented here comprise prolegomena to further work in that area of metaphysics indicated by Gurwitsch as the *problem of conceptualisation*.

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Barry Smith

Annotated Bibliography of Writings on Part-Whole Relations since Brentano

**Compiled with the assistance of
Kevin Mulligan and Peter M. Simons**

We have sought to provide as complete a listing as possible of those writings which either make some direct contribution to the theory of part and whole, or involve explicit application of the theory, particularly in the areas of psychology and linguistics. The earliest work in the bibliography – Stumpf, 1873 – was written by a student of Brentano, and the direct or indirect influence of the latter can be discerned in an overwhelming majority of the items listed. (The diagram overleaf is an attempt to chart this influence.) Given this not wholly arbitrarily selected chronological starting point, it has been necessary to exclude:

(i) the works of 19th century mathematicians such as Bolzano, Riemann, Weierstrass, Cantor and Dedekind, on set theory and manifold theory;

(ii) the writings of Boole, Venn, Peirce and others in the field of algebraic logic (an exception is made in the case of Schröder and his associates in view of their influence upon Husserl);

(iii) work on part-whole relations in Aristotle and the scholastics (including Leibniz and Spinoza), and the writings of 19th century German logicians and philosophers – especially Trendelenburg, Ueberweg, Beneke, Herbart and Lotze – who were influenced by the Aristotelian theory of substance and accident;

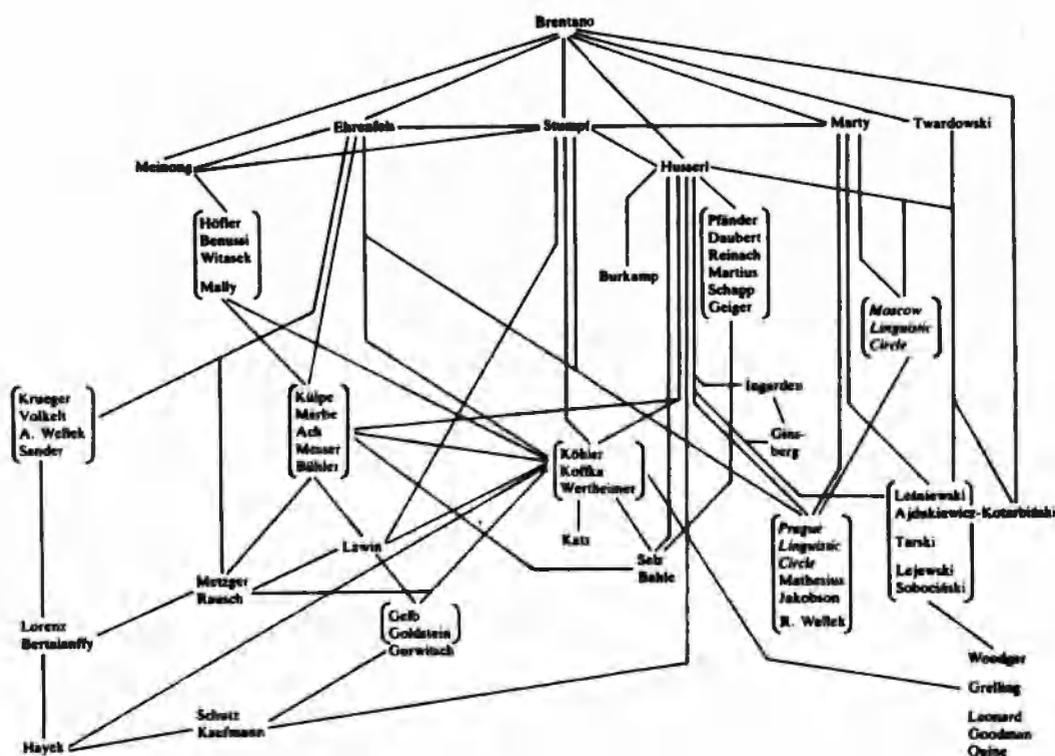
(iv) the writings of Goethe, Tetens, Herder, Humboldt, Hegel, Dilthey (perhaps also Marx, Lukács, Klages, Spann, ...) and other proponents of 18th/19th century organicism or holism.

Because their origins are taken to lie in (iv), and thus outside our designated limits, we have excluded also:

(v) writings of British idealists on internal relations and on the concept of totality, except where, as in the case of Stout, these bear a direct relation to the Brentano-Stumpf-Husserl-tradition, and, with a similar caveat,

(vi) writings on vitalism and emergent properties, and on associated issues in the philosophy of science of the 30's and 40's.

Two further omissions should perhaps be mentioned here. In the light of Rickey's excellent work (1972 ff), we have felt it necessary to list only those works by Leśniewski and his followers which are of immediate relevance to the theory of part and whole. This is in spite of the fact that, within the Leśniewskian framework, *mereology* is founded upon, and thus fully intelligible only against the background of his *protothetic* and *ontology*. Secondly, we have paid little attention to recent work by analytic philosophers at the periphery of part-whole theory, particularly on mass terms (see the bibliography in Pelletier, 1979), and on the ontology of actions and events (see e.g. Thompson, 1977). These and other omissions can be to some extent rectified if the bibliography is used in conjunction with the index to the volume as a whole.



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... [etc.]" An exposition is provided by Chisholm in his 1978.) Eng. trans., *Theory of Categories*, Haag: Nijhoff, 1981.

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cisms; Frege is criticised for failing to do justice to his own insights into the peculiarities of formal concepts).

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- 1921 *Das Ganze und die Summe* (Antrittsrede). Leipzig: Reinicke, 31 pp.
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1890 "Über 'Gestaltqualitäten'", *Vierteljahrsschrift für wissenschaftliche Philosophie*, 14, 242–92 (as repr. in Weinhandl, ed., 1960, 11–43, (Eng. trans. in preparation), e.g. p. 262: "By *Gestaltqualitäten* we understand positive contents of presentations bound up in consciousness with the presence of complexes of presentations, complexes which consist for their part of mutually seperable elements ...". See the exposition in Smith, 1981).

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- 1937 *Grundgedanken der Teiltheorie*, Leipzig: Hirzel (inadequate extensionalist approach differing only trivially from set theory; attempt to provide mereological foundations for mathematical analysis).
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gen kann, ungesättigten Teile, dem sprachlich das Verneinungswort entspricht, und einem Gedanken. Wir können nicht verneinen ohne etwas, was wir verneinen, und dieses ist ein Gedanke. Dadurch, daß der Gedanke den ungesättigten Teil sättigt oder, wie man auch sagen kann, den ergänzungsbedürftigen Teil ergänzt, wird der Zusammenhalt des Ganzen bewirkt.”)

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- Ordnung zum Ausdruck. Die Stellung, die jeder Teilinhalt in der Gesamtarchitektur einnimmt, gibt ihm seine Bedeutung.”).
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- See also Gelb and Goldstein.
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- 1951 *The Structure of Appearance*, 1st ed., Cambridge, Mass.: Harvard University Press. 3rd. ed., Dordrecht: Reidel, 1977. (See esp. ch. 2, § 3, “The Calculus of Individuals”, which presents Goodman’s extensional whole-part theory, an alternative to set theory as methodological basis for the formal theories of qualities, concreta, order, measure and time.)

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See also Leonard and Goodman.
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class Φ of functions are given, one based on the idea of equality, the other on variation:

(1) f is *equiddependent* on Φ iff, whenever the values of all functions in Φ for a given common argument are equal to their values for another common argument, f also takes equal values for the two arguments.

(2) f is *vardependent* on Φ iff, whenever the values in Φ for one common argument differ from those for another common argument in respect of precisely *one* member of Φ , the values of f for these arguments are also different.

In terms of these notions corresponding pairs of definitions of 'independent' and 'interdependent' are given, and logical relations among the various concepts pointed out. The second concept, of variational dependence, has affinities with the Stumpf-Husserl approach to dependence through variation.)

Grelling, K. and Oppenheim, P. 1938

"Der Gestaltbegriff im Lichte der neuen Logik", *Erkenntnis*, 7, 211–25 (logicist treatment of ideas deriving from Ehrenfels, 1890 and from the work of Wertheimer, etc.).

(1939) "Logical analysis of 'Gestalt' as 'Functional Whole'", *Erkenntnis*, 9 (see Grelling, 1939). (The authors propose reserving the term 'Gestalt' for its original meaning of 'shape' or 'form', and propose for the whole-part concept the term 'functional whole' (also, but less satisfactorily referred to as '*Wirkungssystem*' in their 1938). They suggest, as did Husserl, that the concept of a functional whole (Husserl's *pregnant whole*) must be analysed in terms of dependence: "to say that something is not an aggregate, it is sufficient to characterise it as a *functional whole*: this main concept of gestalt theory has rather to be based, as we have done, on the notion of interdependence" (p. 8 of MS). A number of helpful examples explain what the authors have in mind. Despite the startling convergence with Husserl, their basic concept of dependence, unlike that of Hus-

serl, is allowed, as in Grelling 1939, to rest on the unanalysed concept of function.)

See also Rescher and Oppenheim.

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- 1979 "Structures versus Sets: The Philosophical Background of Gestalt Psychology", *Critica. Revista Hispanoamericana de Filosofía*, 9, 3–21 (useful account of Meinong, Ehrenfels, Stumpf, Husserl on the ontology of structured wholes).
- Grzegorzczak, A. 1955 "The systems of Leśniewski in relation to contemporary logical research", *Studia Logica*, 3, 77–95, (but cf. critique in Luschei, 1962, 154–66).
- Gurwitsch, A. 1929 "Phänomenologie der Thematik und des reinen Ich. Studien über Beziehungen von Gestalttheorie und Phänomenologie", *Psychologische Forschung*, 12, 279–381, Eng. trans. in Gurwitsch, 1966, 175–286, (interpretation of Husserl's dependence/independence opposition in Gestalt-theoretical terms).
- 1930 "Ontologische Bemerkungen zur Axiomatik der Euklidischen Geometrie", *Philosophischer Anzeiger*, 4, 78–100 (review of Geiger, 1924).
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- 1949 "Gelb-Goldstein's concept of 'concrete' and 'categorical' attitude and the phenomenology of ideation", *Philosophy and Phenomenological Research*, 10, 172–96, repr. in Gurwitsch, 1966, 359–384.

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- 1952 *The Sensory Order. An Inquiry into the Foundations of Theoretical Psychology*, London: Routledge.
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- 1915 *Die Struktur des logischen Gegenstandes* (*Kantstudien* Ergänzungsheft, 35), Berlin: Reuther and Reichard (early, but highly sophisticated development of a structuralist/contextualist theory of conscious experience, including a discussion of founded objects).
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(Particularly in his glossematic period, Hjelmslev employs the concepts of fusion and one-sided and reciprocal dependence in his linguistic theory.)

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- 1895 "Psychische Arbeit", *Zeitschrift für Psychologie*, 8, 44–103 and 161–230.
- 1912 "Gestalt und Beziehung – Gestalt und Anschauung", *Zeitschrift für Psychologie*, 60, 161–228 (extremely thorough survey of early Gestalt literature, supplementing Gelb, 1911).
- 1917 "Abhängigkeitsbeziehungen zwischen Abhängigkeitsbeziehungen. Beiträge zur Relations- und zur Gegenstandstheorie", *Sitzungsberichte der kaiserlichen Akademie der Wissenschaften in Wien*, 181, 4, 56 pp. (formal theory of relations of compatibility and incompatibility, dependence and independence, constructed by analogy with syllogistic theory).
- 1921 "Tongestalten und lebende Gestalten", *ibid.*, 196, 1, 94 pp. (on determinacy and indeterminacy of Gestalt structures, esp. in relation to the aesthetics of music).
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- 1974 *Jakobson ou le structuralisme phénoménologique*, Paris: Seghers, Eng. trans., *Roman Jakobson's Approach to Language: Phenomenological Structuralism*, Bloomington and London: Indiana University Press, 1975 (important discussion of formalist-structuralist linguistics and of the influence of the *Logical Investigations* and of early phenomenology and experimental psychology on 20th-century linguistics).
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- 1980 “On the poetry and the plurifunctionality of language”, in B. Smith, ed., 1981, 1–44 (esp. the discussion of the opposition between *function* and *aim*: the former is characteristic of dependent parts – footmen, dining tables, gunshot, ... – the latter of independent wholes – human beings, associations, etc.).
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- 1933 *The Nature of Learning*, London: Kegan Paul, (cf. ch. 2. on the concept of system, esp. the discussion of Hertz).
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- (1890) “Logikkalkül und Inhaltslogik”, Beilage III to Hua XXII (1979), 400–405 (on a calculus of *Begriffsgegenstände*).

- 1891 PdA = *Philosophie der Arithmetik, Psychologische und logische Studien*, vol. I (only volume published), Halle: Pfeffer, as repr. in Hua XII (see esp. the discussions of Stumpf, 1873, and of Brentano, e.g. on pp. 19f, 71 ff, 159 ...).
- 1891a Review of vol. I of Schröder, 1890/1905, *Göttingische gelehrte Anzeigen*, 243–78, Hua XXII, 3–43. Eng. trans. by D. Willard, *The Personalist*, 59, 1978, 115–43. (Cf. also Beilage I to Hua XXII, 381–98, “Aus Entwürfen Husserls zu seiner Schröder-Rezension”.)
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- 1900/01 LU = *Logische Untersuchungen*, 1st ed., Halle: Niemeyer, Hua XVIII ff. Vol I, *Prolegomena zur reinen Logik* (critique of psychologism in logic and introduction of the fundamental concepts of formal ontology). Vol II, *Untersuchungen zur Phänomenologie und Theorie der Erkenntnis*, (esp. LU III, “Zur Lehre von den Ganzen und den Teilen”, and IV, “Der Unterschied der selbstständigen und unselbstständigen Bedeutungen und die Idee der reinen Grammatik”).
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- (1905/07) “Seefelder Manuskripte über Individuation (Verlegenheiten Pfänder-Daubert)”, in Hua X, 237–68. (On continuity and change and the unity of objects; moments vs. phases; causal dependence, etc. “Die Unselbstständigkeit, die konkrete Individuen im Zusammenhang mit anderen konkreten Individuen

haben, ist eine ganz andere als die Unselbstständigkeit von Eigenschaften. Eigenschaften sind Substratgegenstände, aber sind Erzeugungen eines Substrats, das in ihnen sozusagen lebt und webt, und das, was es ist, nur in ihnen ist" (p. 260). Cf. also discussion of Meinong's whole-part theory on pp. 216ff.)

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- 1913a "Ideen zu einer reinen Phanomenologie und phänomenologischen Philosophie", in *Jahrbuch für Philosophie und phänomenologische Forschung*, 1, 1–323, Hua III. (On part-whole relations see e.g. §§ 12–15; or § 51, where the *Verflechtung* of consciousness and the natural world is contrasted with the relation between colour and extension.)
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- (1918) "Substrat und Wesen", Beilage to Hua III, 2, pp. 580–82 (see also the associated Beilagen).
- 1929 "Formale und transzendente Logik", *Jahrbuch für Philosophie und phänomenologische Forschung*, 10, 1–298, Hua XVII, Eng. trans., *Formal and Transcendental Logic*, Haag: Nijhoff, 1969.

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- 1952 *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie. Zweites Buch, Phänomenologische Untersuchungen zur Konstitution*, Hua IV (esp. the first section, which illustrates the variety of uses to which the concept of piecing (*Teilung*) was put by Husserl).
- 1952 a *Ideen ... Drittes Buch, Die Phänomenologie und die Fundamente der Wissenschaften*, Hua V.
- 1962 *Die Krisis der europäischen Wissenschaften und die transzendente Phänomenologie. Eine Einleitung in die phänomenologische Philosophie*, W. Biemel, ed., Hua VI; Eng. trans. by D. Carr, Evanston: Northwestern University Press, 1970. (E.g. the discussion in § 22 of Locke for whom "the soul is something self-contained and real by itself, as is a body; in naive naturalism the soul is now taken to be like an isolated space, like a writing tablet ... on which psychic data come and go ... Of course one speaks quite unavoidably, [even] in the Lockean terminology, of perceptions, representations 'of' things, or of believing 'in something', willing 'something', and the like. But no consideration is given to the fact that in the perceptions, in the experiences of consciousness themselves, that of which we are conscious is included *as such* – that the perception is *in itself* a perception of something, of 'this tree'.")
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- 1929 "Rémarques sur l'évolution phonologique du russe comparée à celle des autres langues slaves", *Travaux du Cercle Linguistique de Prague*, II, repr. in SW I, 7–116 (cf. the discussion of laws of foundation on pp. 22 ff).

- 1932 "Zur Struktur des russischen Verbums", *Charisteria Gvilelmo Mathesio quinqvagenario a discipulis et Circuli Lingvistici Pragensis sodalibus oblata*, Prague, repr. in SW II, 3–15.
- 1936 "Beitrag zur allgemeinen Kasuslehre. Gesamtbedeutungen der russischen Kasus", *Travaux du Cercle Linguistique de Prague*, VI, repr. in SW II, 23–71 (e.g. the discussion of meaning-hierarchies in Husserl's *Logical Investigations* on p. 34).
- (1939) "Zur Struktur des Phonems", first published in SW I, 280–310.
- 1940/42 "Kindersprache, Aphasie und allgemeine Lautgesetze", *Språkvetenskapliga Sällskapets Förhandlingar*, repr. in SW I, 328–401, Eng. trans., *Child Language, Aphasia and Phonology*, Haag: Mouton, 1968 (cf. Jakobson's motto to this work from the *Logical Investigations: Alles wahrhaft Einigende sind die Verhältnisse der Fundierung*).
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- 1957 "Shifters, Verbal Categories, and the Russian Verb", Russian Language Project, Department of Slavic Languages and Literature, Harvard University, SW II, 130–47 (especially the discussion of Husserl's *Logical Investigations* treatment of indexical expressions).
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- 1934 “Soziale Kollektive”, *Zeitschrift für Nationalökonomie*, 1, 294–308. (Kaufmann argues that a correct understanding of social part-whole relations dispels the need to believe in mythical social entities. He discusses four senses in which ‘individuals’ might be held to be prior to ‘society’ and concludes that there is no foundation relation in Husserl’s sense between the two.)
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- 1926 "Zur Komplextheorie", *ibid.*, 8, 236–43.
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expressed within the framework of an analytic philosophy of science. Cf. rejoinder by Rescher, *Philosophy of Science*, 20, 1953, 327f.).

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- 1896 “Über die Bedeutung des Weber’schen Gesetzes. Beiträge zur Psychologie des Vergleichens und Messens”, *ibid.*, 11, 81–133, 230–85, 353–404, GA II (esp. § 3 on “Teilbare und unteilbare Grössen” = extensive vs. intensive magnitudes). Cf. Russell, 1899 and 1903, ch. 31.
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- 1915 *Über Möglichkeit und Wahrscheinlichkeit. Beiträge zur Gegenstandstheorie und Erkenntnistheorie*, Leipzig: Barth, repr. as GA VI (cf. *passim* Meinong’s account of implexive existence and of the dependence/independence opposition).
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- 1903 *The Principles of Mathematics*, Cambridge: Cambridge University Press, (esp. chs. 16, "Whole and part", 17, "Infinite wholes", and 31, on intensive magnitudes).
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completely accurate appreciation of Meinong; cf. however the discussion on pp. 206 ff of one-sided and formal and material implication and of objects that are “based on other [objects] as indispensable presuppositions” (pp. 23 ff of reprint).)

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part-whole theory, especially in relation to temporal foundation relations and to the logic of texts and their interpretations; attempt to distinguish between relations proper and *Fundierungsverhältnisse*).

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- 1927 "Die Umgestaltung der Grundanschauungen vom intellektuellen Geschehen", *Kantstudien*, 32, 273–80, partial English translation in Mandler, 1964, 225–34.
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- 1918 "Do finite individuals possess a substantive or an adjectival mode of being?", *Proceedings of the Aristotelian Society, Supplementary Volume*, 127–48 (on whole-part theory and subject-predicate ontology).
- 1921 "The Nature of Universals and Propositions", in *Proceedings of the British Academy*, 157–72, repr. in Stout, 1930, 384–403. ("The position that characters are as particular as the concrete things or individuals which they characterise is common

to me and the nominalists. But I differ from them essentially in maintaining that the distributive unity of a class or kind is an ultimate and unanalysable type of unity. The nominalists, on the contrary, say that it can be explained through the relation of resemblance. This view seems to me entirely indefensible. . . . A relation considered as subsisting between terms presupposes some complex unity within which both the term and relations fall. This complex unity is the *fundamentum relationis*. For example, a relation of 'above and below' as subsisting between *a* and *b* presupposes a spatial complex including both *a* and *b* and the spatial relation between them. In like manner, resemblance presupposes a complex unity of the peculiar type which I call the distributive unity of a class" (p. 387f.)

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- 1883/90 *Tonpsychologie*, 2 vols., Leipzig: Hirzel, repr. Zandvoort: Bonset, 1965.
- 1891 "Psychologie und Erkenntnistheorie", *Abhandlungen der bayerischen Akademie der Wissenschaften*, philos.-philol. Kl., 19, 466–516.
- 1899 "Über den Begriff der Gemütsbewegung", *Zeitschrift für Psychologie*, 21, 47–XX (on dependence relations between emotions and judgments).
- (1906) "Erscheinungen und psychische Funktionen", *Abhandlungen der königlichen preußischen Akademie der Wissenschaften*, phil.-hist. Kl., 4, 40 pp. (publ. 1907).

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- 1907 “Über Gefühlsempfindungen”, *Zeitschrift für Psychologie*, 44, 1–49.
- 1916 “Apologie der Gefühlsempfindungen”, *ibid.*, 75, 1–38.
- 1918 “Die Struktur der Vokale”, *Sitzungsberichte der königlichen preußischen Akademie der Wissenschaften*, 333–58. (Stumpf distinguishes three open problems for the ‘phenomenology’ (in his sense) of the tonal region. “ 1. Wie und unter welchen Bedingungen ist es möglich, daß eine Vielheit einzelner Töne so zur Einheit zusammenschmilzt, wie es doch beim Eindruck eines Vokals der Fall ist? 2. Wie verhält sich die spezifische Qualität eines Vokals zu den Eigenschaften der Teiltöne, aus denen er besteht? 3. Wie verhält sich die Stärke des Gesamtklanges zur Stärke seiner Teiltöne?” (p. 358).)
- 1919 “Erinnerungen an Franz Brentano”, in Kraus, ed., 1919, Eng. trans. in L. L. McAlister, ed. *The Philosophy of Brentano*, London: Duckworth, 1976, 10–46.
- 1924 “[Autobiography]” in R. Schmidt, ed., *Die Philosophie der Gegenwart in Selbstdarstellungen*, 5, 205–65, Eng. trans. in C. Murchison, ed., *History of Psychology in Autobiography*, Worcester, USA: Clark University Press, I, 389–441.
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- elaborating an account of bilateral and multilateral relations of dependence between parts of linguistic structures, e.g. amongst referential deictic pronouns, amongst different sorts of definite description, and in the "mutual dependence of identification of spatio-temporal objects and of spatio-temporal positions" (pp. 451 ff.)
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- (These three works contain what is perhaps the single example of a logically informed critique of Husserlian whole-part theory.)
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- 1955 *Ganzheitspsychologie und Strukturtheorie*, Bern: Francke, (esp. the survey article, "Gestalt- und Ganzheitspsychologie", 49–81).
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- 1912 "Über das Denken der Naturvölker. I. Zahlen und Zahlgebilde", *Zeitschrift für Psychologie*, 60, 321–78 and in Wertheimer, 1925a.
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- 1920 *Über Schlußprozesse im produktiven Denken*, Berlin and Leipzig: De Gruyter, 22 pp., repr. in Wertheimer, 1925 a.
- 1922/23 “Untersuchungen zur Lehre von der Gestalt”, *Psychologische Forschung*, 1, 47–58 (statement of principles); 4 (Stumpf FS), 301–50 (classical study of the laws of organisation governing perception of dot-formations).
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- 1898 *A Treatise on Universal Algebra*, Cambridge: Cambridge University Press.
- 1906 “On mathematical concepts of the material world”, *Philosophical Transactions of the Royal Society of London*, A, 205, 465–525.
- 1917 *The Organization of Thought, Educational and Scientific*, London: Williams and Norgate (especially the treatment of part-whole relations in ch. 7).
- 1919 *An Enquiry Concerning the Principles of Natural Knowledge*, Cambridge: Cambridge University Press, pt. III. “The method of extensive abstraction” (cf. Apostel, 1976, p. 230).

- 1920 *The Concept of Nature*, Cambridge: Cambridge University Press, section IV. "The method of extensive abstraction", (e.g. p. 185: "The relation of 'extending over' is the relation of 'including', either in a spatial or in a temporal sense, or in both. But the mere 'inclusion' is more fundamental than either alternative and does not require any spatio-temporal differentiation").
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- 1934 "Indication, classes, numbers, validation", *Mind*, 43, 281–97, in *Essays in Science and Philosophy*, New York: Philosophical Library, 1948, 227–40.
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- 1968 "On being in the same place at the same time (with some remarks about categories and materialism)", *Philosophical Review*, 77, 90–95 (on the 'is' of material constitution vs. the 'is' of identity).
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