A Hierarchical Representation of Qualitative Shape based on Connection and Convexity

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Abstract. In this paper we consider the problem of representing the shape of a region, qualitatively, within a logical theory of space. Using just two primitive notions, that of two regions connecting, and the convex hull of a region, a wide variety of concave shapes can be distinguished. Moreover, by applying the technique recursively to the inside of a region (i.e. that part of the convex hull not occupied by the region itself), a hierarchical representation at varying levels of granularity can be obtained.

1 Introduction

In this paper we consider the problem of representing the shape of a region, qualitatively, within a logical theory of space, known as RCC theory (Randell and Cohn 1989, Randell, Cui and Cohn 1992, Cohn, Randell and Cui 1994). This logic originally had just one primitive notion, $\mathsf{C}(x, y)$, that of two regions x and y connecting. However, to make an initial attack on this problem, the notion of the convex hull of a region, $\mathsf{conv}(x)$ was introduced (see figure 1); this allowed relations such as topologically-inside[1], scattered-inside and geometrically-inside to be defined – see figure 2. This latter concept was then refined to distinguish tunnel-inside and containable-inside (see figure 3). In 2D these subdivisions of inside reduce to just three, as illustrated in figure 2.

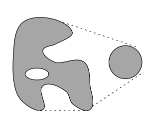

Fig. 1. The convex hull of a region

[1] By the 'inside' of a region, we mean that part of the convex hull not occupied by the region itself.

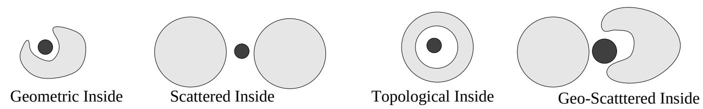

Fig. 2. The four kinds of inside possible in 2D; in each subfigure the darker region is inside the lighter one and the lighter one is outside the darker.

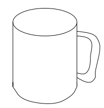

Fig. 3. Two kinds of geometrical inside: tunnel-inside, containable-inside. The mug has a containable inside (where the liquid is normally put) and a tunnel inside (the handle). Actually it also has a third kind of inside since the sum of the mug and its containable and tunnel insides does not comprise the entire convex hull of the mug: there remains that part of the convex hull which one would pass through when inserting ones finger into the handle.

Clearly if one region has another inside it, then it is concave, and depending on the specific kind of inside relation, we can say something about the nature of the concavity. Moreover, it turns out that even without the additional conv primitive, a great deal can be said about the qualitative shape of a region as (Gotts 1994) has shown: the configurations in figure 4 can all be distinguished by first order formulae whose only non logical constant is C.

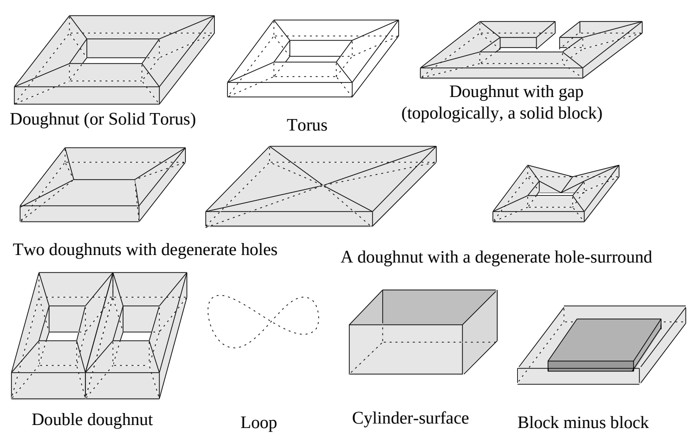

Fig. 4. Gotts can distinguish all these shapes using C alone.

The main concern of this paper however, is to develop techniques for distinguishing shapes such as those shown in figure 5 using C and conv. In the

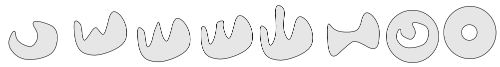

Fig. 5. The proposed approach will distinguish all these shapes

next section we will present a brief overview of RCC theory, before the main development of the new theory.

2 An Overview of RCC Theory

The focus of research on spatial representation and reasoning at Leeds has been to evaluate, extend and implement a theory[2] of space and time based upon Clarke's (1981, 1985) 'calculus of individuals based on connection', and expressed in the many sorted logic LLAMA (Cohn 1987). Our revised and extended theory, now known as 'RCC-theory' has been developed in a series of papers, including (Randell and Cohn 1989), (Randell 1991), (Randell and Cohn 1992), (Randell, Cohn and Cui 1992b), (Cohn et al. 1994), (Bennett 1994), and (Gotts 1994). The most distinctive feature of Clarke's 'calculus of individuals', and of our work, is that extended *regions* rather than points are taken as fundamental. Our formal theory supports regions having either a spatial or a temporal interpretation (temporal 'regions' are periods of time with a non-zero duration, as opposed to temporal 'points', or instants). Informally, these regions may be thought of as infinite in number, and 'connection' may be any relation from external contact (touching without overlapping) to spatial or temporal identity. Spatial regions may have one, two, three, or even more than three dimensions, but in any particular model of the formal theory, all regions are of the same dimensionality. Thus, if we are concerned with a two-dimensional model, such as one in which regions are areas of land, the boundary lines and points at which these regions meet are not themselves considered regions.

The basic part of the formal theory assumes a primitive dyadic relation: $\mathsf{C}(x, y)$, read as 'x connects with y' (where x and y are regions). Two axioms are used to specify that C is reflexive and symmetric. C can be given a topological interpretation in terms of points incident in regions. In this interpretation, $\mathsf{C}(x, y)$ holds when the topological *closures* of regions x and y share at least one point.

Using the relation C, further dyadic relations are defined. These relations are DC (is disconnected from), P (is a part of), PP (is a proper part of), EQ or $=$ (is spatiotemporally identical with), O (overlaps), DR (is discrete from), PO (partially overlaps), EC (is externally connected with), TPP (is a tangential proper part of), and NTPP (is a nontangential proper part of). The relations P,

[2] We use the word 'theory' in its logical/mathematical sense, meaning a set of formal axioms which specify the properties and relations of a collection of entities, not in the natural scientist's sense of an empirically testable explanation of observed regularities.

PP, TPP and NTPP have inverses (here symbolised Pi, PPi, TPPi and NTPPi — a slight change in terminology from some earlier papers).

The complement (compl) of a region, and the sum (sum or $+$), product or intersection (prod or $*$) and difference (diff or $-$) of a pair of regions are also axiomatised in terms of C. The product of two regions may not always be a region: if the two do not overlap, this product is an object of the sort NULL (spatial and temporal regions belong to distinct subsorts of the sort REGION). Similarly, if two regions are EQ, then $\mathsf{diff}(x, y)$ is of sort NULL, as is the complement of the universal region, u. If DC regions are summed, then clearly a multi-piece region will be formed. We will often need to test whether a given region is one-piece or not, which we will denote $\mathsf{CON}(x)$.

$\mathsf{CON}(x) \equiv_{def} \neg\exists(y, z)[x = y + z \wedge \mathsf{DC}(y, z)]$

A 'nonatomic' axiom can be added, ensuring that every region has at least one NTPP:

$\forall x \exists y[\mathsf{NTPP}(y, x)]$.

This has the consequence that space is indefinitely divisible. There are therefore no 'atoms' — regions which cannot be subdivided — in this version of the theory (though see (Randell, Cui and Cohn 1992) for how this might be achieved). Also see (Bennett 1995) for a discussion of other existential axioms that might be added to the theory.

The relations defined in terms of C can be embedded in a relational lattice with the top element interpreted as tautology, and the bottom element as contradiction. This relational lattice is shown as figure 6, together with pictorial representations of the eight 'base relations' (known as RCC-8, which make up the layer immediately above the bottom element:) {DC, EC, PO, TPP, NTPP, EQ, TPPi, NTPPi}. These form a jointly exhaustive and pairwise disjoint (JEPD) set: one and only one of these relations will hold between a pair of regions.

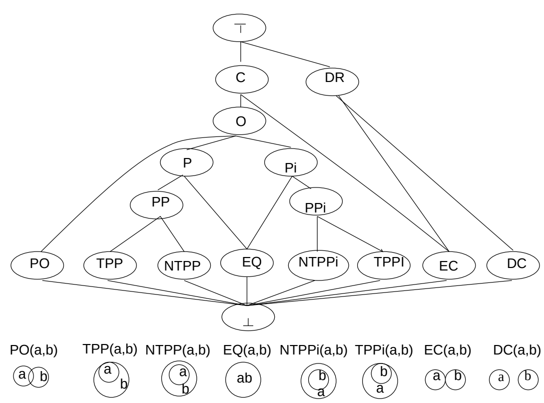

Fig. 6. A lattice defining the subsumption hierarchy of dyadic relations defined in terms of C

In (Randell and Cohn 1989) we first defined the notion of a quasi-manifold, i.e. an n-dimensional region which is well connected in the sense that it cannot be divided into two regions of dimension n, which only connect along a boundary of dimension n-2 or less. Thus the region in figure 7 is not a quasi-manifold. Various definitions (some making use of conv in their formulation) can be found in previous papers from the Leeds group; (Gotts 1994) explores this notion of well connected regions much more fully. Here, we will use $\mathsf{Manifold}(x)$ to predicate that a region has this property.

$\mathsf{Manifold}(x) \equiv_{def}$
$\forall(x_1 x_2)[x = x_1 + x_2 \rightarrow \exists z[\mathsf{O}(z, x_1) \wedge \mathsf{O}(z, x_2) \wedge \mathsf{DC}(z, \mathsf{compl}(x)) \wedge \mathsf{CON}(z)]]$

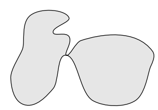

Fig. 7. This region is not a quasi-manifold since it can be divided into two point connected regions: there is no one piece connected region overlapping both parts which is DC to the compl of the region.

Another concept, defined in (Cohn et al. 1994) is the idea of a maximal connected (i.e. one piece) subpart of a region (known as a 'component' in topology). In this paper we will want to insist that each maximal connected subpart of a region is also a quasi-manifold, so we will use the following slightly revised definition:

$\mathsf{MAX\_P}(x, y) \equiv_{def} \mathsf{Manifold}(x) \wedge \mathsf{P}(x, y) \wedge \neg\exists z[\mathsf{PP}(x, z) \wedge \mathsf{P}(z, y) \wedge \mathsf{Manifold}(z)]$

Several axiomatisations of conv have been given in various papers (e.g. (Bennett 1995), (Cohn et al. 1994), (Bennett 1994)). The most recent of these axioms are all repeated below for convenience along with some suggested by Antony Galton (personal communication).

$\forall x[\mathsf{conv}(\mathsf{conv}(x)) = \mathsf{conv}(x)]$
$\forall x[\neg x = \mathsf{conv}(x) \rightarrow \mathsf{TPP}(x, \mathsf{conv}(x))$
$\forall x \forall y[\mathsf{P}(x, y) \rightarrow \mathsf{P}(\mathsf{conv}(x), \mathsf{conv}(y))]$
$\forall x \forall y \mathsf{P}((\mathsf{conv}(x) + \mathsf{conv}(y), \mathsf{conv}(x + y))$
$\forall x \forall y[\mathsf{conv}(x) = \mathsf{conv}(y) \rightarrow \mathsf{C}(x, y)]$ [3]
$\forall x \forall y[\mathsf{conv}(x) * \mathsf{conv}(y) = \mathsf{conv}(\mathsf{conv}(x) * \mathsf{conv}(y))]$
$\forall x \forall y[\mathsf{DC}(x, y) \rightarrow \neg\mathsf{CONV}(x + y)]$
$\forall x \forall y[\mathsf{NTPP}(x, y) \rightarrow \neg\mathsf{CONV}(y - x)]$
$\forall x \forall y[[\mathsf{CONV}(x) \wedge \mathsf{CONV}(y)] \rightarrow \mathsf{CONV}(x * y)]$
$\forall x \forall y \forall z[[\mathsf{EC}(x, y) \wedge \mathsf{CONV}(x + y) \wedge \mathsf{EC}(y, z) \wedge \mathsf{CONV}(y + z) \wedge \mathsf{DC}(x, z)] \rightarrow \mathsf{CONV}(y)]$

[3] Actually this is not necessarily true for infinite regions.

where the predicate $\mathsf{CONV}(x)$ is defined in terms of the primitive function $\mathsf{conv}(x)$ thus:

$\mathsf{CONV}(x) \equiv_{def} x = \mathsf{conv}(x)$

Notice that the above axiomatisation also defines a predicate $\mathsf{CONV}(x)$ which is true for convex regions. A major open question is whether these axioms fully capture the intended meaning of conv and which if any of these axioms are redundant. One possible line of attack would be to introduce an alternative primitive, 'region y is between regions x and z' (cf Tarski's axiomatisation of Geometry (Balbiani, Dugat, del Cerro and Lopez 1994), (Bennett 1995), (Tarski 1929), (Quaife 1989)), which uses a point based betweenness primitive, and define conv in terms of this primitive. Linking this primitive to Tarski's point based betweenness relation may provide a way to verify the completeness of the axiomatisation.

We use conv to define three relations: '$\mathsf{INSIDE}(x, y)$' ('x is inside y'), '$\mathsf{P\text{-}INSIDE}(x, y)$' ('$x$ is partially inside y') and '$\mathsf{OUTSIDE}(x, y)$' ('x is outside y'), each of which also has an inverse. Two functions[4] capturing the concept of the inside and the outside of a particular region are also definable: inside(x) and outside(x). This particular set of relations refines $\mathsf{DR}(x, y)$ in the basic theory. In (Randell, Cui and Cohn 1992, Randell, Cohn and Cui 1992a) we generated a JEPD set of relations by taking the relations given above, their inverses, and the set of relations that result from non-empty intersections. The set of base relations for this particular set were then finally generated by defining an EC and DC variant for each of these relations. A new set of base relations (using the relations defined immediately above) is constructed according to the following schema:

$$\alpha\_\beta\_\gamma(x, y) \equiv_{def} \alpha(x, y) \wedge \beta(x, y) \wedge \gamma(x, y)$$

where: $\alpha \in \{\mathsf{INSIDE}, \mathsf{P\text{-}INSIDE}, \mathsf{OUTSIDE}\}$, $\beta \in \{\mathsf{INSIDEi}, \mathsf{P\text{-}INSIDEi}, \mathsf{OUTSIDEi}\}$, and $\gamma \in \{\mathsf{EC}, \mathsf{DC}\}$ excepting where $\alpha = \mathsf{INSIDE}$, $\beta = \mathsf{INSIDEi}$ and $\gamma = \mathsf{DC}$. E.g. INSIDE_P-INSIDEi_DC(x, y) is true when x is inside y, y is partially x and the two regions are DC. The reason that we exclude the case of two regions being mutually inside each other and DC is that such a configuration of regions is only possible when certain kinds of infinite regions are allowed in one's ontology – which we do not wish to permit (see the fifth axiom in the list above): see (Cohn et al. 1994) for further discussion. This gives a total of 23 base relations instead of the original 8. Further distinctions in INSIDE can be made, as mentioned above which we will not consider further here (Cohn et al. 1994).

3 Using the Convex Hull Operator to Describe Qualitative Shapes

For the present, we will just consider the shapes of 2D regions which are quasi-manifolds. We will also only consider geometric insides (so excluding regions

[4] Note that it does not really make much sense to define a functional analogue of P-INSIDE as this would simply be the sum of the inside and the outside, i.e. the complement of x!

which have interior holes (i.e. have 'topological insides') – this includes the case of an interior hole which is point connected with the rest of the exterior (e.g. a region which meets itself at a point). The technique we will describe is based on identifying each maximal connected part ($\mathsf{MAX\_P}$) of the inside of the region and describing the relationships between the maximal parts of the inside. We will define a predicate $\mathsf{Concavity}(x, y)$ which is true when x is a maximal connected part of the inside of y:

$\mathsf{Concavity}(x, y) \equiv_{def} \mathsf{MAX\_P}(x, \mathsf{inside}(y))$

Thus in figure 8 we can distinguish eight different parts of the inside, named $I_1 ... I_8$.

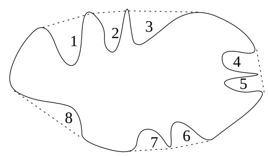

Fig. 8. This shape has 8 maximal subparts of the geometric inside (concavities).

We can now start to describe the relationship between the I_j:

$$\mathsf{EC}(I_1, I_2) \wedge \mathsf{EC}(I_2, I_3) \wedge \mathsf{EC}(I_4, I_5) \wedge \mathsf{EC}(I_6, I_7) \wedge \bigwedge_{\langle i,j \rangle \in \gamma} \mathsf{DC}(I_i, I_j)$$

$$where\ \gamma = \{\langle i, j\rangle :\ i < j\} - \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 4, 5\rangle, \langle 6, 7\rangle\}$$

However this does not distinguish between the shape of figure 8 and that in figure 9 which is different because the single inside (I_8) is between the pair of double insides in figure 9 whilst it is between the triple and the double in figure 8.

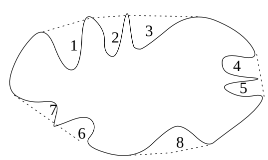

Fig. 9. This shape differs from the previous one in the permutation of the EC-groups of 'insides'.

In order to distinguish these two situations (and other similar ones in general[5]), we may divide the region (whose convex hull we have formed) into n+1 separate

[5] In fact the analysis below only works for regions with four or more concavities.

parts, where n is the number of concavities, such that n of them all EC the n+1st part, and the others each externally connect with the complement of the convex hull just once and with exactly two other of the n parts. The n+1st part is an NTPP of the shape. Suitable partitions for figures 8 and 9 are displayed in figure 10.

Fig. 10. Partitioning can distinguish these two shapes.

Thus $p_1, ...p_n, q$ is a suitable partition of a region p if
$\forall(1 \leq i \leq n)\exists(j,k)[\mathsf{DC}(p_j,p_k) \wedge \mathsf{EC}(p_i,p_j) \wedge \mathsf{EC}(p_i,p_k) \wedge \mathsf{EC1}(p_i, \mathsf{compl}(\mathsf{conv}(x))) \wedge$
$\mathsf{NTPP}(q,p) \wedge q = (p_1 + ... + p_n) \wedge \mathsf{EC}(p_i,q)]$
where
$\mathsf{EC1}(x,y) \equiv_{def} \mathsf{EC}(x,y) \wedge \exists z[\mathsf{CONV}(z) \wedge \neg\mathsf{Manifold}(x - z + y)]$

We can now distinguish the two figures since in figure 10(a), there is a p_i (shown shaded) which ECs both the triple-inside and the single-inside, whereas this is not the case in figure 10(b). It is worth commenting briefly of the definition of $\mathsf{EC1}(x,y)$: the predicate is intended to be true when x and y EC just once (i.e. they have only one boundary in common). This is the case if we can remove part of one of the regions (x) and 'separate' the two regions. The part which is removed must be convex since otherwise one could easily remove the tips of two touching fingers with a single non convex U shaped region. Moreover the separation condition cannot be DC since x may be locally non convex where it diverges from y. Thus we insist that the sum of the two regions minus the part removed is not a quasi-manifold.

However note that we cannot distinguish the shapes in figure 11 which differ purely by virtue of the ordering of their various insides around their perimeters. This is not surprising since if one of them is viewed 'from the other side of the

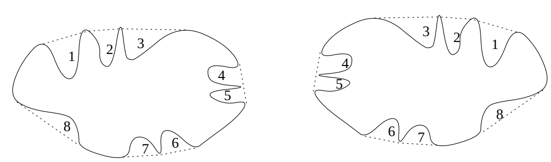

Fig. 11. These two shapes are indistinguishable with our technique.

paper', they are identical. We could imagine introducing orientation information (c.f. (Freksa 1992b)), to distinguish such situations, but will not do this here.

The partitioning technique described above could be used to define a predicate $\mathsf{Adjacent}(x, y)$ which is true when x and y are adjacent (round the perimeter) concavities, whether or not they are DC or EC. In terms of the analysis above it would mean that a single p_i is EC to both. However, we can give a direct definition which does not make use of the above partitioning technique.

$$\begin{aligned}&\mathsf{Adjacent}(i1, i2, x) \equiv_{def} \mathsf{Concavity}(i1, x) \wedge \mathsf{Concavity}(i2, x) \wedge \\ &\quad \exists z[\mathsf{EC}(z, i1) \wedge \mathsf{EC}(z, i2) \wedge \mathsf{PP}(z, x) \wedge \mathsf{CON}(z) \wedge \mathsf{DC}(z, \mathsf{inside}(x) - i1 - i2) \wedge \\ &\qquad \forall(i3, i4)[[\mathsf{Concavity}(i3, x) \wedge \mathsf{Concavity}(i4, x) \wedge \mathsf{DC}(i1 + i2, i3 + i4)] \rightarrow \\ &\qquad\quad \exists w[\mathsf{CON}(w) \wedge \mathsf{PP}(w, x) \wedge \mathsf{EC}(w, i3) \wedge \mathsf{EC}(w, i4) \wedge \mathsf{DR}(w, z)]]]\end{aligned}$$

According to this definition, two concavities are Adjacent iff there is a one-piece part of x that ECs both, but no other concavity, (i.e if the two concavities can be linked together) and every other pair of concavities of x can be similarly linked without any of these links crossing the first link.

There is another distinction we can make; consider the two shapes in figure 12. Both have two indentations, but will have identical qualitative shape descriptions in our formalisation thus far. However we can distinguish them: in the right hand

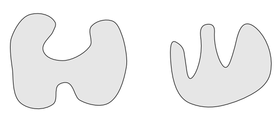

Fig. 12. We want to distinguish these two shapes.

shape we can remove a region (the middle 'finger') and we will still be left with one concavity composed in part of each of the original concavities. In the left hand shape there is no sub-region which can be removed (leaving the region in one piece as a quasi-manifold) and leave just one concavity with the same properties. We can define a predicate to detect the fact that the two insides $i1$ and $i2$ in the right hand shape(x) in figure 12 are on the 'same side' thus:

$$\begin{aligned}&\mathsf{SameSide}(i1, i2, x) \equiv_{def} \mathsf{Concavity}(i1, x) \wedge \mathsf{Concavity}(i2, x) \wedge \\ &\quad \exists z[\mathsf{P}(z, x) \wedge \mathsf{Manifold}(i1 + i2 + z) \wedge \mathsf{Manifold}(x - z) \wedge \\ &\qquad \mathsf{O}(i1, \mathsf{conv}(x - z)) \wedge \mathsf{O}(i2, \mathsf{conv}(x - z))]\end{aligned}$$

Thus if $i1$ and $i2$ are concavities of x then they are on the same side if there is some part of x which when added to $i1$ and $i2$ forms a quasi-manifold whilst leaving the rest of x as a quasi-manifold as well. Moreover, at least part of each concavity must still be a concavity of the remainder of x. Notice that this definition means that the two concavities in each of the shapes in figure 13 are SameSide. If we wanted to distinguish these two cases, then we could modify the final two conjuncts of the definition of SameSide to yield the following predicate which is true of the left hand shape but not the right hand one. In this case there is a point/part of each the three 'arms' which are colinear (a straight edge can EC all three arms): hence we term this predicate SsColinear. Notice that whereas SsColinear is transitive (on its first two arguments) SsNotColinear is not (see figure 14).

$\mathsf{SsColinear}(i1, i2, x) \equiv_{def} \mathsf{Concavity}(i1, x) \wedge \mathsf{Concavity}(i2, x) \wedge$
$\exists z[\mathsf{P}(z, x) \wedge \mathsf{Manifold}(i1 + i2 + z) \wedge \mathsf{Manifold}(x - z) \wedge$
$\mathsf{P}(i1 + i2 + z, \mathsf{inside}(x - z))$

$\mathsf{SsNotColinear}(i1, i2, x) \equiv_{def} \mathsf{SameSide}(i1, i2, x) \wedge \neg\mathsf{SsColinear}(i1, i2, x)$

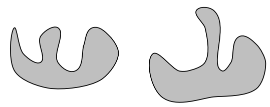

Fig. 13. Can we distinguish these two kinds of same sideness?

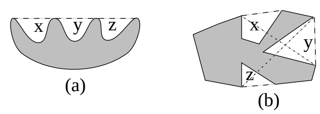

Fig. 14. Examples to demonstrate the transtivity of SsColinear and the lack of transitivity of SsNotColinear. In (a) x, y, z are all SsColinear pairwise. However in (b), although x, y and y, z are SsNotColinear, x, z are not.

It is worth summarising the shape language we have introduced so far. The shape of an object is described by its maximal one-piece inside (concavities); the relationship between any two concavities can be described using four attributes:

- Are they DC or EC?
- Are they SsColinear, SsNotColinear or ¬SameSide?
- Are they Adjacent or not?

Thus there are 12 possible descriptions for each pair of concavities – except that EC implies Adjacent, so DC and Adjacent is impossible, yielding a total of 11 actual JEPD descriptions. Also note that it requires at least four concavities before a pair can be ¬Adjacent.

4 A Hierarchical Representation

We can turn to more fine grained distinctions. One might like to distinguish the two regions depicted in figure 15. The technique described thus far will not distinguish them, but this is easily achieved if we simply apply the technique recursively to each maximal inside of the original shape. Figure 16 depicts the two maximal insides from figure 15 and it is clear the two original shapes are now distinguished. Of course this technique can be applied as often as necessary; consider figure 17.

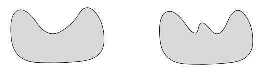

Fig. 15. A hierarchical representation will distinguish these two shapes.

Fig. 16. The insides of the two previous shapes, which are distiguishable by virtue of the shape of *their* insides.

The technique would seem to be ideally suited to describing objects such as coastal regions with many bays – the Norwegian fjords for example.

We can use this shape description technique to test whether two regions will possibly 'plug together'. For instance which of the 'pieces' on the right hand side of figure 18 might 'plug into' the shape depicted on the left hand side of figure 18? We can answer this by determining if a part of one of the 'pieces' has the same qualitative shape description as an inside of the left hand shape.

This might be useful for applications just as (qualitative) jigsaws, locking together of mechanical parts or describing the shape made by a rigid body on a compliant one or, in a GIS context, when reasoning about the original configuration of separated landmasses prior to geological movement.

5 Other Aspects of Qualitative Shape

There are other distinctions in shape which are expressable in our formalism. For example, we can define predicates for convex polygons (i.e. convex closed straight sided contours) of arbitrary degree. The definition for a triangle is as follows:

$$\mathsf{Triangle}(x) \equiv_{def} \mathsf{CON}(x) \wedge \mathsf{CONV}(x) \wedge \exists(u,v,w)[\mathsf{compl}(x) = (u+v+w) \wedge \mathsf{DC}(u,v) \wedge \mathsf{DC}(v,w) \wedge \mathsf{DC}(u,w) \wedge \mathsf{CONV}(u) \wedge \mathsf{CONV}(v) \wedge \mathsf{CONV}(w)]$$

This definition states that a triangle is a connected convex region, whose complement can be partitioned into three disjoint convex regions. Similar definitions can of course be written for quadralaterals, pentagons, hexagons etc.

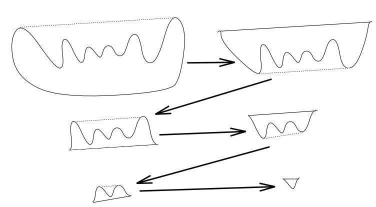

Fig. 17. The technique can be applied recursively as often as necessary to distinguish shapes.

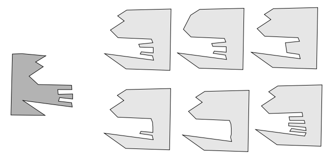

Fig. 18. Which of these shapes might 'plug' into the leftmost one?.

Given the definition for a triangle, it is of course possible to test for a region being an arbitrary polygon using the well known property that any polygon can be represented as the sum of triangles.

$\mathsf{Polygon}(x) \equiv_{def} \mathsf{Triangle}(x) \vee$
$[\mathsf{CON}(x) \wedge \exists y[\mathsf{PP}(y,x) \wedge \mathsf{Triangle}(y) \wedge \mathsf{Polygon}(x-y)]]$

Convex polygons can of course be tested for separately with $\mathsf{CONV}(x)$.

Although symmetry is of course not definable quantitatively within this system, we could define a notion of qualitative symmetry (i.e. any shape which is qualitatively symmetrical would be continuously deformable to one which was quantiatively symmetrical and which has the same qualitative description). For example, a 180 degree qualitative rotational symmetry exists if a region x can be partitioned into two DC and CON regions $x1$ and $x2$ each of which has the same description in our language of shape. We must ensure that the partition between these two regions takes the form of a straight line, i.e. that $\mathsf{CONV}(\mathsf{conv}(x) - \mathsf{conv}(x1)) \wedge \mathsf{CONV}(\mathsf{conv}(x) - \mathsf{conv}(x2))$

6 Changes in Shape

In earlier papers (Cohn, Randell, Cui and Bennett 1993, Cohn et al. 1994) we presented the transition networks[6] for our sets of relations. These networks tell us that if a particular relation holds between a pair of regions at some particular time, then, assuming continuous motion and deformation[7], what the 'next' possible relationship that might hold could be. For example figure 19 diagrams the situation for RCC8. Here, we are not interested in relationships between regions, but rather, a shape description of a single region. Since the number of possible shape descriptions is rather large (in fact unbounded since a region may have arbitrarily many concavities), it is not as easy to draw a similar diagram for the shape descriptions defined in this paper. So in figure 20 we give a diagram which shows the possible continuous transformations for regions (without holes) with up to two top level concavities – *assuming a single local deforming process only*[8] (otherwise, for example, the left hand convex shape could be transformed directly to any of the others).

[6] (Freksa 1992a) terms these conceptual neighbourhoods, and (Egenhofer and Al-Taha 1992) have a very closely related notion: closest topological distance.

[7] In the GIS context, processes such as erosion and flooding would yield these kind of changes.

[8] In the sense of (Leyton 1988), e.g. making an indentation.

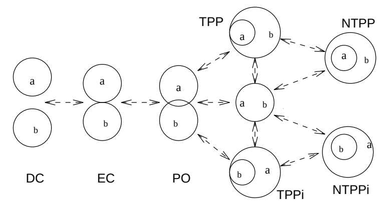

Fig. 19. The continuous transitions (conceptual neighbourhood) for RCC8

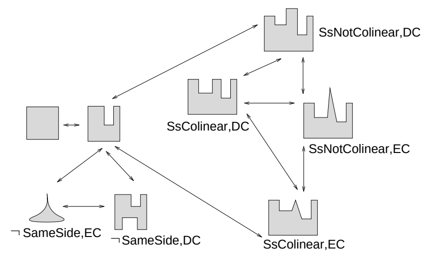

Fig. 20. A diagram showing the continous transformations possible between regions with up to 2 concavities.

7 Related Work

Perhaps the most closely related work to the underlying RCC8 calculus and the description of holes, is that of (Egenhofer, Clementini and Di Felice 1994, Egenhofer 1994). This work is not based on a logical framework and the notion of connection and convexity as ours is, but rather on regions defined in terms of sets of points and relations are defined using matrices which compare the set intersections of the interior and boundary of two regions. In the first of these two papers an analysis of 2D regions with holes is presented whilst in the latter the model is refined to consider not just whether the intersection is empty or not but its dimensionality; moreover regions are broken into components and complex relationships are built by composing the relationships between the sequence of the components of two regions.

The well known work of (Leyton 1988) is highly relevant to this research. Leyton describes the shape of 2D[9] regions by noting the sequence of four different kinds of curvature extrema along the perimeter of a region. He also associates a process with each of these kinds of curvature extrema: protusion, indentation, squashing and internal resistence. He gives a set of 6 rewrite rules which apply to a single curvature extrema which provide a set of transformation rules for evolving shapes over time (e.g. 'squashing continues till it indents', 'internal resistence continues till it protudes'). These give a notion somewhat similar to the notion of 'conceptual neighbourhood' described above. He also shows how these rules can be applied to give the possible process paths between two given

[9] He does in fact consider the 3D case as well, but not at the same level of detail.

shapes, analogously to our work on qualitative spatial simulation (Cui, Cohn and Randell 1992). In many ways the work presented here is very similar to Leyton's. One crucial difference is that he has no notion of convex hull and so cannot distinguish between SsColinear and SsNotColinear; our approach will however distinguish these two shapes. His approach does make distinctions our approach cannot though: for example all convex objects have identical shape descriptions in our approach whereas there are three different kinds of one piece convex object having a maximum of eight different curvature extrema.

Another approach to the representation of shape is to use voronoi regions rather than convex hulls. (Edwards 1993) outlines some of the possibilities.

It turns out that the idea of describing shape using convex hulls and a hierarchical decomposition of the concavities has already been briefly explored in the computer vision literature (Sklansky 1972), under the terminology of *concavity trees*.

Finally, the work of (Casati and Varzi 1994) is worth mentioning; they present an excellent treatise on the nature of holes, and an appendix starts to give a formal axiomatisation of this which is continued in (Varzi 1993).

8 Final Comments

This paper has presented a relatively informal description of a possible approach to qualitative shape description using a conceptually very simple formalism with just two primitive concepts, both of which are well known, natural, and easy to compute if necessary.

It should be clear that the technique outlined above could also be used to describe the shape of holes in a region: for example consider the holes in the region displayed in figure 21. Equally, the extension to 3D and non manifolds should be possible though the details still need to be worked out.

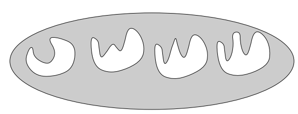

Fig. 21. The technique can easily be extended to differentiate these holes.

Future work will concentrate on the continued formalisation of these ideas, on theoretical aspects such as complexity analysis and on their practical application. Possible refinements of 'scattered-inside', i.e. the shape of multipiece objects should also be investigated. The conceptual neighborhood diagram presented in section 6 needs to be placed on a sound theoretical footing by building in proper theory of spatial processes and their effect on the shape of regions.

Finally, a proper evaluation of the utility of the formalism should be carried out: can it really be used to describe and distinguish between the shapes of a wide variety of real objects? Certain objects clearly have a natural and obvious description in the formalism: consider, for example, the teeth of a tenon saw – these are a series of concavities which are all SsColinear and EC. How well does the formalism describe other commonplace objects?

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References

Balbiani, B., Dugat, V., del Cerro, L. F. and Lopez, A.: 1994, *Eléments de géométrie mécanique*, Editions Hermes.

Bennett, B.: 1994, Spatial reasoning with propositional logics, *in* J. Doyle, E. Sandewall and P. Torasso (eds), *Principles of Knowledge Representation and Reasoning: Proceedings of the 4th International Conference (KR94)*, Morgan Kaufmann, San Francisco, CA.

Bennett, B.: 1995, Modal logics for qualitative spatial reasoning, *Bulletin of the Interest Group on Propositional and Predicate Logics (IGPL)*. To appear.

Casati, R. and Varzi, A.: 1994, *Holes and Other Superficialities*, MIT Press, Cambridge, MA.

Clarke, B. L.: 1981, A calculus of individuals based on 'connection', *Notre Dame Journal of Formal Logic* **23**(3), 204–218.

Clarke, B. L.: 1985, Individuals and points, *Notre Dame Journal of Formal Logic* **26**(1), 61–75.

Cohn, A. G.: 1987, A more expressive formulation of many sorted logic, *Journal of Automated Reasoning* **3**, 113–200.

Cohn, A. G., Randell, D. A. and Cui, Z.: 1994, Taxonomies of logically defined qualitative spatial relations, *in* N. Guarino and R. Poli (eds), *Formal Ontology in Conceptual Analysis and Knowledge Representation*, Kluwer. To appear.

Cohn, A. G., Randell, D. A., Cui, Z. and Bennett, B.: 1993, Qualitative spatial reasoning and representation, *in* N. P. Carreté and M. G. Singh (eds), *Qualitative Reasoning and Decision Technologies*, CIMNE, Barcelona, pp. 513–522.

Cui, Z., Cohn, A. G. and Randell, D. A.: 1992, Qualitative simulation based on a logical formalism of space and time, *Proceedings AAAI-92*, AAAI Press, Menlo Park, California, pp. 679–684.

Edwards, G.: 1993, The voronoi model and cultural space: Applications to the social sciences and humanities, *in* A. U. Frank and I. Campari (eds), *Spatial Information Theory: A Theoretical Basis for GIS*, Vol. 716 of *Lecture Notes in Computer Science*, Springer Verlag, Berlin, pp. 202–214.

Egenhofer, M.: 1994, Topological similarity, Proc FISI Workshop on the Topological Foundations of Cognitive Science.

Egenhofer, M. J. and Al-Taha, K. K.: 1992, Reasoning about gradual changes of topological relationships, *in* A. U. Frank, I. Campari and U. Formentini (eds), *Theories* *and Methods of Spatio-temporal Reasoning in Geographic Space*, Vol. 639 of *Lecture Notes in Computer Science*, Springer-Verlag, Berlin, pp. 196–219.

Egenhofer, M. J., Clementini, E. and Di Felice, P.: 1994, Toplogical relations between regions with holes, *Int. Journal of Geographical Information Systems* **8**(2), 129–144.

Freksa, C.: 1992a, Temporal reasoning based on semi-intervals, *Artificial Intelligence* **54**, 199–227.

Freksa, C.: 1992b, Using orientation information for qualitative spatial reasoning, *in* A. U. Frank, I. Campari and U. Formentini (eds), *Proc. Int. Conf. on Theories and Methods of Spatio-Temporal Reasoning in Geographic Space*, Springer-verlag, Berlin.

Gotts, N. M.: 1994, How far can we 'C'? defining a 'doughnut' using connection alone, *in* J. Doyle, E. Sandewall and P. Torasso (eds), *Principles of Knowledge Representation and Reasoning: Proceedings of the 4th International Conference (KR94)*, Morgan Kaufmann.

Leyton, M.: 1988, A process grammar for shape, *Artificial Intelligence* p. 34.

Quaife, A.: 1989, Automated development of tarski's geometry, *J. Automated Reasoning* **5**(1), 97–118.

Randell, D. A.: 1991, *Analysing the Familiar: Reasoning About Space and Time in the Everyday World*, PhD thesis, University of Warwick.

Randell, D. A. and Cohn, A. G.: 1992, Exploiting lattices in a theory of space and time, *Computers and Mathematics with Applications* **23**(6-9), 459–476. Also appears in "Semantic Networks", ed. F. Lehmann, Pergamon Press, Oxford, pp. 459-476, 1992.

Randell, D. A., Cohn, A. G. and Cui, Z.: 1992a, Computing transitivity tables: A challenge for automated theorem provers, *Proceedings CADE 11*, Springer Verlag, Berlin.

Randell, D. A., Cohn, A. G. and Cui, Z.: 1992b, Naive topology: Modelling the force pump, *in* P. Struss and B. Faltings (eds), *Advances in Qualitative Physics*, MIT Press, pp. 177–192.

Randell, D. A., Cui, Z. and Cohn, A. G.: 1992, A spatial logic based on regions and connection, *Proc. 3rd Int. Conf. on Knowledge Representation and Reasoning*, Morgan Kaufmann, San Mateo, pp. 165–176.

Randell, D. and Cohn, A.: 1989, Modelling topological and metrical properties of physical processes, *in* R. Brachman, H. Levesque and R. Reiter (eds), *Proceedings 1st International Conference on the Principles of Knowledge Representation and Reasoning*, Morgan Kaufmann, Los Altos, pp. 55–66.

Sklansky, J.: 1972, Measuring concavity on a rectangular mosaic, *IEEE Trans. on Computers* **C-21**(12), 1355–1364.

Tarski, A.: 1929, Les fondaments de la géométrie des corps, *Ksiega Pamiatkowa Pierwszego Polskiego Zjazdu Matematycznego* pp. 29–33. A suplement to *Annales de la Société Polonaise de Mathématique*. English translation, 'Foundations of the Geometry of Solids', in A. Tarski, *Logic, Semantics, Metamathematics*, Oxford Clarendon Press, 1956.

Varzi, A. C.: 1993, Spatial reasonng in a holey world, *Proceedings of the Spatial and Temporal Reasoning workshop, IJCAI-93*, pp. 47–59.