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Taxonomies of Logically Defined Qualitative Spatial Relations *

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Abstract.

This paper develops a taxonomy of qualitative spatial relations for pairs of regions, which are all logically defined from two primitive (but axiomatised) notions. The first primitive is the notion of two regions being *connected*, which allows eight jointly exhaustive and pairwise disjoint relations to be defined. The second primitive is the convex hull of a region which allows many more relations to be defined. We also consider the development of the useful notions of composition tables for the defined relations and networks specifying continuous transitions between pairs of regions. We conclude by discussing what kind of criteria to apply when deciding how fine a taxonomy to create.

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1 Introduction

Although the use of interval temporal logics has been an active research area in AI for some time, the analogous development of ontologies for space and spatial logics based on regions has only relatively recently started to become a serious research activity (e.g. Pribbenow and Schlieder (1992), Narayannan (1992)). Various approaches have been promulgated; for example one can simply use Allen's (1983) temporal relations on each of the cartesian axes to specify the qualitative relationship between two regions (e.g. Hernández (1992), Mukerjee and Joe (1990)), but this has the disadvantage of either requiring knowledge about the absolute orientation of the two regions or their orientation relative to a fixed viewpoint. For many applications one might only have local information available. Qualitative orientation representation and reasoning has been explicitly investigated by Zimmerman and Freksa (1993) and Mukerjee and Joe (1990) amongst others.

Given a qualitative spatial knowledge there are various kinds of reasoning that have been investigated or are desirable. For example one can perform a qualitative spatial simulation (Cui, Cohn and Randell 1992), or reason spatially about physical systems such as a force pump (Randell, Cohn and Cui 1992b), or the heart (Gotts, Hunter, Hamlet and Vincent 1989). Another obvious application area is natural language understanding (see e.g. Vieu (1991) or Aurnague (1991)). Other work can be found in, for example Pribbenow and Schlieder (1992) or Narayannan (1992).

In previous work (Randell and Cohn 1989, Randell and Cohn 1992, Randell, Cui and Cohn 1992) we have started to develop sets of *jointly exhaustive and pairwise disjoint* (henceforth: JEPD) sets of binary relations for pairs of spatial regions. This work has been based in sorted first order logic and is based on a perhaps surprisingly sparse set of primitives; from a single primitive binary relation C(x,y)[1], 'x is connected to y', which is axiomatised to be reflexive and symmetric, a set of eight JEPD relations can be defined.[2] This work is based on that of Clarke (1981, 1985).

By adding a further function, conv(x), the convex hull of a region, and suitably axiomatising it, many more relations can be defined, for example one region may be inside, or partly inside, or outside another region's convex hull whilst not overlapping at all with the other region. We had started to develop a set of such relations, e.g. (Randell, Cui and Cohn 1992) defines 22 JEPD relations; the purpose of this paper is to explore the possible options in much more detail yielding a much larger set of JEPD relations. Of course for some purposes, a small, coarse grained set of relations may be sufficiently descriptive but given the complexities in the 3D spatial world, with the many ways in which various shaped regions can interact with each other, it is easy to see that many subtleties will be missed by a taxonomy of just 8 or 22 relations. Ultimately of course, no finite taxonomy will be adequate, but an extended set of relations (or a calculus of relations in which new finer grained taxonomies, with known properties, can be easily built), would be useful in many situations. For example, although

[1] A word on notation: predicates will always start with a capital letter, functions and constants with a lower case letter; this should help easy any confusion when we later define predicates and functions with the same name.

[2] Previously we had nine relations, splitting the equality relation into two depending on whether the regions were topologically closed or not, but we now prefer not to make the distinction between topologically closed, open or semi-open regions – see (Randell, Cui and Cohn 1992) for further details and discussion. It is also worth pointing out that Egenhofer and Franzosa (1991) have developed an isomorphic set of eight spatial predicates but from a different mathematical foundation.

the 22 relation taxonomy can describe one region being inside the convex hull of another, it cannot distinguish between, say, the food inside a pressure cooker with the lid off and with the lid on (which may be very important for all kinds of reason, e.g. in the latter case the cat may not be able to steal it, or because the food may not boil over [become *partly inside*] – the impossibility of this kind of transition can be detected by one of our reasoning processes described later).

There are many ways in which finer grained qualitative distinctions could be made. In this paper we will simply confine ourselves to refining the simple notions of inside, partially inside and outside to be found in (Randell, Cui and Cohn 1992), though we briefly consider other possible dimensions of refinement at the end of the paper.

The structure of the rest of the paper is as follows: first we briefly review our previous work in the area and then we develop a rich taxonomy of qualitative spatial relations using only the existing two primitives. Then we present a extended set of envisioning axioms and discuss the consequences of the extended ontology on the provision of a composition/transitivity table. We conclude with a discussion about how fine grained a taxonomy should be.

2 Previous Work

Here we briefly review material to be found in (Randell, Cui and Cohn 1992) but omitting formal definitions and contenting ourselves with simply describing the relations defined there. The basic ontological entity we consider is a *region*; note that boundaries and points are not regions.[3] Also note that we only consider regions which do not have missing interior points or lines or exterior 'spokes', i.e. assuming a point set theoretic interpretation of regions, the interior of any region must equal the interior of the closure, and the closure of any region must be identical to the closure of the interior. Regions in the theory support either a spatial or temporal interpretation, though we will only consider the spatial interpretation here. Informally, these regions may be thought to be potentially infinite in number, and any degree of connection between them is allowed in the intended model, from external contact to identity in terms of mutually shared parts. The formalism supports two or three dimensional interpretations (or higher dimensions!).

The basic part of the formalism[4] assumes one primitive dyadic relation: $\mathsf{C}(x,y)$ read as 'x connects with y'. The relation $\mathsf{C}(x,y)$ is axiomatised to be reflexive and symmetric. We can give a topological model to interpret the theory, namely that $\mathsf{C}(x,y)$ holds when the topological closures of regions x and y share a common point.[5]

Using $\mathsf{C}(x,y)$, a basic set of dyadic relations are defined: '$\mathsf{DC}(x,y)$' ('x is disconnected from y'), '$\mathsf{P}(x,y)$' ('x is a part of y'), '$\mathsf{PP}(x,y)$ ('x is a proper part of y'), '$x = y$' ('x is identical with y'), '$\mathsf{O}(x,y)$' ('x overlaps y'), '$\mathsf{DR}(x,y)$' ('x is discrete from y') '$\mathsf{PO}(x,y)$' ('x partially overlaps y'), '$\mathsf{EC}(x,y)$' ('x is externally connected with y)', '$\mathsf{TPP}(x,y)$' ('x is a tangential proper part of y') and '$\mathsf{NTPP}(x,y)$' ('x is a nontangential proper part of y'). The relations: P,PP,TPP

[3] However we believe that from a modelling point of view, such mathematical abstractions are not necessary and one can use special kinds of regions such as *skins* and *atoms* – see (Randell, Cui and Cohn 1992).

[4] We use a sorted logic; for the most part this need not concern us here; important sortal restrictions will be mentioned as appropriate.

[5] In Clarke's theory and in our original theory (Randell and Cohn 1989, Randell and Cohn 1992) when two regions x and y connect, they are said to share a point in common; thus the interpretation of the connects relation here and in (Randell, Cui and Cohn 1992) is weaker.

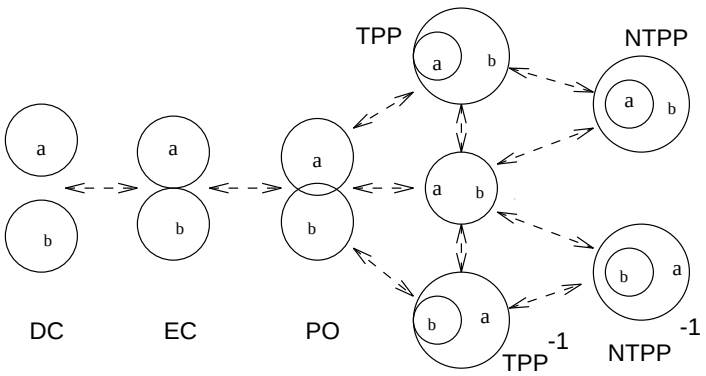

Figure 1: A pictorial representation of the base relations and their direct topological transitions.

and NTPP being non-symmetrical support inverses. For the inverses we use the notation Φ^{-1}, where $\Phi \in \{$P,PP,TPP and NTPP$\}$. Definitions for any of the inverse predicates are all of the form $\Phi^{-1}(x,y) \equiv_{def} \Phi(y,x)$ Of the defined relations, DC,EC,PO,=,TPP,NTPP and the inverses for TPP and NTPP are provably JEPD.

A pictorial representation of the base relations defined above is given in figure 1. This figure also depicts the direct (i.e. 'continuous') possible transitions between the base relations (cf Freksa's (1992) *conceptual neighbourhoods*). These transitions are alternatively expressed as 'envisioning axioms' (Randell 1991)[6] and are used as the basis of the qualitative simulation program in (Cui et al. 1992).

It is natural to want to compose regions or otherwise define new regions given existing ones: e.g. one might want to name the region which is the *sum* of the regions occupied by a cup and saucer (sum(cup,saucer)); or to pick out the region occupied by the non mouldy part of an apple, i.e. compute a *difference* (diff(apple,mouldy-part)); or to pick out the *intersection* of two regions defined by two rotating arms (prod(arm1,arm2)); finally we might want to name the *complement* of a region (compl(a)). Thus we define a set of Boolean functions. The functions: 'compl(x)', 'prod(x,y)' and 'diff(x,y)' are partial but are made total in the sorted logic by simply specifying sort restrictions and by introducing a new sort called NULL and an axiom relating it to the rest of the calculus. The sorts NULL and REGION are disjoint.

Given the ability to construct the sum of two arbitrary regions it is easy to see that regions can be divided into two kinds depending on whether there are topologically connected (i.e. in one piece) or disconnected (in more than one piece). Such scattered regions may be used to model, for example, a cup broken into several pieces. We therefore define a monadic predicate CON to distinguish the former kind of region.

An additional axiom is also required which stipulates that every region has a nontangential proper part.[7] This axiom mirrors a formal property of Clarke's theory, where he stipulates that every region has a nontangential part, and thus an interior (remembering that in Clarke's theory a topological interpretation is assumed).

A primitive function 'conv(x)' ('the convex-hull of x') is defined and axiomatised. We also can define a predicate CONV(x) which is true for convex regions.

We use conv to define three relations: INSIDE(x,y)' (x is inside y'), 'P-INSIDE(x,y)' ('x is

[6]Each link in the diagram corresponds to an axiom which expresses that if R1(x,y) is true then either R1(x,y) will continue to be true for ever, or x or y will disappear (become NULL) or R2(x,y) will be the next relationship to be true of x and y in the future.

[7]A consequence of this axiom is that there can be no *atomic* regions, i.e. regions which contain no subparts. For a discussion of how such regions can be introduced into the language, see (Randell, Cui and Cohn 1992).

partially inside y') and '$\mathsf{OUTSIDE}(x,y)$' ('x is outside y'), each of which also has an inverse. Two functions[8] capturing the concept of the inside and the outside of a particular region are also definable:$\mathsf{inside}(x)$ and $\mathsf{outside}(x)$. This particular set of relations refines $\mathsf{DR}(x,y)$ in the basic theory. In (Randell, Cui and Cohn 1992, Randell, Cohn and Cui 1992a) we generated a JEPD set of relations by taking the relations given above, their inverses, and the set of relations that result from non-empty intersections. The set of base relations for this particular set were then finally generated by defining an EC and DC variant for each of these relations. A new set of base relations (using the relations defined immediately above) are constructed according to the following schema:

$\alpha_\beta_\gamma(x,y) \equiv_{def} \alpha(x,y) \wedge \beta(x,y) \wedge \gamma(x,y)$

where: $\alpha \in \{\mathsf{INSIDE}, \mathsf{P\text{-}INSIDE}, \mathsf{OUTSIDE}\}$, $\beta \in \{\mathsf{INSIDE}^{-1}, \mathsf{P\text{-}INSIDE}^{-1}, \mathsf{OUTSIDE}^{-1}\}$, and $\gamma \in \{\mathsf{EC}, \mathsf{DC}\}$ excepting where $\alpha = \mathsf{INSIDE}$, $\beta = \mathsf{INSIDE}^{-1}$ and $\gamma = \mathsf{DC}$. This gives a total of 23 base relations instead of the original 8.[9] As an example of the use of the relations, the sequence in figure 2 can be described thus:

(i): $\mathsf{OUTSIDE_OUTSIDE}^{-1}\mathsf{_DC}(x,y)$,
(ii): $\mathsf{P\text{-}INSIDE_OUTSIDE}^{-1}\mathsf{_DC}(x,y)$,
(iii): $\mathsf{INSIDE_OUTSIDE}^{-1}\mathsf{_DC}(x,y)$,
(iv): $\mathsf{INSIDE_OUTSIDE}^{-1}\mathsf{_EC}(x,y)$,
(v): $\mathsf{PO}(x,y)$. [10]

3 Refining the Taxonomy

The first way in which we can refine the set of relations is to consider PO as well as DC and EC at the same time as INSIDE: intuitively x could be 'inside' y and partly overlapping y as depicted in figure 2(v). To capture this we need to modify the definition of INSIDE and P-INSIDE as specified below. For later notational convenience, we will also modify the definition of OUTSIDE in a similar manner.

$\mathsf{INSIDE}(x,y) \equiv_{def} \neg\mathsf{P}(x,y) \wedge \mathsf{P}(x,\mathsf{conv}(y))$
$\mathsf{P\text{-}INSIDE}(x,y) \equiv_{def} \neg\mathsf{P}(x,y) \wedge \mathsf{PO}(x,\mathsf{conv}(y)) \wedge \exists w[\mathsf{P}(w,\mathsf{conv}(y)) \wedge \neg\mathsf{P}(w,y) \wedge \mathsf{PO}(w,x)]$
$\mathsf{OUTSIDE}(x,y) \equiv_{def} \neg\mathsf{P}(x,y) \wedge \neg\exists w[\mathsf{P}(w,\mathsf{conv}(y)) \wedge \neg\mathsf{P}(w,y) \wedge \mathsf{PO}(w,x)]$

This is useful particularly in the context of qualitative spatial simulation (Cui et al. 1992) where the new base relations now allow Figure 2(v) to be described more accurately: $\mathsf{INSIDE_OUTSIDE}^{-1}\mathsf{_PO}$(x,y).

With this modification there are now 31 base relations (the original 8, less DC, EC and PO, plus the EC, DC and PO versions of the allowable combinations of INSIDE, OUTSIDE, P-INSIDE and their inverses).

[8] Note that it does not really make much sense to define a functional analogue of P-INSIDE as this would simply be the sum of the inside and the outside, i.e. the complement of x!

[9] This figure of 23 JEPD relations is one more than in our previous publications (and an earlier version of this paper): the extra relation is $\mathsf{INSIDE_INSIDE}^{-1}\mathsf{_EC}$ which we had thought was not physically realisable. However, either of the configurations in figure 4 models this relation.

[10] Alternatively, of course, the sequence could be described as: (i): $\mathsf{OUTSIDE_OUTSIDE}^{-1}\mathsf{_DC}(y,x)$, (ii): $\mathsf{OUTSIDE_P\text{-}INSIDE}^{-1}\mathsf{_DC}(y,x)$, (iii): $\mathsf{OUTSIDE_INSIDE}^{-1}\mathsf{_DC}(y,x)$, (iv): $\mathsf{OUTSIDE_INSIDE}^{-1}\mathsf{_EC}(y,x)$, (v): $\mathsf{PO}(y,x)$.

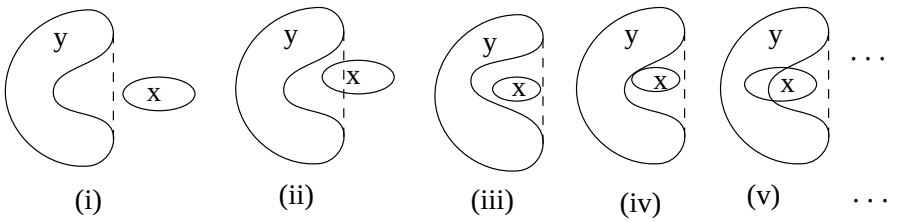

Figure 2: The natural sequence of x moving from being outside y, to partly inside, to inside, to inside but touching and finally to partially overlapping... Note that the dashed line denotes the extent of the convex hull of y.

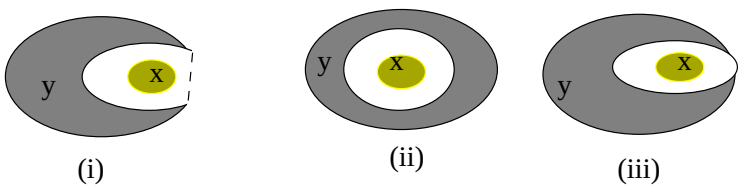

Figure 3: The Geometrical (i) and Topological inside (ii). Is (iii), where the 'arms' of y touch at a point a special case of (i) or (ii)?

Now we turn to considering ways in which the INSIDE relation could be specialised. The first specialisation is to differentiate between the *topological* and the *geometrical* insides: the former has the property that to pass from the topological inside of a region to the outside involves cutting through the region itself. In two dimensions the prototypical situations are illustrated in figure 3. Figure 3(i) illustrates the topological inside, figure 3(ii), the geometrical inside. Figure 3(iii) is a limiting case where the two 'arms' of the enclosing region meet at a point. It is a somewhat moot point as to whether one prefers to view (iii) as a specialisation of geometrical inside (as we did in (Randell, Cui and Cohn 1992)) or whether it makes more sense both conceptually and in practice to define it as a specialisation of topological inside since one still has to cut through the enclosing object to reach the outside (although one does not have to cut through any sub region of it, since the boundary is not part of an object as a boundary is not a region). We choose this alternative here and this is reflected in the definitions below.[11] It is also worth pointing out that although, intuitively, the geometric inside of an object such as a cup would seem to define the region available for filling with liquid, the geometric inside is actually more general than this: e.g. the region 'in' the handle is also part of the geometric inside of the cup. We will return to this 'problem' again shortly.

$$\begin{aligned}
&\mathsf{TOP\text{-}INSIDE}(x,y) \equiv_{def} \mathsf{INSIDE}(x,y) \wedge \\
&\quad [\forall z[[\mathsf{CON}(z) \wedge \mathsf{C}(z,x) \wedge \mathsf{C}(z,\mathsf{outside}(y)] \rightarrow \mathsf{O}(z,y)]] \vee \\
&\quad [\mathsf{CON}(\mathsf{sum}(\mathsf{inside}(y),\mathsf{outside}(y))) \wedge \neg \mathsf{CON}'(\mathsf{sum}(\mathsf{inside}(y),\mathsf{outside}(y)))] \\
&\mathsf{GEO\text{-}INSIDE}(x,y) \equiv_{def} \mathsf{INSIDE}(x,y) \wedge \neg \mathsf{TOP\text{-}INSIDE}(x,y)
\end{aligned}$$

[11] The definition of CON′ corrects the definition to be found in (Randell, Cui and Cohn 1992). Also note, for each INSIDE relation, we are defining a function which yields the sum fusion of all the regions in the particular inside relation. Each such definition is of the same form as the two functions defined below for geo-inside and top-inside. To save space, we will henceforth assume that such functions are automatically but implicitly, defined for each new refinement of INSIDE. Also note that $\alpha(\overline{x}) =_{def} \iota y[\Phi[\alpha(\overline{y})]$ means $\forall \overline{x}[\Phi(\alpha(\overline{x})]]$; thus, e.g., the definition for $\mathsf{top\text{-}inside}(x,y)$ is translated out (in the object language) as: $\forall xyz[\mathsf{C}(z,\mathsf{top\text{-}inside}(x,y)) \leftrightarrow \exists w[\mathsf{P}(w,x) \wedge \mathsf{P}(w,y) \wedge \mathsf{C}(z,w)]]$.

Figure 4: Two configurations depicting regions which are mutually inside each other.

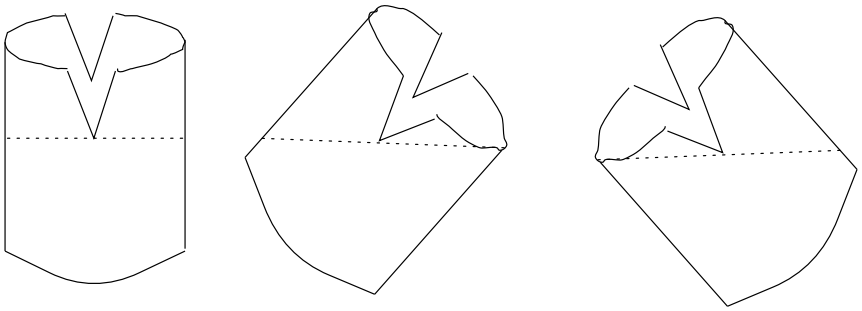

Figure 5: The containable inside of a region may not be completely fillable at any one time.

$\mathsf{CON}'(x) \equiv_{def} \mathsf{CON}(x) \wedge \forall yz[\mathsf{sum}(y,z) = x \rightarrow$
$\exists vw[\mathsf{P}(v,y) \wedge \mathsf{P}(w,z) \wedge \mathsf{P}(\mathsf{conv}(\mathsf{sum}(v,w)),x)]]$
$\mathsf{top\text{-}inside}(x) =_{def} \iota y \forall z \mathsf{C}(y,z) \leftrightarrow \exists w[\mathsf{TOP\text{-}INSIDE}(w,x) \wedge \mathsf{C}(z,w)]$
$\mathsf{geo\text{-}inside}(x) =_{def} \iota y \forall z \mathsf{C}(y,z) \leftrightarrow \exists w[\mathsf{GEO\text{-}INSIDE}(w,x) \wedge \mathsf{C}(z,w)]$

Note that TOP-INSIDE is only defined for $y \neq$ us for sortal reasons which is also intuitively correct as the universal region is convex and so has no inside. The case illustrated by figure 3(iii) is subsumed under the definition of TOP-INSIDE, but since we believe it is probably not pragmatically useful to distinguish the two cases we have not refined the definition into two relations here. In order to give the revised schema for generating JEPD base relations specialising EC, DC and PO we have to ascertain which combinations of the relations are possible. There is not space to treat this theoretically here but the main difficulty is in deciding which combinations of the inside and inside inverse relations exist. In no case is a DC variant possible and it turns out that all the EC variants are possible except TOP-INSIDE_TOP-INSIDE^{-1}; figure 4 depicts models for the relations that are possible. Note that TOP-INSIDE_GEO-INSIDE^{-1} and GEO-INSIDE_TOP-INSIDE^{-1} are only possible given the revised definition of TOP-INSIDE above; if figure 3(iii) was defined to be a specialisation of GEO-INSIDE then only GEO-INSIDE_GEO-INSIDE^{-1} would be possible. The schema is thus:

$\alpha_\beta_\gamma(x,y) \equiv_{def} \alpha(x,y) \wedge \beta(x,y) \wedge \gamma(x,y)$

where: $\alpha \in$ {TOP-INSIDE, GEO-INSIDE, P-INSIDE, OUTSIDE}, $\beta \in$ {TOP-INSIDE^{-1}, GEO-INSIDE^{-1}, P-INSIDE^{-1}, OUTSIDE^{-1}}, and $\gamma \in$ {EC, DC, PO} excepting where $\alpha \in$ {TOP-INSIDE, GEO-INSIDE}, $\beta \in$ {TOP-INSIDE^{-1},GEO-INSIDE^{-1}} and $\gamma =$ DC or where $\alpha =$ TOP-INSIDE, $\beta =$ TOP-INSIDE^{-1} and $\gamma \in$ {PO, EC}. This now gives a total of 8-3+(4*4*3)-(2*2*1) - 2 = 47 base relations.

3.1 Refining TPP

A natural corollary of refining EC is to refine TPP: if there are many ways that two discrete regions can touch, depending on whether one is inside or outside the other, then there are many ways in which a tangential proper part of a region (which by definition touches the complement of the superior region) can touch the complement, depending on whether it is touching the outside, or the various kinds of inside. Thus we can distinguish TPP_GEO, TPP_TOP and TPP_OUT since the complement of a region now divides up into three JEPD regions: the geometric and topological inside and the outside. The definitions of these specialisations of TPP are quite natural.

$\mathsf{TPP_GEO}(x,y) \equiv_{def} \mathsf{TPP}(x,y) \wedge \mathsf{EC}(x,\mathsf{geo\text{-}inside}(y))]$
$\mathsf{TPP_TOP}(x,y) \equiv_{def} \mathsf{TPP}(x,y) \wedge \mathsf{EC}(x,\mathsf{top\text{-}inside}(y))]$
$\mathsf{TPP_OUT}(x,y) \equiv_{def} \mathsf{TPP}(x,y) \wedge \mathsf{EC}(x,\mathsf{outside}(y))]$

However, these relations are not pairwise disjoint since a tangential proper part of a region could simultaneously touch (i.e. EC) all the different parts of the complement. Thus the set of pairwise disjoint base relations which jointly exhaust the original TPP definition are given by the following schema.[12]

$\mathsf{TPP_}\alpha\mathsf{GEO_}\gamma\mathsf{TOP_}\delta\mathsf{OUT}(x,y) \equiv_{def}$
$\quad \alpha\mathsf{TPP_GEO}(x,y) \wedge \delta\mathsf{TPP_TOP}(x,y) \wedge \gamma\mathsf{TPP_OUT}(x,y)$
where $\alpha, \delta, \gamma \in \{\neg, \Lambda\}$ and Λ represents the empty string.

Thus TPP is refined into 8 new base relations and similarly for TPP^{-1} giving 14 extra base relations (8+8-2).

4 Envisioning Axioms

We now turn to consider the extension required to the envisioning axioms previously summarised by figure 1. As before we will represent these axioms diagrammatically. It is easiest to specify the possible transitions using relatively high level predicates rather than in terms of the base predicates. First we will consider the transitions whose name includes OUTSIDE, P-INSIDE, G-INSIDE, TOP-INSIDE, or GEO-INSIDE. The transition network for these predicates is displayed in figure 6. Intuitively, one might have thought that TOP-INSIDE would be isolated from the rest of the network since a region topologically inside another would have to partially overlap the containing region in order to move to the outside or geometric inside, but it is clear that if the containing region alters its topology from a torus to a simply connected region (as depicted in figure 7, then a direct transition from to TOP-INSIDE GEO-INSIDE is indeed possible. Of course, it would be important in many applications to have restricted continuity networks which built in assumptions concerning the rigidity, shape or topology of the regions involved, but we will not do this here.

[12] We have taken a liberty with the usual syntax of predicate calculus here and for notational convenience allow a negation symbol to be part of a predicate name (e.g. $\mathsf{TPP_}\neg\mathsf{GEO_}\neg\mathsf{TOP_OUT}$ is a predicate symbol generated by the above definition. Since $\neg$ is never the initial symbol of such a name, no confusion will arise). Of course negation symbols appearing in the right hand side of the definition as instantiations of α, δ or γ are however real logical negations.

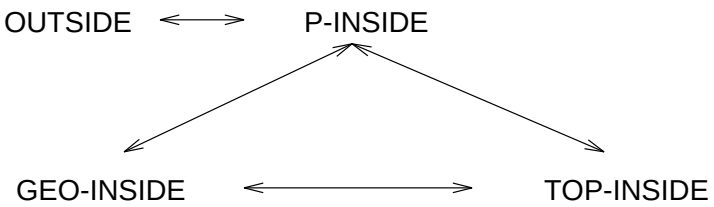

Figure 6: The transition network for the 4 high level inside/outside relations.

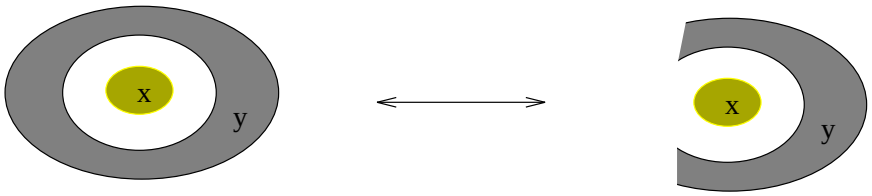

Figure 7: The transition from TOP-INSIDE to GEO-INSIDE is possible since regions might change their topology.

In order to determine the possible transitions for a predicate with a multipart name (such as OUTSIDE_GEO-INSIDE^{-1}_EC) one simply determines the allowable transitions for each part of the name; thus in the above example, OUTSIDE can only transition to P-INSIDE, GEO-INSIDE^{-1} to P-INSIDE^{-1} or to TOP-INSIDE^{-1} and EC to PO or DC. In the case that a sub-name transitions from PO to TPP, TPP^{-1} or =, then of course the rest of the sub-names are dropped, e.g. OUTSIDE_GEO-INSIDE^{-1}_PO can transition to = or to any TPP or TPP^{-1} relation.

We also have to consider the transition network for the TPP specialisations. It turns out that this subnetwork is an 8 clique, ie every transition is possible. The only restriction on transitions to/from this subnetwork is that only a specialisation of TPP_α can transition to α-INSIDE_PO, where $\alpha \in$ {GEO,TOP,OUT}.

5 Further Work

There are clearly many ways in which the work presented here could be extended; one obvious way is to further refine the taxonomy by introducing more distinctions; we will discuss this further in the next subsection. Also of importance is to derive composition tables for the extended set of relations; this is discussed further in (Randell, Cohn and Cui 1992a) and is the subject of active research at present.

Another aspect that deserves greater investigation is to look at special cases of the transition network; for example one might know that some or all of the regions involved are one piece, or only have one concavity, or by introducing a new primitive to allow a notion of relative size to be expressed (a qualitative concept exploited very frequently in the qualitative reasoning community) one can easily see that certain transitions could be outlawed (e.g. if x is bigger than y, then the transition from PO(x,y) to TPP(x,y) is not possible). Other kinds of knowledge that could be exploited in this way include the empirical notion of rigidity (so that one would know that a region will not deform to change its shape or size). For example, Galton (1993) has developed specialisations of our continuity network for the eight JEPD relations by assuming

that regions do not change their shape or size in any "significant" manner. This allows him to classify the possible set of interactions between two regions into six different categories and produce a specialised continuity network for each of these cases.

Technical work that is currently in progress includes integrating the ideas expressed here into our qualitative simulation program (Cui et al. 1992), finish building the extended composition tables and formally verifying the pairwise disjointness and mutual exhaustiveness of the new taxonomies.

We should also address the question of the correctness of our definitions and axioms seriously. In general it is extremely difficult to be sure that a formalisation correctly captures one's intuitions about a particular set of concepts. One can try to ensure that the formalisation is at least consistent by finding a a model. The best that can probably be done to check whether a formalisation is conceptually adequate is to isolate and prove a set of theorems which capture an important set of properties which are true in the intended interpretation of the formalisation. Clarke (1981, 1985) proves a large set of theorems for his calculus which show its properties quite well. For the revised formulation with just eight JEPD relations we proved some important theorems in (Randell, Cui and Cohn 1992). However an important task for the future is to validate the new definitions in this paper using this technique.

We are also working on a modal spatial logic (Cohn 1993) and a translation from our first order formulation to intuitionistic propositional logic (Bennett 1993) which seems to have certain computational advantages.

5.1 Further Refinements of Inside

As mentioned above, the notion of geometric inside does not adequately characterise the notion of containerhood. We had thought that the notion of the *containable inside* of a region (intuitively the part of the geometric inside which could contain liquid, if appropriately oriented), was not definable with the primitives available but we now believe we have a definition. First we define the notion of a 'lid' which is a region which converts (part of) a geometric inside to a topological inside (i.e. the geometric inside is the topological inside of the sum of the container and the lid) and then use this to define when one region is part of the containable inside of another. The 'containable inside' of a region is the sum fusion of all such regions.[13] The three arguments to '$\mathsf{LID}(w,x,y)$' are the lid w, the container y and the contained region x (i.e. the part of the geometric inside which w is a lid of).

$$\mathsf{LID}(w,y,x) \equiv_{def} \mathsf{CONV}(w) \wedge \mathsf{P}(x,\mathsf{geo\text{-}inside}(y)) \wedge \mathsf{P}(x,\mathsf{top\text{-}inside}(\mathsf{sum}(w,y)))$$
$$\mathsf{CONT\text{-}INSIDE}(x,y) \equiv_{def} \mathsf{P}(x,\mathsf{geo\text{-}inside}(y)) \wedge \exists w\, \mathsf{LID}(w,y,x)$$

Note that the restriction of a lid to be convex is crucial (a convex region will of course also be one piece): for example this ensures that there can be no lid for a straight tube, because one would either require two separate lids for each end, or the lid would have to be concave to 'wrap round' both ends; this ensures that the handle of a cup has no containable inside.

[13] Note that one might not be able to completely fill con-inside(x) with a liquid; for example consider a beaker where a v shaped wedge has been cut out of the top as depicted in figure 5. con-inside(beaker) cannot ever be completely full of liquid; however, when tipped, as indicated in the figure, different parts of the con-inside will contain liquid.

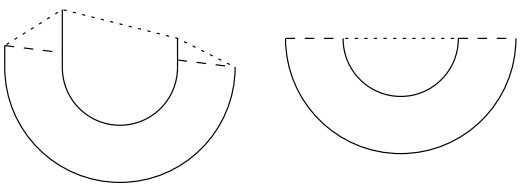

Figure 8: The pipe and containable insides of two U-tubes. The dashed lines indicate the extent of the containable space specified by the definition above. The dotted lines give the extent of the pipe-inside.

This definition effectively refines the notion of the geometric inside into the containable inside and the rest of the geometrical inside, which we will call the *pipe-inside*.[14] Although the geometric inside splits into two disjoint parts, we need three base relations to cover the possible configurations since a region might be geometrically inside another but neither wholly inside the containable inside or the pipe inside (i.e. it might be partly inside each of the regions); thus we define G-INSIDE(x,y) to cover this third case.

$$\mathsf{PIPE\text{-}INSIDE}(x,y) \equiv_{def} \mathsf{GEO\text{-}INSIDE}(x,y) \wedge \neg \mathsf{PO}(x, \mathsf{con\text{-}inside}(y))$$
$$\mathsf{G\text{-}INSIDE}(x,y) \equiv_{def} \mathsf{GEO\text{-}INSIDE}(x,y) \wedge \mathsf{PO}(x, \mathsf{con\text{-}inside}(y))$$

It is worth pointing out that bent pipes (e.g. U-tubes) can act as containers and our definition will indeed give such regions a containable inside; figure 8 illustrates two configurations (which are cross sections through 3D regions).[15]

The primary aim of the ontology presented here has been to develop a theory of relations between spatial regions with concavities; this work has strong connections with the conceptualisation of objects with holes developed by Casati and Varzi (1993). They develop a classification of holes; there are three main types: one of these corresponds to the notion of topological inside developed here. The other two specialise geometric inside in a slightly different way to the present ontology: they distinguish between tunnels (perforations) and indentations. It would be easy using the apparatus developed in this paper to make these distinctions in our theory. However a full integration of the two theories would certainly be of interest.

So far we have not differentiated between one piece and multipiece regions: all our binary relations are equally applicable to both. However, one might want to distinguish[16] the situation depicted in figure 9 where the region x is inside the convex hull of the multi piece region y but is not inside any one piece proper part of y. The predicate SCAT-INSIDE(x,y) defined below captures this concept. This refines the notion of PIPE-INSIDE so we also redefine PIPE-INSIDE to take account of this. The auxiliary predicate MAX-P(x,y) is true when x is a maximal connected (ie one-piece) sub region of y.

[14] This might not always be a very intuitive name for this kind of geometric inside: for example consider (in three dimensional space) a region consisting of 6 perpendicular spokes radiating out from a centre (as in the axes of an x, y, z graph): such a region has a geometric inside but no containable inside, and one would not call the object a pipe! This opens up the possibility of further refining the notion of geometric inside so that the pipe-inside was split into the cases which one would want to consider to be the inside of a pipe and the cases such as the spoked region above. We will not pursue this possibility further in this paper. Also note in passing that in two dimensions every region has a null pipe-inside.

[15] Of course a U tube is a pipe, but since it is bent it has a containable inside; it also has a pipe-inside which is the region 'outside' the tube, but inside the convex hull of the tube.

[16] This situation was suggested to us by Simone Pribbenow.

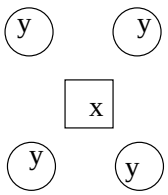

Figure 9: x is inside y, but without being inside any one-piece subpart of y.

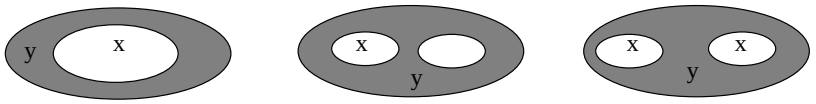

Figure 10: Examples of NTS.

$$\text{SCAT-INSIDE}(x,y) \equiv_{def} \text{INSIDE}(x,y) \wedge$$
$$\forall w \text{MAX-P}(w,x) \rightarrow \neg\exists z[\text{MAX-P}(z,y) \wedge \text{INSIDE}(w,z)]$$
$$\text{MAX-P}(x,y) \equiv_{def} \text{CON}(x) \wedge \text{P}(x,y) \wedge \neg\exists z[\text{PP}(x,z) \wedge \text{P}(z,y) \wedge \text{CON}(z)]$$
$$\text{PIPE-INSIDE}(x,y) \equiv_{def} \text{GEO-INSIDE}(x,y) \wedge \neg\text{PO}(x,\text{con-inside}(y)) \wedge \neg\text{SCAT-INSIDE}(x,y)$$

The geometric inside is now split into four JEPD cases: CONT-INSIDE, PIPE-INSIDE, G-INSIDE and SCAT-INSIDE. This adds a further 3 base relations to the previous total.

A refinement of TOP-INSIDE is also possible. In (Randell and Cohn 1989) we defined a predicate NTS(x,y) which was true when x was non tangentially surrounded by y. This is effectively a special case of TOP-INSIDE(x,y), for example when x *is* the topological inside of y. The original definition now has to be changed slightly to allow for the elimination of the distinction between open and closed intervals.

$$\text{NTS}(x,y) \equiv_{def} \exists z \text{NTPP}(x,z) \wedge y = \text{prod}(z,\text{compl}(x))$$

The region z will be the sum of x and y in the case that the predicate is true. This effectively specialises the notion of topological inside to the case where one region completely fills a maximally connected part of the topological inside of another region; i.e. a region y may have a multipiece topological inside (e.g. in the case of a car where the boot[17] and passenger compartments form two separated parts of the topological inside, assuming all the doors and vents are shut); if the boot was completely filled with luggage, then the car would non tangentially surround the luggage. Examples of NTS appear in figure 10.

It might also be useful to define similar notions for the other kinds of inside, in particular for containable inside, since one might want to know whether a particular container is full. A complication is that the containable inside might be multi piece (i.e. there are several concavities which could act as containers). One way to proceed is to define the notion of a maximal container with respect to a region and then define a predicate FILLS(x,y) (i.e. every maximal one piece subpart of x completely fills a maximal one piece part of the containable inside of y).

$$\text{MAX-CONT-INSIDE}(x,y) \equiv_{def} \text{MAX-P}(x,\text{con-inside}(u))$$
$$\text{FILLS}(x,y) \equiv_{def} \forall z \text{MAX-P}(z,x) \rightarrow \text{MAX-CONT-INSIDE}(z,y)$$

[17] Or *trunk* in American English.

6 Discussion

In the preceding sections we have seen how a perhaps surprisingly complex and expressive ontology for describing qualitative spatial relationships can be logically defined from just 2 primitives. Including the refinements offered by CONT-INSIDE, SCAT-INSIDE or NTS, there are well over 100 base relations, all of which are JEPD. It is clear that it would not be difficult to continue defining new predicates expressing finer distinctions. For example one could introduce the notion of 'touching at two separate places', or one could define notions of JUST-INSIDE (where x ECs the outside of y) or JUST-OUTSIDE (where x ECs the inside of y) (see Randell and Cohn (1992)).

In the face of a seemingly limitless scale of ever decreasing granularity, how can we decide what is a 'useful' set of primitives, i.e. when should we stop the enterprise? This is a difficult question; indeed Joskowicz (1992) has argued that there is no general purpose commonsense spatial reasoner and thus questions such as this one just cannot be answered in general – it will depend on the domain and the task in hand and perhaps many other considerations. What we shall attempt to do here is to start to propose some criteria by which one might make such decisions.

First, if we are really concerned purely with qualitative reasoning, then one may not wish to consider refinements such as the one suggested above of 'touching at two separate places' which has a 'metric' component (i.e. involves counting rather than existence/non existence predication). Another, perhaps more interesting criterion, is to examine the effect of a proposed distinction on the transition network: if the proposed refinement cannot be exploited by the transition network, i.e. if the new nodes form a clique (or perhaps a 'near clique') such as happened with our proposed introduction of the specialisations of TPP, then we may not consider it worth while making such a distinction.[18] Of course this argument depends on the purpose to which we intend to put our spatial description, but if it involves dynamic reasoning (i.e. reasoning about evolving spatial configurations over time), then this seems like a reasonable criterion. This criterion would also seem to rule out refining P-INSIDE in the same way we refined GEO-INSIDE (when we introduced the containable and pipe insides) as the transition network would again be a clique for the subnetwork for P-INSIDE.

A further criterion that seems reasonable to consider is that any proposed refinement of an existing predicate should have at least one property which is not true of the 'parent' predicate. Eg if Q is refined into R and S, then there should be some property T such that $\forall x$ R(x) $\rightarrow$ T(x) or $\forall x$ S(x) $\rightarrow$ T(x) but not $\forall x$ Q(x) $\rightarrow$ T(x). Actually this criterion overlaps with the previous one since, under this consideration, we would accept the refinement of INSIDE to TOP-INSIDE and GEO-INSIDE, since TOP-INSIDE cannot transition to P-INSIDE (but INSIDE can). The distinguishing property could either be a 'topological property' (such as we have just mentioned) or some domain specific property (e.g. a liquid will stay in an appropriately oriented cont-inside (but not in an arbitrary geo-inside).)

Sometimes there would seem to be only pragmatic or domain dependent reasons for deciding whether to make a refinement (e.g. our decision to exclude the refinement suggested by figure 3(iii)). Equally the decision whether to make distinctions such as the CONT-INSIDE or to refine the notion of PIPE-INSIDE so as to capture the real notion of a pipe better, will ultimately depend on whether our domain has containers and pipes in it.

[18] There is still a small advantage to be gained since, as we have already noted, not every TPP specialisation can transition to every PO specialisation.

Another obvious criterion is a computational one: one might take account of the decidability or tractability of reasoning within the chosen taxonomy. This is, of course, a standard consideration in the many so called 'description languages' (or taxonomic languages) that have been designed in recent years such as KL-ONE and its derivatives. Depending on what computation we want to perform, we can consider what the computational cost is. At present, we have expressed all the definitions in (sorted) FOL, so this question is not entirely trivial to answer. An interesting idea to pursue would be to discover what fragment of the conceptual hierarchy we have presented here can be expressed in a language with known decidability or tractability properties.

In the temporal domain, Ladkin (1986) notes that once multi-piece temporal intervals are considered, then the number of possible relations is infinite. He provides a finite taxonomy of relations for such a temporal calculus by insisting that no relation depend on the number of maximally connected subintervals: new relations can only be defined by quantifying over all the maximally connected subintervals (using a predefined set of quantifiers[19]) or by considering the initial and final maximally connected subintervals. One could imagine generalising[20] this idea to the spatial case, though there is no corresponding notion of initial and final subregions (unless one introduces primitive orderings based on the three orthogonal axes).

It is worth pointing out that in some applications there may be no single level of granularity which is appropriate: rather, a hierarchical approach with reasoning taking place at many different levels according to the particular situation may be best. We are conducting research on this topic.

One last comment is that some of these criteria may be equally applicable to domains other than the spatial case, ie these may be useful criteria for deciding how fine grained a taxonomy to create in almost any domain.

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[19] These are: *mostly, always, partially, sometimes* and disjunction.

[20] Most of the predicates we have defined in this paper have not taken explicit account of the fact that regions can be multi piece (except for SCAT-INSIDE).

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