RICHARD CARTWRIGHT

SCATTERED OBJECTS

According to Hobbes, "a body is that, which having no dependence on our thought, is coincident or coextended with some part of space".[1] Bodies in Hobbes' sense are material objects in ours; so at any rate I shall assume. And I shall assume also that his definition is correct at least in its implication that coincidence with some part of space is required of anything that is to count as a material object. But what is to count as a part of space?

By a *region of space*, or simply a *region*, let us agree to understand any set of points of space. And by a *receptacle* let us understand a region of space with which it is possible some material object should be, in Hobbes' phrase, coincident or coextended. Plainly, not every region is a receptacle. The null region is not; neither is any region that consists of a single point or, for that matter, of any finite number of points. Nor are higher cardinalities by themselves sufficient: no region exceeds a straight line in sheer number of members;[2] yet straight lines, along with curves and surfaces, are not receptacles. How, then, are receptacles to be characterized?

Let p be any point of space. By an *open sphere about* p is meant a region the nembers of which are all and only those points that are less than some fixed distance from p. In other words, a region A is an open sphere about the point p if and only if there is a positive real number r such that A is the set of all those points whose distance from p is less than r. A region that is an open sphere about some point or other is called simply an *open sphere.*

Every open sphere is, I suggest, a receptacle. There are of course neither minimal nor maximal open spheres: given any open sphere, no matter how large or small, there is a larger and a smaller. My suggestion will thus disturb those for whom material objects are "moderate-sized specimens of dry goods".[3] But surely not all material objects *are* moderate-sized. Heavenly bodies are bodies, some of them very large; and antibodies are bodies, extremely small ones. Given these actualities, why impose bounds on the possibilities?

Keith Lehrer (ed.), Analysis and Metaphysics, 153–171.

Others will be disturbed because they think of receptacles as closed. Let me explain. A point p is said so be a *boundary point* of a region A if and only if every open sphere about p has a non-null intersection with both A and the complement of A (where the *complement* of a region is the set of points of space not in the region). Otherwise put, p is a boundary point of A just in case every open sphere about p has in it points of A and points of the complement of A. To illustrate, let S be the open sphere of radius r about p and let q be a point whose distance from p is exactly r. Then, every open sphere about q will intersect both S and the complement of S; and hence q is a boundary point of S. In fact, the boundary points of S are precisely those points that are like q in that their distance from p is exactly r. A point whose distance from p is less than r will be the center of an open sphere included in S; and a point whose distance from p is greater than r will be the center of an open sphere included in the complement of S. Now, a region, spherical or otherwise, is said to be *open* just in case none of its boundary points is a member of it and *closed* just in case all its boundary points are members of it. We have just seen that an open sphere is, appropriately enough, an open region: an open sphere and its surface have no points in common. And it is precisely this that will cause some to resist the suggestion that every open sphere is a receptacle. Their intuitions tell them that a receptacle should be closed. Descartes' told him otherwise. After explaining that what he calls the "external place" of a body is "the superficies of the surrounding body", he remarks that "by superficies we do not here mean any portion of the surrounding body, but merely the extremity which is between the surrounding body and that surrounded".[4] I shall follow Descartes, though I should have no idea how to defend my choice; indeed, the issue seems hardly worthy of serious dispute. There is, however, a possible misconception that needs to be cleared away, a misconception perhaps latent in Descartes' use of 'between'. If receptacles are open, it might seem that bodies never touch, since something – if only a very fine something – is always in between. But this *is* a misconception. On either view body x touches body y when and only when at least one boundary point of the region occupied by x is also a boundary point of the region occupied by y. The only issue is whether such a boundary point must belong to the regions occupied by x and y. And it is this issue that seems hardly worthy of serious dispute.

I shall assume, then, that every receptacle is an open region. But not every receptacle is an open sphere; bodies do, after all, come in other shapes. To allow for the endless possibilities, it will perhaps be suggested that an open region be counted a receptacle provided only that it is non-null. Receptacles would thus come to be identified with non-null open regions, spherical or otherwise. A good many unwanted regions would thereby be excluded: regions with only a finite number of points, curves, and surfaces, for example. But the suggestion will not do. Consider a region the members of which are all the points of an open sphere S save for a single point p. Consider, that is, $S-\{p\}$, where S is an open sphere and p is a point in S. We are reluctant, I think, to suppose this a receptacle. Surely no material object could occupy all the points of an open sphere save one. It is not that objects never have holes; it is rather that holes are never so small. Yet $S-\{\text{p}\}$ is open, for it contains none of its boundary points.

Only some then among open regions are receptacles. Which ones? To investigate the question we shall need the notions of the interior and the closure of a region. By the *interior* of a region is meant the set of all points in the region that are not boundary points of the region. Note that a region is open if and only if it is identical with its interior, for no boundary point of an open region is a member of the region and interiors themselves are always open. The *closure* of a region is the union of the region with the set of all its boundary points. Just as a region is open if and only if it is identical with its interior, so a region is closed if and only if it is identical with its closure; for a closed region includes the set of its boundary points and closures themselves are always closed. Now consider again the region $S-\{p\}$. The point p is a boundary point of the region, for every open sphere about p intersects both $S-\{p\}$ and its complement. But as boundary points go p is peculiarly situated, for it is also a member of the interior of the closure of $S-\{p\}$. Close $S-\{p\}$ and you pick up p along with the points on the surface of S; take the interior of the resulting region and you keep p, though you lose the points on the surface of S. In view of this peculiarity of situation, let us say that p is an 'inner' boundary point of $S-\{p\}$, where in general an *inner boundary point* of a region is a boundary point of the region that is also a member of the interior of the closure of the region. It is possession of an inner boundary point that leads us to exclude the region $S-\{p\}$ from the class of receptacles; and, accordingly, I suggest that at least a necessary condition for a non-null open region to qualify as a receptacle is that it have no inner boundary points.

It is easily shown that open regions having no inner boundary points are precisely those regions that are identical with the interiors of their closures. And a region that is identical with the interior of its closure is known as an *open domain.*[5] So the present suggestion comes to this: a region of space is a receptacle only if it is a non-null open domain. Another example may serve to clarify the suggestion. Image an open sphere cut by a plane. Let the open region on one side of the plane be A and that on the other be B. Both A and B are open domains, but their union is not; for the points on the plane other than those on the surface of the sphere are inner boundary points of $A \cup B$. Otherwise put, since the points on the plane that are not on the surface of the sphere are members of the interior of the closure of $A \cup B$ but not of $A \cup B$ itself, $A \cup B$ is not identical with the interior of its closure and is therefore not an open domain.[6] Thus $A \cup B$ is not a receptacle: no object can be coincident or coextended with it. This is not to exclude the possibility of cracks; it is simply to insist that cracks are never so fine. Of course, the interior of the closure of $A \cup B$ is a receptacle. It is in fact the open sphere with which we began. Thus a body can occupy a region that *includes* $A \cup B$. But such a region must include as well the set of inner boundary points of $A \cup B$.

The proposition that every receptacle is a non-null open domain is not apt to meet with serious opposition. But what of its converse? Is every non-null open domain a receptacle? Here there is likely to be controversy. The issue turns on the notion of connectedness, and we therefore need to see exactly what this notion is.

It is customary to say that two regions are *separated* if and only if the intersection of either with the closure of the other is null. Thus, in the example just discussed, the regions A and B are separated: take the closure of either and you pick up no points of the other. That is, no point or boundary point of either is a member of the other. Obviously, if two regions are separated, their intersection is null. But two regions with a null intersection need not be separated. Thus the intersection of A with the closure of B is null; yet A and the closure of B are not separated, for there are boundary points of A in the closure of B. Now, a region is said to be *disconnected* if and only if it is the union of two non-null separated regions; and a region is *connected* if and only if it is not disconnected. Thus, keeping to the same example, $A \cup B$ is disconnected. In contrast, the interior of the closure of $A \cup B$ is connected, for there do not exist two non-null separated regions of which it is the union. It is a connected open domain. But it is by no means the case that all open domains are connected. Consider, for example, two open spheres that touch at a single point. The closure of either intersected with the other is null, and the two are therefore separated. Hence their union is disconnected. But it is an open domain: none of its boundary points is inner, even the point of contact. Or consider two open spheres situated at some distance from each other. Their union is evidently a disconnected open domain.

Connected open domains, as long as they are non-null, presumably present no problems. Each is a receptacle. But disconnected open domains are another matter. Are they receptacles? I shall defend the position that they are, though admittedly I have no conclusive argument.

Let us say that a material object is *scattered* just in case the region of space it occupies is disconnected. That there are scattered material objects seems to me beyond reasonable doubt. If natural scientists are to be taken at their word, all the familiar objects of everyday life are scattered. But I have in mind nothing so esoteric. Rather, it seems to me a matter of simple observation that among material objects some are scattered. Consider, for example, my copy of McTaggart's *The Nature of Existence.* There surely *is* such a thing; and it is a *material* thing, even a moderate-sized specimen of dry goods. After all, it is made of paper and certain other materials; it weights roughly three and a quarter pounds; it is bound in a hard black cover; it occupies a certain region of space, into which it was recently moved; and so on. But it is scattered, for Volume I is in Cambridge and Volume II is in Boston. Each volume occupies, or at least to the ordinary eye appears to occupy, a connected open domain; but these regions are separated, and hence their union is disconnected. This example will bring to mind hosts of similar ones. Let me mention two others of a somewhat different kind. There is at the moment a pipe on my desk. Its stem has been removed, but it remains a pipe for all that; otherwise no pipe could survive a thorough cleaning. So at the moment the pipe occupies a disconnected region of space, a region which appears to common sense to be the disconnected open domain that is the union of the connected open domains occupied by the two parts. Consider, finally, some printed inscription: the token of 'existence' on the title page of my copy of McTaggart's *The Nature of Existence*, for example. Presumably it is a material object – a "mound of ink", as some say. But evidently it occupies a disconnected region of space.

If there are scattered objects, then some disconnected open domains are receptacles. It does not follow that all are. Still, once some have been admitted, it seems arbitrary to exclude any – just as it seems arbitrary to impose limits on the size or shape of receptacles. And it should be remembered that to call a region a receptacle is not to say that some object is in fact coincident or coextended with it but only that this is not impossible. All this inclines me to identify receptacles with non-null open domains.[7]

An interesting question remains, however. To introduce it, let me mention an objection that is apt to be brought against the contention that my copy of *The Nature of Existence* is a scattered material object. Some will be inclined to say, with Leibniz, that my copy of *The Nature of Existence* is a "being by aggregation", not a "true unity".[8] Leibniz would not himself have taken this to imply that it is not a material object – only that it is not what he called an "individual substance". His notion of individual substance aside, however, some will still be inclined to say that my copy of *The Nature of Existence* is a mere 'plurality' or 'aggregate' or 'assemblage' of material objects and not properly speaking a single material object in its own right. It is no more correct, they will say, to suppose there is one thing composed or made up of my copy of Volume I and my copy of Volume II than to suppose there is one thing composed or made up of, say, the Eiffel Tower and Old North Church. We do speak of my copy of *The Nature of Existence* as if it were a single thing, and there is no parallel to this in the case of the Eiffel Tower and Old North Church. But it will be claimed that this is reflective merely of our special human interests, not of the metaphysical status of the entities involved. The two volumes are a mere assemblage, just as are the tower and the church.

The obscurity of the objection makes a direct response difficult. What exactly is meant by "a mere plurality or aggregate or assemblage"? And what sense is to be made of the claim that my copy of *The Nature of Existence* – or anything else, for that matter – is not 'one' thing? Furthermore, one wonders how far the objection is to be carried. The alleged defect in my copy of *The Nature of Existence* is surely not simply that the region occupied by Volume I is at some distance from that occupied by Volume II. Even were the two volumes side by side, separated only by a plane, they would presumably still be said not to constitute or compose a single material object. The interior of the closure of the union of the region occupied by the one volume with that occupied by the other would be a receptacle, but an unoccupied one. But then why not argue, as Leibniz did, that *no* material thing is properly speaking *one* thing? Any connected open domain can be cut by a plane in such a way as to leave two open domains whose union is disconnected. Therefore, Leibniz reasoned, every corporeal object is in theory divisible and what is in theory divisible is only a being by aggregation.[9]

In spite of its obscurity and the uncertainty of its extent of applicability, the objection brings to the surface a question of some interest. *Is* there a material object composed of the Eiffel Tower and Old North Church? In general, is it the case that for each non-null set of material objects there is a material object composed of the members of the set? The question needs sharper formulation, and for that some additional technical terminology is required.

A set M of material objects will be said to *cover* a region A if and only if A is included in the union of the receptacles occupied by members of M. If A simply is that union, then obviously M covers A. For example, the set the members of which are the Eiffel Tower and Old North Church covers the region which is the union of the receptacle occupied by the Eiffel Tower and the receptacle occupied by Old North Church. In particular, if x is any material object, the set having x as sole member covers the region occupied by x. Clearly, if M covers A, then M covers any region included in A. Hence a given region may be covered by more than one set. The region occupied by Old North Church, for example, is covered by the set having Old North Church as sole member and also by the set the members of which are the Eiffel Tower and Old North Church. If M covers A, then A is included in the interior of the closure of the union of the receptacles occupied by members of M. But notice that M may cover A and yet fail to cover the interior of the closure of A. Thus, although a set the members of which are two books situated side by side covers the union of the receptacles occupied by the books, it does not cover the interior of the closure of that union.

If and only if a set covers a region, the region itself will be said to be *covered*. It should be noticed that if each member of a collection of regions is covered, so is the union of the collection. Indeed, so is any region included in the union of the collection.

Given the notion of a covered region, a proposition I shall call the Covering Principle can be formulated: *if A is any non-null covered open region, there exists exactly one material object x such that the region occupied by x is the interior of the closure of A*. Our question is whether this principle, or more especially a certain consequence of it, is true.

A preliminary word of explanation. Given a non-null covered open region A, the Covering Principle guarantees that A is the region occupied by a unique material object if and only if A is the interior of its closure, that is, if and only if A is an open domain. This accords with our requirement that only open domains be counted receptacles. It is in fact easily shown that the Covering Principle runs no risk of violating that requirement; for the interior of the closure of any set is an open domain. But then, it may be asked, why limit the principle to non-null covered *open* regions? The answer is that otherwise there would be a conflict with the requirement that receptacles be non-null; for the interior of the closure of a non-open region may well be the null set.[10]

Notice now the power of the principle. To use a familiar and convenient metaphor, it provides for the generation by composition of new objects from old in somewhat the fashion of the Power Set Axiom in Set Theory. Given three objects in separated receptacles A, B, and C, there will exist four others. For, if A, B, and C are covered, so are each of $A \cup B$, $A \cup C$, $B \cup C$, and $A \cup B \cup C$; and if A, B, and C are separated, these unions are distinct from one another and from each of A, B, and C. In general, given a set M of n objects situated in pair-wise separated receptacles, there will exist $2^n - (n+1)$ further objects, each compounded of members of M. A dozen dollar bills in your wallet makes for 4083 additional objects in your wallet – none of them dollar bills, however. And this is by no means the end. For the Covering Principle provides for generation of objects by division as well as by composition. Let A be a covered receptacle, and for purposes of simplification suppose it connected. Then A is the interior of the closure of the union of two connected and separated receptacles B and C, so situated that B lies on one side of a plane that intersects A while C lies on the other. Clearly, the same is in turn true of B and C, and of the receptacles into which they are thus divided, and so on without end. So there corresponds to A an infinity of connected and pair-wise separated receptacles each of which is covered. The Covering Principle provides that each of these receptacles is the region of space occupied by a unique material object. Now, it is easily seen that if any region is covered, at least one connected receptacle is covered. Hence, by the Covering Principle, if there is one material object, there are infinitely many.[11]

Our present concern is less with division than composition. It will therefore be of use to extract from the Covering Principle an appropriately weaker principle, one directed squarely at the issue of scattered objects. First a definition. A material object x will be said to *fuse* a set M of material objects just in case the receptacle occupied by x meets two conditions: (i) it includes the receptacles occupied by members of M, and (ii) it is included in every receptacle that includes the receptacles occupied by members of M. More simply, x fuses M if and only if the region occupied by x is the smallest receptacle that includes the receptacles occupied by members of M. In the case of any collection of receptacles, there is always a smallest receptacle that includes each member of the collection – namely, the interior of the closure of the union of the collection. So we might as well have said: x fuses M if and only if the region occupied by x is precisely the interior of the closure of the union of the receptacles occupied by members of M. My copy of *The Nature of Existence*, for example, fuses the set whose members are my copy of Volume I and my copy of Volume II; and the object, if such there be, composed of the Eiffel Tower and Old North Church fuses the set whose members are the Eiffel Tower and Old North Church. Notice that any material object fuses the set having that object as sole member. And notice also a sort of transitivity: if x fuses a set the members of which in turn fuse other sets, x fuses the union of those other sets.

Can distinct objects fuse the same set? Not if the Covering Principle is true. If x and y fuse M, the receptacle occupied by x is the very same as that occupied by y; and the Covering Principle tells us that no receptacle is the region occupied by more than one object. Can there be a non-null set of material objects which no material object fuses? Again, not if the Covering Principle is true. For the union of the receptacles occupied by members of the set is a covered non-null open region the interior of the closure of which is the smallest receptacle that includes the receptacles occupied by members of the set. Thus the Covering Principles entails what I shall call the Fusion Principle: *if M is any non-null set of material* *objects, there is exactly one material object x such that x fuses M*. According to this principle, each non-null set of material objects has a unique *fusion*: a material object so situated that its receptacle is the interior of the closure of the union of the receptacles occupied by members of the set. If the Fusion Principle is true, there really is a material object – exactly one, in fact – composed of the Eiffel Tower and Old North Church. It is composed of them in the sense that the region it covers is the union of the regions they cover.

I have taken the word 'fusion' from the exposition given by Leonard and Goodman of the so-called calculus of individuals.[12] And it may be instructive at this point to digress briefly from our main concerns in order to make contact with the principal ideas of that calculus.

Let E be a non-empty set, and let R be a relation that is reflexive in E, anti-symmetric in E, and transitive in E. (We are to think of R as a part-whole relation among elements of E, though of course that plays no role in the abstract development.) Two elements of E are said to *overlap* just in case they have a part in common; that is, if x and y are in E, x overlaps y if and only if some element of E bears R to both x and y. Now, the ordered pair (R, E) is a *mereology* just in case two further conditions are satisfied: (i) if x and y are members of E such that every member of E that overlaps x also overlaps y, then x is part of y; (ii) there exists a function f from the collection of non-empty subsets of E into E such that, for each non-empty subset A of E, a member of E overlaps $f(A)$ if and only if it overlaps some member of A.[13] As thus defined, mereologies are natural models of the Leonard-Goodman calculus.

Examples of mereologies are readily available. In fact, if B is the set of non-zero elements of a complete Boolean algebra and R is the inclusion relation among elements of B, (B, R) is a mereology in which the Boolean join plays the part of the merological function f.[14] More pertinent examples are provided by the following small theorem:

> *Let N be a non-empty family of non-empty open domains of a topological space. And suppose N is such that: (i) the interior of the closure of the union of each non-empty subset of N is itself in N: (ii) if A and B are members of N such that A-B is non-empty, then the interior of the closure of $A-B$ is in N. Then $(N, \subseteq)$ is a mereology with respect to which the mereological*

> *function f is the function that assigns to each non-empty subset of N the interior of the closure of its union.*

Notice that the theorem holds for any topological space – that is, for any space defined simply via a specification of the subsets that are to count as open, where the notion of an open set is subject only to the usual condition that among the open sets are to be found all unions of collections of open sets and all intersections of finite collections of open sets. Of greater interest for our purposes is the following corollary. Assume the Covering Principle; and assume that each material object occupies a unique receptacle, where receptacles are non-empty open domains of a topological space. Let M be a non-empty set of material objects that satisfies two closure conditions: (i) the fusion of each non-empty subset of M is itself in M; (ii) if x and y are elements of M such that the receptacle of x minus the receptacle of y is non-empty, then the material object that occupies the interior of the closure of the receptacle of x minus the receptacle of y is in M. (Note that the existence of this object is a consequence of the Covering Principle.) Then, if P is the relation that an element x of M bears to an element y of M just in case the receptacle of x is included in the receptacle of y, (M, P) is a mereology with respect to which the mereological function f is the function that assigns to each non-empty subset of M its fusion.

So much for connections with the calculus of individuals. Let us return to our main themes.

As already noticed, every material object is the fusion of at least one set, namely, the set having that object as sole member. Commonly, an object will be the fusion of other sets as well. A scattered object, for example, will be the fusion of the set having itself as sole member; but it will also be the fusion of the set of those objects that occupy maximal connected receptacles included in the receptacle of the scattered object. Indeed, if the Covering Principle is true, every object will be the fusion of endlessly many sets. The Covering Principle provides for fission as well as fusion, and what is obtained at any stage by fission is a set of which the original object is the fusion. Any given object occupies a receptacle; and covered receptacles are, as we have seen, endlessly divisible into further covered receptacles. The given object will be the fusion of the set of objects occupying the sub-receptacles obtained at any stage of the division

– provided, of course, the division is exhaustive, in the sense that the receptacle occupied by the object is the interior of the closure of the union of those sub-receptacles. To think of an object in this way will seem more or less natural depending on our willingness to count as genuine the alleged occupants of the various sub-receptacles. Two halves of an intact baseball will perhaps seem material objects only in some contrived sense, and the baseball itself will then not naturally be thought of as the fusion of a set the members of which are the two halves. Similarly with bottles, doughnuts, and sheets of paper. But it is otherwise with automobiles, books of matches, and salami sandwiches. In these cases we take rather easily to the idea that the object is the fusion of a set of other objects – not just any set of alleged objects yielded by the Covering Principle, of course, but a set consisting of what are in the natural way thought of as parts of the object.

To become quite specific, consider some particular book of macthes, and for ease of reference call it 'Charlie'. It is altogether natural to think of Charlie as consisting of twenty matches, a paper base to which they are attached, a surrounding paper cover, and an appropriately placed metal staple. That is to say, Charlie is quite naturally thought of as the fusion of the set that has these various objects as members. Thus, calling the set in question 'A', we are inclined to assert

(1) Charlie = the fusion of A.

Of course, we are not prepared to regard every set the members of which are twenty matches, a paper cover, and so on as having a book of matches as its fusion. The objects in the set must be properly put together. But the objects in A are properly put together. And the region of space Charlie occupies is the interior of the closure of the union of the receptacles occupied by members of A.

But now let us remove a single match from Charlie and place it some distance from him, while putting him back where he was – that is, putting him in a receptacle properly included in the receptacle he earlier occupied. Charlie, we should all agree, has undergone a change. He has lost a part, as material objects often do. He now consists of the various objects he consisted of before, save for the removed match. The receptacle he now occupies is the interior of the closure of the union of the receptacles occupied by members of $A-\{z\}$, where z is the match that has been removed. Just as we were earlier inclined to assert (1), so we are now inclined to assert

(2) Charlie = the fusion of $(A - \{z\})$.

But we can hardly deny

(3) the fusion of $A \neq$ the fusion of $(A - \{z\})$.

And so we seem to be in violation of the principle that no one thing is identical with diverse things.

It will no doubt be suggested at once that the appearance of paradox is removed once time is properly taken into account. Charlie *was* identical with the fusion of A but is *now* identical with the fusion of $(A - \{z\})$; or, avoiding tensed verbs:

(4) at t, Charlie = the fusion of A

whereas

(5) at t', Charlie = the fusion of $(A - \{z\})$,

where it is to be understood that t' is appropriately later than t. If it is pointed out that (3), (4), and (5) together entail

(6) at t', Charlie $\neq$ the fusion of A,

the response will be that this is no cause for alarm, since (4) and (6) are perfectly compatible.

But is it really possible for both (4) and (6) to be true? Their conjunction appears to imply that there is a certain object – namely, the fusion of A – with which Charlie is identical at t but not at t'. And this surely is impossible. It is impossible for Charlie to have been identical with one object, the fusion of A, and then to have become identical with another object, the fusion of $(A - \{z\})$. No object can be identical with something for a while and then become identical with something else. Once identical with one thing, never identical with another.[15]

It will be pointed out that the conjunction of (4) and (6) does not imply that there is an object with which Charlie is identical at t but not at t'. According to (4), Charlie has at t the property of being sole fuser of A;

and, according to (6), he lacks that property at t'. But this no more requires Charlie to have been temporarily identical with the fusion of A than the fact that Lyndon Johnson had and then lost the property of being president of the United States requires him to have been temporarily identical with a certain object with which Richard Nixon became identical. Thus (4) amounts to

(7) (x) (x fuses A at t iff x = Charlie),

(5) amounts to

(8) (x) (x fuses $A - \{z\}$ at t' iff x = Charlie),

and (6) amounts to

(9) $\sim$ (x) (x fuses A at t' iff x = Charlie).

And it is evidently quite possible that all these should be true.

There is reason to doubt, however, whether this ends the matter. If the Fusion Principle is true, some object is the fusion of A at t', a certain scattered object we may call 'Harry'. Now why should we not say that Harry fused A at t? We have treated Charlie as a continuant, an object that endures for a period of time during which it undergoes change. It would seem only fair to treat Harry in the same way. Like Charlie, Harry underwent a certain change. He occupied a connected receptacle at t and a disconnected one at t'. Harry became a scattered object.

It would appear, then, that Harry has as good a claim to having been sole fuser of A at t as does Charlie. If (7) is true, so it would appear is

(10) (x) (x fuses A at t iff x = Harry).

Now, from (7) and (10) it presumably follows that at t Charlie and Harry were identical. But they are not identical now. And so once more we seem to have on our hands a temporary identity.

And another is in the offing. For consider Sam, the object which at t occupied the receptacle now occupied by Charlie. Sam is right where he was at t. He has of course undergone a change: at t he and z were in contact, the boundaries of their receptacles intersected; and this is no longer the case. But his position has not changed. The receptacle he now occupies is the one he occupied at t – namely, the interior of the closure of the union of the receptacles occupied by members of $A-\{z\}$. In short, Sam is at t' the fusion of $(A-\{z\})$. Or, to adopt the preferred form,

(11) (x) (x fuses $A - \{z\}$ at t' iff x = Sam).

But from (8) and (11) it presumably follows that at t' Charlie and Sam are identical, which they certainly were not at t. Though now identical, Charlie and Sam were once diverse. Or so it seems.

How are these temporary identities to be avoided? Perhaps some will say that there really is no such object as Harry: Charlie exists and so does the removed match, but those two objects do not compose or make up a single scattered object. But if there are scattered objects at all – and I have urged that there are – why object to Harry? There would appear to be no difference in principle between Harry, on the one hand, and my copy of *The Nature of Existence*, on the other. It has to be conceded that there is no readily available response to a request to say what sort or kind of object Harry is. But it is not clear to me that this is indicative of anything more than a paucity of readily available schemes of classification, a paucity resulting from quite parochial concerns of human beings. It is not out of the question that objects composed in the way Harry is should come to be of some interest; we should not then be at a loss to find an appropriate kind or sort.

Short of denying outright the existence of Harry, it might be contended that he begins to exist only at t', that he starts his career with Charlie's loss of z. This suggestion does have the merit of preserving the Fusion Principle while removing the necessity to puzzle over the apparent temporary identity of Charlie with Harry. But I see nothing else to be said for it. Bodies do from time to time become scattered. What reason is there to suppose this is not the situation with Harry? And in any case, what is to be done about Sam? There is no plausibility at all in an outright denial of his existence, and it seems obvious enough that his duration coincides with Charlie's. To deny the existence of Harry or to claim that he begins to exist only upon z's removal from Charlie simply leaves the problem of Sam untouched.

An alternative suggestion, one that not only preserves the Fusion Principle but also has the required generality, is that Charlie is really identical with Harry. On this view, Charlie fuses A at t and also at t'. He does not lose a part; he becomes scattered. As for Sam, well, once Charlie is thought of as scattered at t', we are free to think of Sam as fusing $A-\{z\}$ at t' without thereby implying a temporary identity of Sam with Charlie. There is simply no time at which Charlie and Sam occupy the same receptacle.

In spite of its neatness, I think this view will seem less than wholly satisfactory. We are all, I believe, inclined to think that after the removal of z Charlie survives as a non-scattered object. If asked to give his present location, we should indicate a certain connected receptacle, the one occupied by Sam. Perhaps our stake in Charlie's non-scattered persistence is not especially great, but it is there all the same; and certainly in other, analogous cases the view under discussion would seem quite unacceptable. If a branch falls from a tree, the tree does not thereby become scattered; and a human body does not become scattered upon loss of a bit of fingernail.

At this point some will despair of preserving the Fusion Principle. They will see no alternative to saying that Charlie and Harry, though distinct, nevertheless occupy the same receptacle at t and that Charlie and Sam, though again distinct objects, share a receptacle at t'. To take this position is to sacrifice the Fusion Principle by denying that exactly *one* thing fuses a given non-empty set of material objects. Both Charlie and Harry, according to this view, fuse A at t; neither has the property of being *sole* fuser of A at t. Similarly, neither Charlie nor Sam is sole fuser of $A-\{z\}$ at t'; for at t' Charlie and Sam are spatially coincident.

This view seems to me to put undue strain on one's metaphysical imagination. Locke wrote:

> ... never finding, nor conceiving it possible, that two things of the same kind should exist in the same place at the same time, we rightly conclude, that, whatever exists anywhere at any time, excludes all of the same kind, and is there itself alone.[16]

Are not Charlie and Harry two things of some one appropriate kind? Notice, furthermore, that it is not that just *two* material objects will, on this view, occupy the same receptacle at the same time; for it takes only a little ingenuity to find material objects other than Charlie and Harry with an equal claim to occupancy of that receptacle at t. To give some indication of the procedure involved, let us remove a second match from Charlie, place it some distance from z and from Charlie, and again put Charlie back where he was. Charlie has lost another part. In thus putting Charlie back where he was while leaving the position of z unchanged, we have also put Harry back where *he* was; he now occupies a receptacle properly included in the receptacle he occupied at t'. Harry too has lost a part. As a result, he has lost the property, which he had at t', of being sole fuser of A. But something now has that property, a certain scattered object whose receptacle is the union of two receptacles: the one occupied by Harry and the one occupied by the second removed match. Let us call that object 'Bill'. Now, there is no more reason to suppose that Bill just now came into existence than there is to suppose that Harry came into existence at t'. Indeed, there is no reason to deny that there is a material object which occupied a connected receptacle at t, became somewhat scattered at t', and has just now had its degree of scatter increased. Bill has as good a claim to occupancy of Charlie's receptacle at t as do Charlie and Harry. So, if Charlie and Harry shared a receptacle at t, they shared it with Bill.

If the Fusion Principle is to be retained, is there an alternative to acquiescence in the view that Charlie fails to survive in non-scattered form? I think there is. The view I have in mind involves recourse to what are sometimes called 'temporal parts' or 'stages' of objects. In the case at hand the suggestion would be that although Charlie and Harry are distinct objects, as is revealed by their divergent careers, a certain temporal part of Charlie is identical with a certain temporal part of Harry: Charlie's t-stage, as we might call it, is identical with Harry's t-stage. Similarly, although Charlie and Sam are distinct objects, Charlie's t'-stage is identical with Sam's t'-stage. No stage of Sam is identical with any stage of Harry, though it happens that each stage of Harry has some stage of Sam as a spatial part. What was loosely spoken of earlier as the fusion of A at t is now to be thought of as the fusion of the set of t-stages of members of A; and this object is simply Charlie's t-stage – that is, Harry's t-stage. Similarly, Harry's t'-stage is the fusion of the set of t'-stages of members of A; and Sam's t'-stage – that is Charlie's t'-stage – is the fusion of the set of t'-stages of members of $A - \{z\}$. Charlie, Harry, and Sam thus come to be conceived as distinct four-dimensional objects, which happen on occasion to share a common temporal part.

Philosophers as divergent in their outlooks as McTaggart and Quine have found the doctrine of temporal parts congenial or even obviously true.[17] But there are others who exhibit something less than overwhelming enthusiasm for it.[18] To these latter I can say only that, if they are drawn to the Fusion Principle and are at the same time reluctant to think that Charlie fails to survive in non-scattered form, they had better learn to live with temporal parts.

Massachusetts Institute of Technology

NOTES

[1] *De Corpore*, II. 8. 1.

[2] Cantor, 'Ein Beitrag zur Mannigfaltigkeitslehre', *Journ. für die Reine und Angewandte Math.* **84** (1878), 242–258.

[3] The phrase, though not the view, is J. L. Austin's. See *Sense and Sensibilia*, Oxford, 1962, p. 8.

[4] *Principles of Philosophy*, Part II, Principle XV. (Translation by Haldane and Ross.)

[5] Kuratowski, *Topology*, Volume I, New York and London, 1966, p.75. A common alternative is 'regular open set'.

[6] Compare Halmos, *Lectures on Boolean Algebras*, Princeton, N.J., 1963, p. 14.

[7] The identification of receptacles with non-null open domains was suggested to me by remarks made by Tarski in 'Foundations of the Geometry of Solids', included in his *Logic, Semantics, and Metamathematics*, Oxford, 1956, pp. 24–29.

[8] See, e.g., his Letter to Arnauld, April 30, 1687, in George R. Montgomery (trans.), *Leibniz: Discourse on Metaphysics, Correspondence with Arnauld, and Monadology*, LaSalle, Ill., 1945, esp. pp. 189–191.

[9] "Every extended mass may be considered as a composite of two or of a thousand others, and the only extension there is, is that by contact. Consequently, we shall never find a body of which we can say that it is really one substance; it will always be an aggregate of several." Leibniz, Draft of the letter of Nov. 28–Dec. 8, 1686 to Arnauld, in Montgomery (*op. cit.*), pp. 149–157. The quotation is from pp. 154–5.

[10] For instance, the interior of the closure of a region containing a single point. Any set the interior of the closure of which is null is called *nowhere dense*.

[11] "The least corpuscle is actually subdivided *in infinitum* and contains a world of other creatures which would be wanting in the universe if that corpuscle was an atom, that is, a body of one entire piece without subdivision." Leibniz, Fourth Letter to Clarke, in Leroy E. Loemker (trans. and ed.), *Leibniz: Philosophical Papers and Letters*, Second Edition (1970), pp. 687–691. The quotation is from p. 691.

[12] Henry S. Leonard and Nelson Goodman, 'The Calculus of Individuals and its Uses', *Journal of Symbolic Logic* **5** (1940), 45–55. I have not had access to the earlier expositions given by Lesniewski, for references to which see the bibliography in Eugene C. Luschei, *The Logical Systems of Lesniewski*, Amsterdam, 1962.

[13] More economical characterizations are known. See, e.g., Tarski's 'Foundations of the Geometry of Solids'.

[14] See Tarski, *Logic, Semantics, and Metamathematics*, p. 333, footnote.

[15] Compare David Wiggins, *Identity and Spatio-Temporal Continuity*, Oxford, 1967, p. 68.

[16] *Essay concerning Human Understanding*, Bk. II, Ch.xxvii, Section 1. And compare Aquinas: "nec est possibile, secundum naturam, duo corpora esse simul in eodem loco,

qualiacumque corpora sint". (*Summa Theologiae*, I, 67, 2, *in corpore.*) See also Aristotle, *Physics*, 209^{a}6.

[17] See McTaggart, *The Nature of Existence*, Vol. I, Cambridge, 1921, p. 176; and Quine, *From a Logical Point of View*, Second Edition, revised, New York, 1963, pp. 65–79.

[18] Thus C. D. Broad: "It is plainly contrary to common sense to say that the phases in the history of a thing are parts of the thing". (*Examination of McTaggart's Philosophy*, Vol. I, Cambridge, 1933, pp. 349–350.)