Mereology 2

Thomas Bittner
Thomas.bittner@ifomis.uni-leipzig.de

Remarks:

- The course web-page was (is??) down
- Somebody hacked into Barry's Ontology site and deleted most of its content
- Please send me email –then I will send you the course notes

Talks ???

- First come – first serve
- History of Part-whole
- Holes

Overview

1. Extending ground mereology
2. The weak supplemention principle
3. The proper part principle
4. The strong supplementation principle
5. Extensional mereology (EM)
6. Theorems of EM
7. Summary

Ground mereology - **M**

- Axioms
  - M1 P xx
  - M2 P xy & P yx ⇒ x = y
  - M3 P xy & P yz ⇒ P xz
- Defined relations:
  - Overlap
  - Underlap
  - Proper part

Partial ordering is not parthood

- Ground mereology captures some aspects of parthood
- Bur admits models which conflict with our intuitions about parthood

⇒ More axioms are needed

Extending ground mereology

- Adding Principles asserting the existence of entities given the existence of other entities
  - Whenever an entity has one proper part then it has more than one proper part
  - Given two entities then there exists an entity which is the sum of them
  - Given a set of entities then there exists an entity that is the sum of the entities in that set
  - Products, complements, …

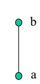
Our example model

- Structure $A_1 = (S_1, R_1)$ with
  - $S_1 = \{a,b\}$
  - $R_1 = \{(a,a), (a,b), (b,b)\}$
- We interpreted P as R_1
- We then verified that the axioms
  - P xx
  - P xy & P yx ⇒ x = y
  - P xy & P yz ⇒ P xz

  hold

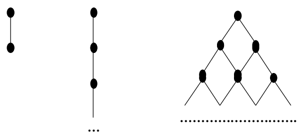
A nicer way of representing models

- Structure $A_1 = (S_1, R_1)$ with
  - $S_1 = \{a,b\}$
  - $R_1 = \{(a,a), (a,b), (b,b)\}$
  - R_1 interpreted as P

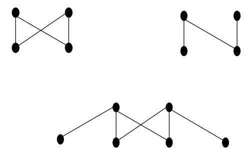


- We represent entities of our domain as points in the plane
- If *P xy* holds then we connect them by a line
- The first argument is further down than the second argument
- We do not explicit represent *P xx*

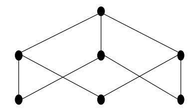
More models of ground mereology



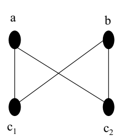
More models of ground mereology (2)



More models of ground mereology (3)

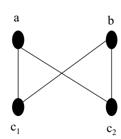


Exercise: list the pairs constituting the relation R_P



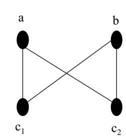
- S = {a,b, c_1 c_2}
- R_P = {
  - (c_1,a), (c_1,b), (c_2,a), (c_2,b),
  - (a,a),(b,b), (c_1,c_1), (c_1,c_1)
  - }

Exercise: which entities overlap?



- O xy iff (∃z)(R zx & R zy)
- O = {
  - (a,b), (b,a),
  - (c_1,a), (c_1,b), (c_2,a), (c_2,b),
  - (a,a),(b,b), (c_1,c_1), (c_1,c_1)
  - }

Exercise: which entities are disjoint?



- D xy iff ¬ O xy
- D = { (c_1,c_2), (c_2,c_1)}

Counterintuitive models

A whole with a
Single proper part

PP xy ⇒ (∃z)(PP zy & ¬(z=x))

If *x* is a proper part of *y* then
there is a *z* which is a proper part of *y* and *z* and *x* are not identical

Counterintuitive models

PP xy ⇒ (∃z)(PP zy & ¬(z=x))

R_{PP} = {(a,b)}

Counterintuitive models

PP xy ⇒ (∃z)(PP zy & ¬(z=x))

R_{PP} = {(a,b)}

z	PP ab	⇒	PP zb	&	¬(z=a)
a					
b					

Counterintuitive models

$PP\ xy \Rightarrow (\exists z)(PP\ zy\ \&\ \neg(z=x))$

$R_{PP} = \{(a,b)\}$

z	PP ab	⇒	PP zb	&	¬(z=a)
a	T	F	T	F	F
b					

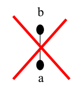
Counterintuitive models

$PP\ xy \Rightarrow (\exists z)(PP\ zy\ \&\ \neg(z=x))$

$R_{PP} = \{(a,b)\}$

z	PP ab	⇒	PP zb	&	¬(z=a)
a	T	F	T	F	F
b	T	F	F	F	T

Counterintuitive models



$PP\ xy \Rightarrow (\exists z)(PP\ zy\ \&\ \neg(z=x))$

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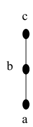
z	PP ab	⇒	PP zb	&	¬(z=a)
a	T	F	T	F	F
b	T	F	F	F	T

Counterintuitive models

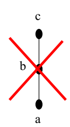


A whole consisting of three nested proper parts

$R_{PP} = \{(a,b),(b,c),(a,c)\}$

z	PP ab	⇒	PP zb	&	¬(z=a)
a	T	F	T	F	F
b	T	F	F	F	T
c	T	F	F	F	T

Counterintuitive models



A whole consisting of three nested proper parts

$R_{PP} = \{(a,b),(b,c),(a,c)\}$

z	PP ab	⇒	PP zb	&	¬(z=a)
a	T	F	T	F	F
b	T	F	F	F	T
c	T	F	F	F	T

Counterintuitive models

Infinite chain of nested proper parts

Is this model ruled out by
$PP\ xy \Rightarrow (\exists z)(PP\ zy\ \&\ \neg(z=x))$?

Counterintuitive models

Infinite chain of nested proper parts

This model is NOT ruled out by
$PP\ xy \Rightarrow (\exists z)(PP\ zy\ \&\ \neg(z=x))$
there is always a z with PP zx

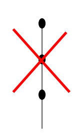
Counterintuitive models

Infinite chain of nested proper parts

This model is NOT ruled out by
$PP\ xy \Rightarrow (\exists z)(PP\ zy\ \&\ \neg(z=x))$
there is always a z with PP zx

How about
$PP\ xy \Rightarrow (\exists z)(PP\ zy\ \&\ \neg\ P\ zx)$?

Counterintuitive models



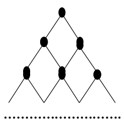
Infinite chain of nested proper parts

This model is NOT ruled out by
$PP\ xy \Rightarrow (\exists z)(PP\ zy\ \&\ \neg(z=x))$
there is always a z with PP zx

How about
$PP\ xy \Rightarrow (\exists z)(PP\ zy\ \&\ \neg\ P\ zx)$?

Since the chain is infinite there is no such z
⇒ the model is ruled out

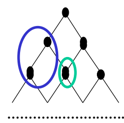
Counterintuitive models



Infinite tree of nested proper parts

Is this model ruled out by
PP xy $\Rightarrow$ $(\exists z)$(PP zy & $\neg$ P zx) ?

Counterintuitive models

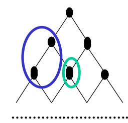


Infinite tree of nested proper parts

This model is NOT ruled out by
PP xy $\Rightarrow$ $(\exists z)$(PP zy & $\neg$ P zx)

For all x and y with PP xy
there is always a z with P zx

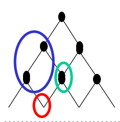
Weak supplementation principle



Infinite tree of nested proper parts

Is his model ruled out by
PP xy $\Rightarrow$ $(\exists z)$(PP zy & $\neg$ O zx)
???

Weak supplementation principle

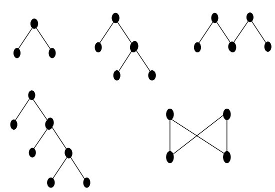


Infinite tree of nested proper parts

This model is ruled out by
PP xy $\Rightarrow$ $(\exists z)$(PP zy & $\neg$ O zx)

For all x and y with PP xy
there is never a z wich does
not overlap O zx

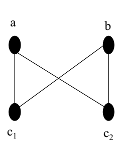
Models of the WSP



Minimal mereology - **MM**

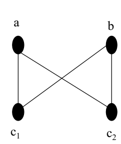
- Axioms
  - M1 P xx
  - M2 P xy & P yx $\Rightarrow$ x = y
  - M3 P xy & P yz $\Rightarrow$ P xz
  - WSP PP xy $\Rightarrow$ $(\exists z)$(PP zy & $\neg$ O zx)

Models of WSP



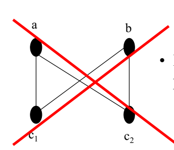
- R_{PP} = {(c_1,a), (c_2,a), (c_1,b), (c_2,b)}
- Proper parts of a: c_1,c_2
- Proper parts of b: c_1,c_2
- Two DISTINCT entities which share the same proper parts !!

Two distinct entities which share the same proper parts



- A statue and the clay
- City of Vienna and the federal state of Vienna

Two distinct entities which share the same proper parts



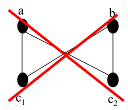
- But not for distinct physical objects

The proper part principle (PPP)

- If
  - *x* has some proper part and
  - Every proper part of *x* is a proper part of *y*
- Then *x* is a part of *y*
- $((\exists z)\mathrm{PP}\ zx\ \&\ (\forall z)(\mathrm{PP}\ zx \Rightarrow \mathrm{PP}\ zy)) \Rightarrow \mathrm{P}\ xy$

The proper part principle (PPP)

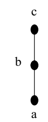
- If
  - *x* has some proper part and
  - Every proper part of *x* is a proper part of *y*
- Then *x* is a part of *y*



- Proper parts of a: c_1,c_2
- Proper parts of b: c_1,c_2
- (a,b) is not in R_P

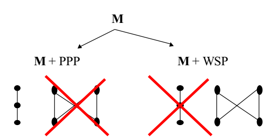
The proper part principle (PPP)

- If
  - *x* has some proper part and
  - Every proper part of *x* is a proper part of *y*
- Then *x* is a part of *y*



- Proper parts of a: {}
- Proper parts of b: {a}
- Proper parts of c: {a,b}
- R_P={(a,b),(b,c),(a,c),…}

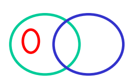
PPP and WSP are independent



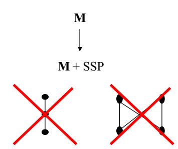
M + PPP not |-- WSP   **M** + WSP not |-- PPP

The strong supplementation principle (SSP)

- If x is not a part of y then
  - There exists a z such that
    - z is a part of x and
    - z does not oberlap y
- $\neg \mathrm{P}\ xy \Rightarrow (\exists z)(\mathrm{P}\ zx\ \&\ \neg \mathrm{O}\ zy)$



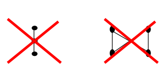
SSP (cont)



Assignment: give truthtables that show that these are not models
Of SSP !

SSP (cont)

M + SSP rules out:



- The theory formed by **M** + SSP is strictly stronger than the theories formed by **M** + WSP and **M** + PPP
  - **M** + WSP not |-- SSP
  - **M** + PPP not |-- SSP
- The theory formed by **M** + SSP is strictly stronger than the theories formed by **M** + WSP + PPP
  - **M** + PPP + WSP not |-- SSP

M + PPP + WSP not |-- SSP

- Find a structure that is a model of
  M + PPP + WSP but not of SSP
- All half-open, half closed intervals of the real line:
  [0,1), [1,2), …,(0,1],(1,2]
- **M** + PPP + WSP are satisfied:
  - PP interpreted as subset relation among sets $\subset$
  - M is satisfied ($\subset$ is a partial order)
  - If all proper subsets of *x* are proper subsets of *y* then *x* is a subset of *y* – therefore PPP holds
  - If $x \subset y$ then there exists a *z* such that $x \cup z = y$ and $x \cap z = \varnothing$

M + PPP + WSP not |-- SSP

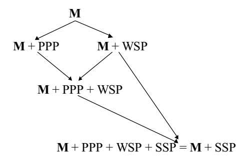
- All half-open, half closed intervals of the real line: [0,1), [1,2), …,(0,1],(1,2]
- SSP is NOT satisfied
  - $\neg\ [0,1) \subseteq (0,1]$
  - $\neg(\exists z)\ (z \subseteq [0,1)\ \&\ \neg\ \mathrm{O}\ z\ (0,1]$
    - How about {0}or {1}?
    - A singelton set cannot be half open or half closed

SSP (cont)

M + SSP rules out:

- The theory formed by **M** + SSP is strictly stronger than the theories formed by **M** + WSP and **M** + PPP
  - **M** + WSP not |-- SSP
  - **M** + PPP not |-- SSP
  - **M** + PPP + WSP not |-- SSP
- But:
  - **M** + SSP |-- WSP   Assignment due Sep. 17
  - **M** + SSP |-- PPP   Assignment due Sep. 10

Hierarchy of theories



Extensional mereology - **EM**

- Axioms
  - M1   P xx
  - M2   P xy & P yx ⇒ x = y
  - M3   P xy & P yz ⇒ P xz
  - SSP   ¬P xy ⇒ (∃z)(P zx & ¬ O zy)

Some theorems of EM

- M+SSP |-- (z)(O zx ⇒ O zy) ⇒ P xy
- M+SSP |-- x = y ⇔(∀z)(O zx ⇔ O zy)
  - Extensionality of overlap
  - Identity of two entities is determined by the entities they overlap
- M+PPP |-- ((∃z)PP zx & (∀z)(PP zx ⇔ PP zy)) ⇒ x = y

(z)(O zx ⇒ O zy) ⇒ P xy

1.	¬P xy ⇒ (∃z)(P zx & ¬ O zy)	SSP
2.	¬(∃z)(P zx & ¬ O zy) ⇒ ¬¬ P xy	1 transp
3.	(z) ¬(P zx & ¬ O zy) ⇒ P xy	2 DN, QN
4.	(z)(P zx ⇒ O zy) ⇒ P xy	3 Imp
5.	(z)(O zx ⇒ O zy)	ass
6.	O zx ⇒ O zy	5 UI
7.	P xz ⇒ O xz	Theorem
8.	P xz ⇒ O zy	7,6 HS
9.	(z)(P xz ⇒ O zy)	8 UG
10.	P xy	9, 4 MP
11.	(z)(O zx ⇒ O zy) ⇒ P xy	5-10 CP

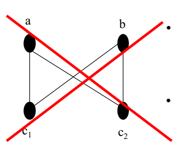
(∀z)(O zx ⇔ O zy) ⇒ x=y

1.	(∀z)(O zx ⇔ O zy)	ass
2.	O zx ⇔ O zy	1 UI
3.	O zx ⇒ O zy & O zy ⇒ O zx	2 Eq
4.	(∀z)(O zx ⇒ O zy)	(3 simp)UG
5.	(z)(O zx ⇒ O zy) ⇒ P xy	Theorem UI
6.	P xy	4,5 MP
7.	(z)(O zy ⇒ O zx)	(3 simp)UG
8.	(z)(O zy ⇒ O zx) ⇒ P yx	Theorem UI
9.	P yx	7,8 MP
10.	P xy & P yx	6,9 conj
11.	x = y	10, M2 MP
12.	(∀z)(O zx ⇔ O zy) ⇒ x=y	1- 11 CP

((∃z)PP zx & (∀z)(PP zx ⇔ PP zy)) ⇒ x = y

1.	((∃z)PP zx & (∀z)(PP zx ⇔ PP zy))	ass
2.	(∀z)(PP zx ⇔ PP zy)	1 simp
3.	PP zx ⇔ PP zy	2 UI
4.	PP zx ⇒ PP zy & PP zy ⇒ PP zx	3 Eq
5.	(z)(PP zx ⇒ PP zy)	(4 simp) UG
6.	((∃z)PP zx & (z)(PP zx ⇒ PP zy)	(1 simp),5 conj
7.	P xy	6, PPP MP
8.	(z)(P zy ⇒ PP zx)	(4 simp) UG
9.	(∃z)PP zx	1 simp
10.	PP zx	
11.	PP zx & P xy	10, 7 conj
12.	PP zy	≈11, M3 MP
13.	(∃z) PP zy	12 EG
14.	(∃z) PP zy & (z)(PP zy ⇒ PP zx)	13, 8 conj
15.	P yx	14, PPP MP
16.	P xy & P yx	7,15 conj
17.	x = y	16, M2 MP
18.	((∃z)PP zx & (∀z)(PP zx ⇔ PP zy)) ⇒ x = y	1-17 CP

Two distinct entities which share the same proper parts



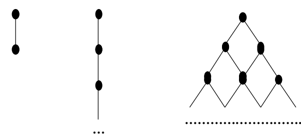
- This cannot be a model of PPP since by PPP we have extensionality of proper parthood:
- Two entities that have the same proper parts must be identical

Summary

Extending ground mereology

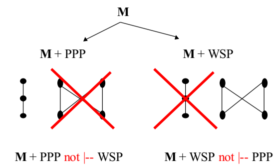
- Adding Principles asserting the existence of entities given the existence of other entities
  - Whenever an entity has one proper part then it has more than one proper part
  - Given two entities then there exists an entity which is the sum of them
  - Given a set of entities then there exists an entity that is the sum of the entities in that set
  - Products, complements, …

Models of ground mereology ruled out by WSP

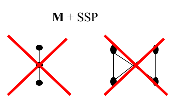


WSP $\quad PP\ xy \Rightarrow (\exists z)(PP\ zy\ \&\ \neg\ O\ zx)$

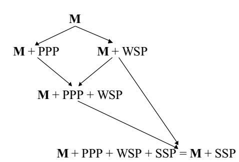
PPP and WSP are independent



SSP
$\neg P\ xy \Rightarrow (\exists z)(P\ zx\ \&\ \neg O\ zy)$



Hierarchy of theories



Extensional mereology - **EM**

- Axioms
  - M1 $\quad P\ xx$
  - M2 $\quad P\ xy\ \&\ P\ yx \Rightarrow x = y$
  - M3 $\quad P\ xy\ \&\ P\ yz \Rightarrow P\ xz$
  - SSP $\quad \neg P\ xy \Rightarrow (\exists z)(P\ zx\ \&\ \neg\ O\ zy)$

Some theorems of EM

- M+SSP |-- $(z)(O\ zx \Rightarrow O\ zy) \Rightarrow P\ xy$
- M+SSP |-- $x = y \Leftrightarrow (\forall z)(O\ zx \Leftrightarrow O\ zy)$
  - Extensionality of overlap
  - Identity of two entities is determined by the entities they overlap
- M+PPP |-- $((\exists z)PP\ zx\ \&$
  $(\forall z)(PP\ zx \Leftrightarrow PP\ zy)) \Rightarrow\ x = y$

Assignments due Sep. 10

- M |-- $(z)(P\ zx \Leftrightarrow P\ zy) \Leftrightarrow x = y$
- M |-- $P\ xy \Rightarrow (z)(O\ zx \Rightarrow O\ zy)$
- **M** + SSP |-- PPP
  fill in the gaps in Simon's proof on pg. 29 of 'Parts'
- Give truthtables that show that the given structures are not models Of SSP !