Mereology 5

Thomas Bittner
bittner3@buffalo.edu

Ground mereology - **M**

- Axioms
  - M1 $P\,xx$
  - M2 $P\,xy \,\&\, P\,yx \Rightarrow x = y$
  - M3 $P\,xy \,\&\, P\,yz \Rightarrow P\,xz$
- Defined relations:
  - Overlap
  - Underlap
  - Proper part

Extending ground mereology

- Adding Principles asserting the existence of entities given the existence of other entities
  - Whenever an entity has one proper part then it has more than one proper part
  - Given two overlapping entities then there exists an entity which is the product of them and given two entities then there exists an entity which is the sum of them
  - Given a set of entities then there exists an entity that is the sum of the entities in that set

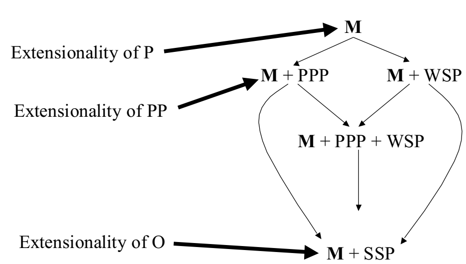
Whenever an entity has one proper part then it has more than one proper part

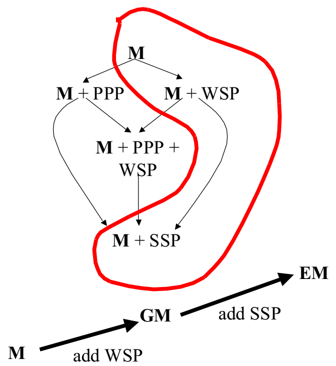
- WSP
  - $PP\,xy \Rightarrow (\exists z)(P\,zy \,\&\, \neg O\,zx)$
- PPP
  - $((\exists z)PP\,zx \,\&\, (\forall z)(PP\,zx \Rightarrow PP\,zy)) \Rightarrow P\,xy$
- SSP
  - $\neg P\,xy \Rightarrow (\exists z)(P\,zx \,\&\, \neg O\,zy)$
- RP
  - $\neg P\,xy \Rightarrow (\exists z)(z = x\text{-}y)$

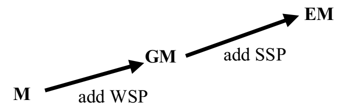
Extensionality (relation to identity)

Of parthood	Of proper parthood	Of overlap
$(z)(P\,zx \Leftrightarrow P\,zy) \Leftrightarrow x = y$	$(\exists z)PP\,zx \,\&\, (z)(PP\,zx \Leftrightarrow PP\,zy) \Leftrightarrow x = y$	$(z)(O\,zx \Leftrightarrow O\,zy) \Leftrightarrow x = y$

Hierarchy of theories



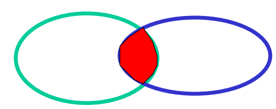




Extending ground mereology

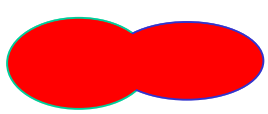
- Adding Principles asserting the existence of entities given the existence of other entities
  - Given two overlapping entities then there exists an entity which is the product of them and
  - given two entities then there exists an entity which is the sum of them
- Closure principles

Binary products



- If the two entities a and b overlap then the product of a and b is an entity c which is such that for any w if w is a part of c then w is part of a and part of b:
  prod(abc) ≡ (∀w)(P wc ⇔ Pwa & Pwb)

Binary sums



- If the two entities a and b underlap then the sum of a and b is an entity c which is such that for any w: if w overlaps c then w overlaps a or w overlaps b and vice versa:
  sum(abc) ≡ (∀w)(O wc ⇔ O wa or O wb)

The binary product axiom

- If two entities x and overlap then there exists an entity z which is such that whatever is part of z is also part of x and y and vice versa
- A_{prod} O xy ⇒(∃z) prod(xyz)
- A_{prod} ensures that products for overlappers always exist

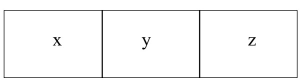
The binary sum axiom

- If two entities x and underlap then there exists an entity z which is such that whatever is overlaps z is also overlaps x or y and vice versa
- A_{sum} U xy $\Rightarrow$($\exists$z) sum(xyz)
- A_{sum} ensures that sums for underlappers always exist

The ι operator

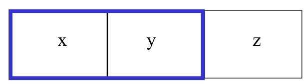
- Use the definitions
  - a*b ≡(ιz)(∀w)(P wz ⇔ Pwa & Pwb)
  - a+b ≡(ιz)(∀w)(O wz ⇔ O wa or O wb)
- Write the axioms as
  - A_* O xy ⇒ (∃z)(z = x*y)
  - A_+ U xy ⇒ (∃z)(z = x+y)

example



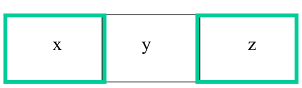
example

u = x + y



example

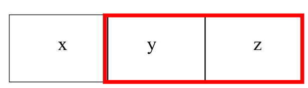
v = x + z



v consists of two disconnected pieces

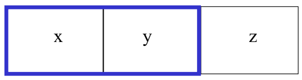
example

w = y + z



example

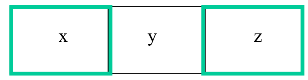
u = x + y



x is proper part of u

example

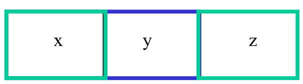
v = x + z



x is proper part of v

example

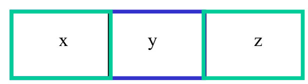
u = x + y
v = x + z



u overlaps v

example

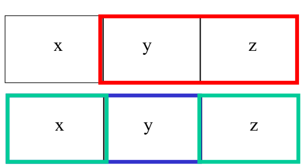
u = x + y
v = x + z



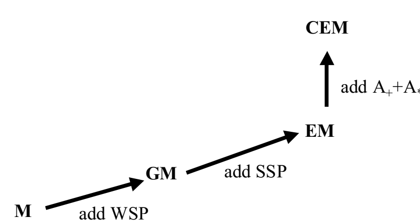
u * v = x

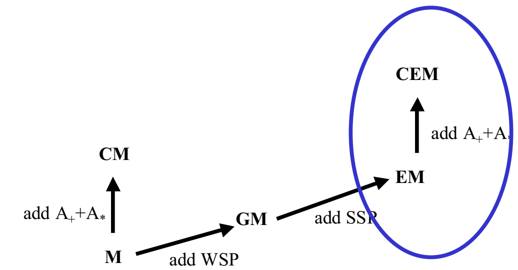
example

u = x + y
v = x + z
w = y + z



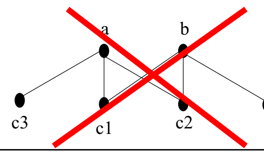
x + w = u + v = x + y + z

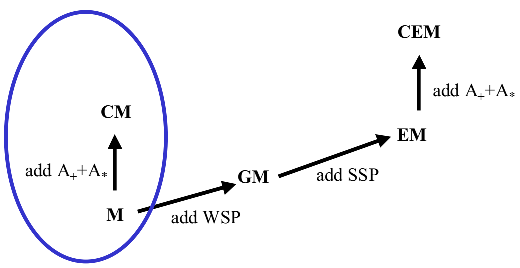




The binary product axiom (2)

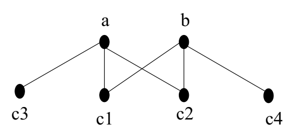
- A_{prod} $\quad$ $O\,xy \Rightarrow (\exists z)\ prod(xyz)$
- A_{prod} ensures that products for overlappers always exist
- From extensionality of parthood it follows that that products are unique:
  $prod\ (xyz_1)\ \&\ prod\ (xyz_2) \Rightarrow z_1{=}z_2$



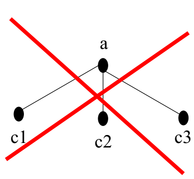


The binary product axiom (3)

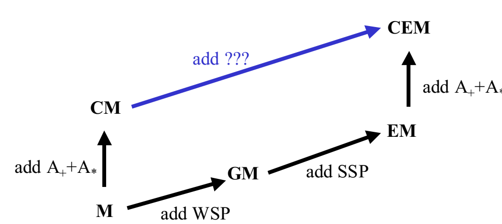
- A_{prod} $\quad$ $O\,xy \Rightarrow (\exists z)\ prod(xyz)$
- A_{prod} ensures that products for overlappers always exist
- No extensionality therefore no unique products
- prod (abc_1) and prod (abc_2) hold



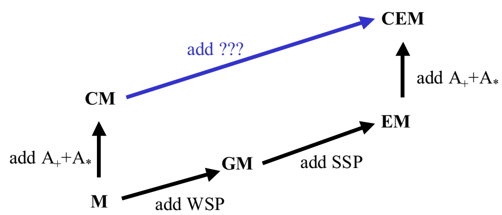
A model ruled out by A_{sum}

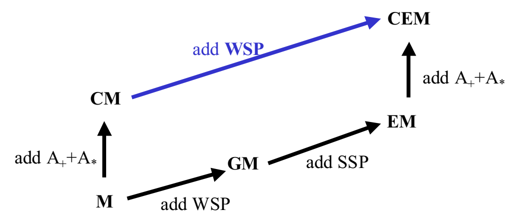


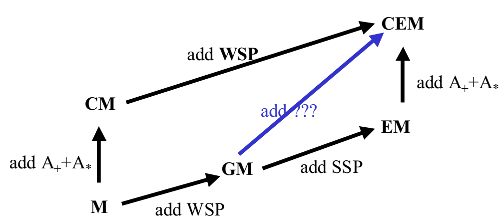
- c_1 and c_2 underlap but NOT sum(c_1c_2a):
- Not everything that overlaps a also overlaps c_1 or c_2 : c_3



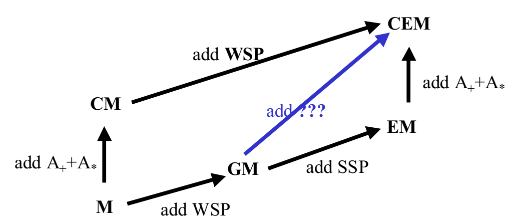
M+A_*+WSP |-- SSP



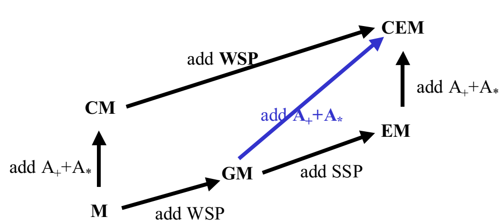




M+WSP+A_* |-- SSP



M+WSP+A_* |-- SSP



M+WSP+A_* |-- SSP

Lemmata

1. O xy ⇒ P (x*y)x — last class
2. ¬ P xy ⇒ (¬ O xy ⇒ (∃z)(P zx & ¬ O zy)) — at home
3. ¬ P xy ⇒ (O xy ⇒ (∃z)(P zx & ¬ O zy)) — last class
4. P zx ⇒ (O zy ⇒ O z(x*y)) — at home
5. PP xy or x = y ⇒ P xy — done
6. P xy & ¬x=y ⇒ PP xy — done

Theorem

M+WSP+A_* |-- ¬ P xy ⇒ (∃z)(P zx & ¬ O zy)

at home

M+WSP+A$_*$ |-- SSP

Lemmata

1. O xy ⇒ P (x*y)x — last class
2. ¬ P xy ⇒ (¬ O xy ⇒ (∃z)(P zx & ¬ O zy)) — at home
3. ¬ P xy ⇒ (O xy ⇒ (∃z)(P zx & ¬ O zy)) — last class
4. P zx ⇒ (O zy ⇒ O z(x*y)) — at home
5. PP xy or x = y ⇒ P xy — done
6. P xy & ¬x=y ⇒ PP xy — done

Theorem

M+WSP+A$_*$ |-- ¬ P xy ⇒ (∃z)(P zx & ¬ O zy)

at home

L1: O xy ⇒ P (x*y)x

…

3. z=x*y
4. (∃u)(sum(xyu) & (v)(sum(xyv) ⇒ u=v) & u=z) — 3 Dι
5. sum(xyu) & (v)(sum(xyv) ⇒ u=v) & u=z
6. sum(xyu) — 5 simp
7. (w)(P wu ⇔ (P wx & P wy)) — 6 D_{sum}
8. ….

3. z=x*y
4. (∃u)(prod(xyu) & (v)(prod(xyv) ⇒ u=v) & u=z) — 3 Dι
5. prod(xyu) & (v)(prod(xyv) ⇒ u=v) & u=z
6. prod(xyu) — 5 simp
7. (w)(P wu ⇔ (P wx & P wy)) — 6 D_{sum}
8. ….

M+WSP+A$_*$ |-- SSP

Lemmata

1. O xy ⇒ P (x*y)x — last class
2. ¬ P xy ⇒ (¬ O xy ⇒ (∃z)(P zx & ¬ O zy)) — at home
3. ¬ P xy ⇒ (O xy ⇒ (∃z)(P zx & ¬ O zy)) — last class
4. P zx ⇒ (O zy ⇒ O z(x*y)) — at home
5. PP xy or x = y ⇒ P xy — done
6. P xy & ¬x=y ⇒ PP xy — done

Theorem

M+WSP+A$_*$ |-- ¬ P xy ⇒ (∃z)(P zx & ¬ O zy)

at home

M+WSP+A$_*$ |-- SSP

O xy ⇒ P (x*y)x (using A$_*$)

↘

¬ P xy ⇒ (O xy ⇒ (∃z)(P zx & ¬ O zy))

↗

P zx ⇒ (O zy ⇒ O z(x*y)) (using A$_*$)

M+WSP+A$_*$ |-- SSP

Lemmata

1. O xy ⇒ P (x*y)x — last class
2. ¬ P xy ⇒ (¬ O xy ⇒ (∃z)(P zx & ¬ O zy)) — at home
3. ¬ P xy ⇒ (O xy ⇒ (∃z)(P zx & ¬ O zy)) — last class
4. P zx ⇒ (O zy ⇒ O z(x*y)) — at home
5. PP xy or x = y ⇒ P xy — done
6. P xy & ¬x=y ⇒ PP xy — done

Theorem

M+WSP+A$_*$ |-- ¬ P xy ⇒ (∃z)(P zx & ¬ O zy)

at home

~~¬ P xy~~ ⇒ (¬ O xy ⇒ (∃z)(P zx & ¬ O zy))

¬ O xy ⇒ (∃z)(P zx & ¬ O zy)

1. ¬ O xy — ass
2. P xx — M1
3. P xx & ¬ O xy — 1,2 conj
4. (∃z)(P xx & ¬ O xy) — 3 EG
5. ¬ O xy ⇒ (∃z)(P zx & ¬ O zy) — 1-4 CP

M+WSP+A_* |-- SSP

Lemmata

1.	O xy ⇒ P (x*y)x	last class
2.	¬ P xy ⇒ (¬ O xy ⇒ (∃z)(P zx & ¬ O zy))	done
3.	¬ P xy ⇒ (O xy ⇒ (∃z)(P zx & ¬ O zy))	last class
4.	P zx ⇒ (O zy ⇒ O z(x*y))	at home
5.	PP xy or x = y ⇒ P xy	done
6.	P xy & ¬x=y ⇒ PP xy	done

Theorem

M+WSP+A_* |-- ¬ P xy ⇒ (∃z)(P zx & ¬ O zy)

at home

M+WSP+A_* |-- SSP

Lemmata

1.	O xy ⇒ P (x*y)x	last class
2.	¬ P xy ⇒ (¬ O xy ⇒ (∃z)(P zx & ¬ O zy))	done
3.	¬ P xy ⇒ (O xy ⇒ (∃z)(P zx & ¬ O zy))	last class
4.	P zx ⇒ (O zy ⇒ O z(x*y))	at home
5.	PP xy or x = y ⇒ P xy	done
6.	P xy & ¬x=y ⇒ PP xy	done

Theorem

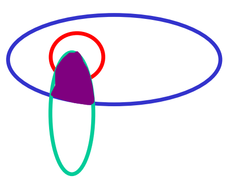
M+WSP+A_* |-- ¬ P xy ⇒ (∃z)(P zx & ¬ O zy)

at home

P zx ⇒ (O zy ⇒ O z(x*y))

or equivalent (by Exp)

(P zx & O zy) ⇒ O z(x*y)



(P zx & O zy) ⇒ O z(x*y)

0.	P zx & O zy ⇒ O xy	Th
1.	P zx & O zy	ass
2.	O xy	1,0 MP
3.	(∃v)(v= x*y)	2, A_* MP
4.	v= x*y	
5.	(w)(P wv ⇔ (P wx & P wy))	4 D_*
6.	O zy	1 simp
7.	(∃u)(P uz & P uy)	6 D_O
8.	P uz & P uy	
9.	P uz & P zx	(8 simp), (1 simp) conj
10.	P ux	9, M3 MP
11.	P ux & P uy	10, (8 simp) conj
12.	P uv ⇔ (P ux & P uy)	5 UI
13.	P uv	12, 13 MP
14.	P u(x*y)	13, 4 Id
15.	P u(x*y) & P uz	14, (8 simp) conj
16.	(∃u)(P u(x*y) & P uz)	15 EG, 8-15 EI, 4-15 EI
17.	O (x*y)z	16 D_O
18.	(P zx & O zy) ⇒ O z(x*y)	1-17 CP

M+WSP+A_* |-- SSP

Lemmata

1.	O xy ⇒ P (x*y)x	last class
2.	¬ P xy ⇒ (¬ O xy ⇒ (∃z)(P zx & ¬ O zy))	done
3.	¬ P xy ⇒ (O xy ⇒ (∃z)(P zx & ¬ O zy))	last class
4.	P zx ⇒ (O zy ⇒ O z(x*y))	done
5.	PP xy or x = y ⇒ P xy	done
6.	P xy & ¬x=y ⇒ PP xy	done

Theorem

M+WSP+A_* |-- ¬ P xy ⇒ (∃z)(P zx & ¬ O zy)

at home

M+WSP+A_* |-- SSP

Lemmata

1.	O xy ⇒ P (x*y)x	last class
2.	¬ P xy ⇒ (¬ O xy ⇒ (∃z)(P zx & ¬ O zy))	done
3.	¬ P xy ⇒ (O xy ⇒ (∃z)(P zx & ¬ O zy))	last class
4.	P zx ⇒ (O zy ⇒ O z(x*y))	done
5.	PP xy or x = y ⇒ P xy	done
6.	P xy & ¬x=y ⇒ PP xy	done

Theorem

M+WSP+A_* |-- ¬ P xy ⇒ (∃z)(P zx & ¬ O zy)

at home

M+WSP+A_* |-- ¬ P xy ⇒ (∃z)(P zx & ¬ O zy)

1.	¬ P xy	ass
2.	O xy or ¬ O xy	Ex. Middle
3.	O xy ⇒ (∃z)(P zx & ¬ O zy)	1, L3 MP
4.	¬ O xy ⇒ (∃z)(P zx & ¬ O zy)	L2
5.	O xy ⇒ (∃z)(P zx & ¬ O zy) & ¬ O xy ⇒ (∃z)(P zx & ¬ O zy)	3,4 conj
6.	(∃z)(P zx & ¬ O zy) or (∃z)(P zx & ¬ O zy)	2, 5 CD
7.	(∃z)(P zx & ¬ O zy)	6 taut
8.	¬ P xy ⇒ (∃z)(P zx & ¬ O zy)	1-7 CP

M+WSP+A_* |-- SSP

Lemmata

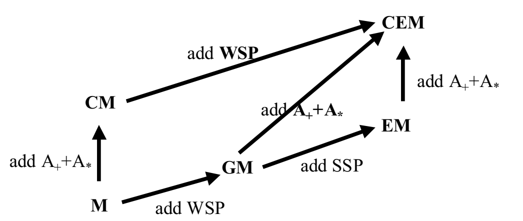
1.	O xy ⇒ P (x*y)x	last class
2.	¬ P xy ⇒ (¬ O xy ⇒ (∃z)(P zx & ¬ O zy))	done
3.	¬ P xy ⇒ (O xy ⇒ (∃z)(P zx & ¬ O zy))	last class
4.	P zx ⇒ (O zy ⇒ O z(x*y))	done
5.	PP xy or x = y ⇒ P xy	done
6.	P xy & ¬x=y ⇒ PP xy	done

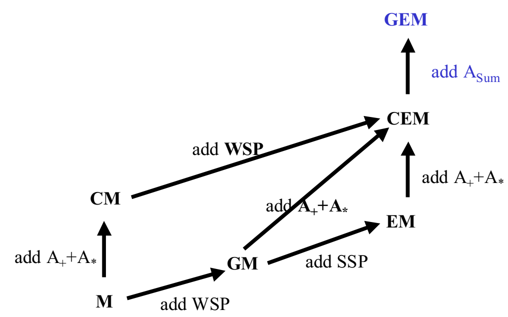
Theorem

M+WSP+A_* |-- ¬ P xy ⇒ (∃z)(P zx & ¬ O zy)

done

M+WSP+A_* |-- SSP ✓



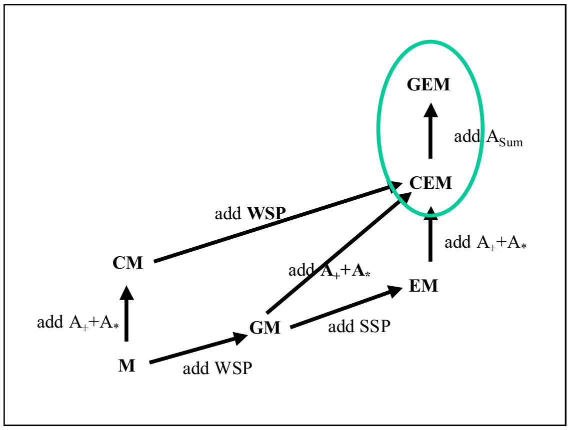


The summation axiom

- z Sum x φ means:
  - z is the sum of all x that satisfy φ
- z Sum x φ ≡
  - (w)(O wz ⇔(∃x)(φ(x) & O xw))
  - Anything overlaps z iff there exists an entity x that satisfies φ and that overlaps w
- The summation axiom
  - (∃x)φ(x) ⇒ (∃z) z Sum x φ
  - Whatever φ there is if there is one thing that satisfies φ then there exists the sum of all φ-ers

Strength of the summation axiom

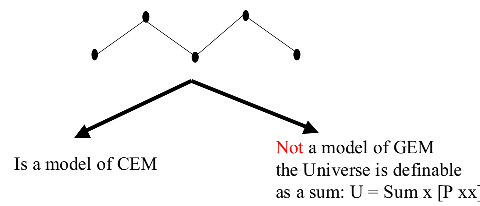
- x+y ≡ Sum z (P zx or P zy)
- x * y ≡ Sum z (P zx & P zy)
- x – y ≡ Sum z (P zx & ¬O zy)
- ~x ≡ Sum z (¬O zx)
- U = Sum z (P zz)

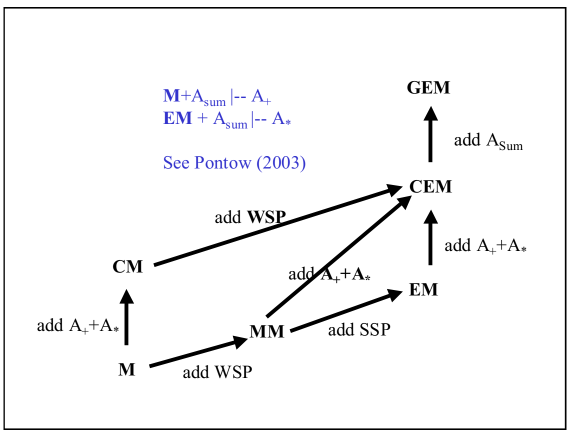


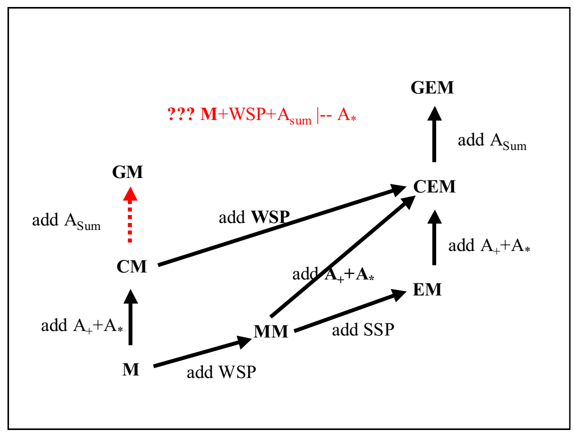
CEM vs. GEM

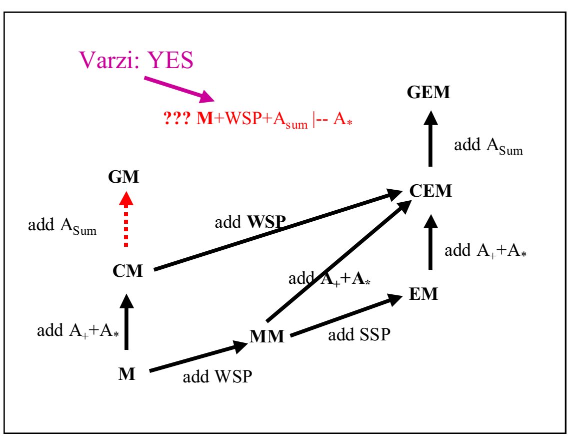
GEM is strictly stronger than CEM

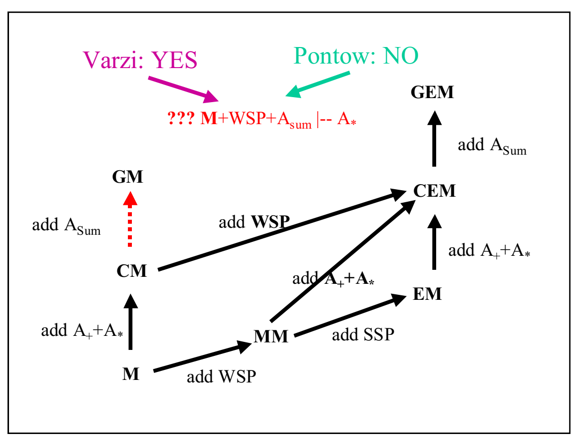
- Obvious in infinite domains
- But also in finite domains





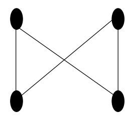






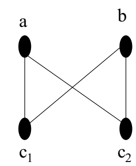
Pontow’s counter model

- In order to show that A_* is not derivable from GM = **M**+WSP+A_{sum} **M**+WSP+A_{sum} NOT |-- A_*
- He gives a model that
  - satisfies M, WSP, A_{sum}
  - and does NOT satisfy A_*
  - and does not satisfy SSP

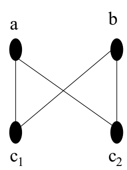


- Complicated: model has to satisfy A_{Sum} for arbitrary formulas ϕ

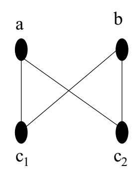
Assignment



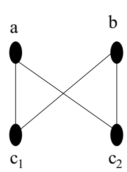
Show that this structure is not a model for A_*



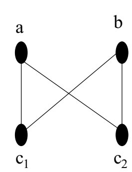
z	w	P wz	⇔	P wa	&	P wb
a	a	P aa	F	P aa	F	P ab
a	b					
a	c1					
a	c2					



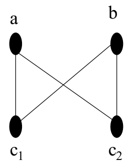
z	w	P wz	⇔	P wa	&	P wb
b	a	P ab	T	P aa	T	P ab
b	b					
b	c1					
b	c2					



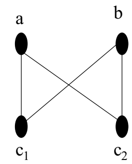
z	w	P wz	⇔	P wa	&	P wb
b	a	P ab	T	P aa	T	P ab
b	b	Pbb	F	P ba	F	P bb
b	c1					
b	c2					



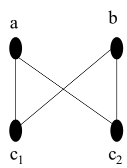
z	w	P wz	⇔	P wa	&	P wb
c1	a	P ac1	T	P aa	F	P ab
c1	b					
c1	c1					
c1	c2					



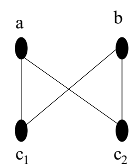
z	w	P wz	⇔	P wa	&	P wb
c1	a	P ac1	T	P aa	F	P ab
c1	b	P bc1	T	P ba	F	P bb
c1	c1					
c1	c2					



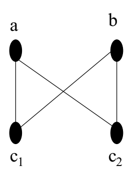
z	w	P wz	⇔	P wa	&	P wb
c1	a	P ac1	T	P aa	F	P ab
c1	b	P bc1	T	P ba	F	P bb
c1	c1	P c1c1	T	P c1a	T	P c1b
c1	c2					



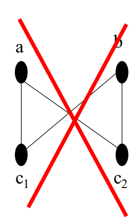
z	w	P wz	⇔	P wa	&	P wb
c1	a	P ac1	T	P aa	F	P ab
c1	b	P bc1	T	P ba	F	P bb
c1	c1	P c1c1	T	P c1a	T	P c1b
c1	c2	P c2c1	F	P c2a	T	P c2 b



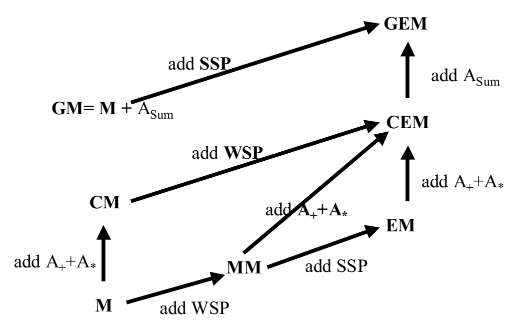
z	w	P wz	⇔	P wa	&	P wb
c2	a					
c2	b					
c2	c1	P c2c1	F	P c1a	T	P c1b
c2	c2					



z	w	P wz	⇔	P wa	&	P wb
a	a	P aa	F	P aa	F	P ab
a	b					
a	c1					
a	c2					
b	a					
b	b	Pbb	F	P ba	F	P bb
b	c1					
b	c2					
c1	a	P ac1	T	P aa	F	P ab
c1	b	P bc1	T	P ba	F	P bb
c1	c1	P c1c1	T	P c1a	T	P c1b
c1	c2	P c2c1	F	P c2a	T	P c2 b
c2	a					
c2	b					
c2	c1	P c2c1	F	P c1a	T	P c1b
c2	c2					



z	w	P wz	⇔	P wa	&	P wb
a	a	P aa	**F**	P aa	F	P ab
a	b					
a	c1					
a	c2					
b	a					
b	b	Pbb	**F**	P ba	F	P bb
b	c1					
b	c2					
c1	a					
c1	b					
c1	c1					
c1	c2	P c2c1	**F**	P c2a	T	P c2 b
c2	a					
c2	b					
c2	c1	P c2c1	**F**	P c1a	T	P c1b
c2	c2					



SSP implies RP ???

Varzi, A (2003), Mereology, pg. 15

The corresponding closure principles can therefore be stated thus:

(P.8) $\neg Pyx \rightarrow \exists z(z = x - y)$ Remainder

(P.9) $\exists z \neg Pzx \rightarrow \exists z(z = \sim x)$ Complementation

SSP

The first of these is equivalent to (P.5), but the second is independent of any of the principles considered so far. In many versions, a closure theory also involves a postulate to the effect that the domain has an upper bound—that is, there is something (the "universal individual") of which everything is part:

Assignment: prove that SSP implies RP or show that this is impossible

In which context ?????

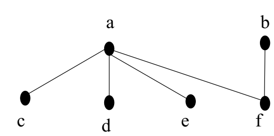
SSP implies RP

M+SSP NOT |-- RP

M+SSP + A_{+} |-- RP

M+SSP NOT |-- RP

Find a structure that models M and SSP but does not satisfy RP:

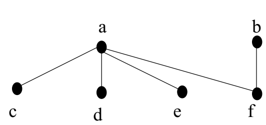


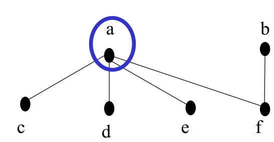
Satisfies M + SSP

$\neg$ P xy $\Rightarrow$ $(\exists z)(z = x\text{-}y)$

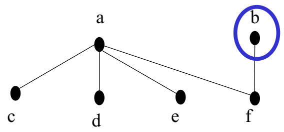
$\neg$ P xy $\Rightarrow(\exists z)(\forall w)$(P wz $\Leftrightarrow$ P wx & $\neg$O wy)

- Assign a to x and b to y
- Then $\neg$ P ab holds for the model below
- We need to show that $(\exists z)(\forall w)$(P wz $\Leftrightarrow$ P wa & $\neg$O wb) does not hold

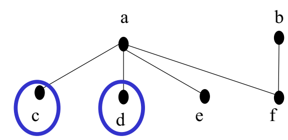




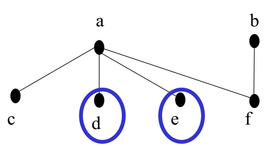
z	w	P wz	⇔	P wa	&	¬O wb
a	a	P aa	F	P aa	F	¬O ab



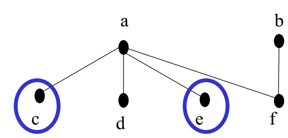
z	w	P wz	⇔	P wa	&	¬O wb
a	a	P aa	F	P aa	F	¬O ab
b	b	P bb	F	P ba	F	¬O bb



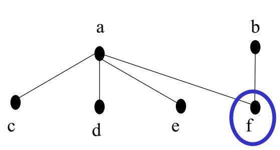
z	w	P wz	⇔	P wa	&	¬O wb
a	a	P aa	F	P aa	F	¬O ab
b	b	P bb	F	P ba	F	¬O bb
c	d	P dc	F	P da	T	¬O db



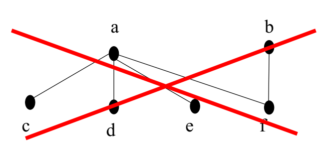
z	w	P wz	⇔	P wa	&	¬O wb
a	a	P aa	F	P aa	F	¬O ab
b	b	P bb	F	P ba	F	¬O bb
c	d	P dc	F	P da	T	¬O db
d	e	P ed	F	Pea	T	¬O eb



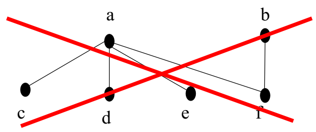
z	w	P wz	⇔	P wa	&	¬O wb
a	a	P aa	F	P aa	F	¬O ab
b	b	P bb	F	P ba	F	¬O bb
c	d	P dc	F	P da	T	¬O db
d	e	P ed	F	Pea	T	¬O eb
e	c	P ce	F	P ca	T	¬O cb



z	w	P wz	⇔	P wa	&	¬O wb
a	a	P aa	F	P aa	F	¬O ab
b	b	P bb	F	P ba	F	¬O bb
c	d	P dc	F	P da	T	¬O db
d	e	P ed	F	Pea	T	¬O eb
e	c	P ce	F	P ca	T	¬O cb
f	f	P ff	F	P fa	F	¬O fb



z	w	P wz	⇔	P wa	&	¬O wb
a	a	P aa	F	P aa	F	¬O ab
b	b	P bb	F	P ba	F	¬O bb
c	d	P dc	F	P da	T	¬O db
d	e	P ed	F	Pea	T	¬O eb
e	c	P ce	F	P ca	T	¬O cb
f	f	P ff	F	P fa	F	¬O fb

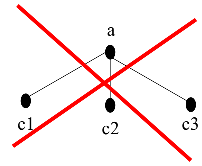


This proves: M+SSP NOT |-- RP

But:

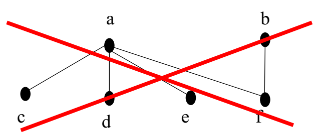
- what happens if we add A_{sum} ???
- M+SSP + A_{sum} |-- RP ???

Remmber:
A model ruled out by A_{sum}



- c_1 and c_2 underlap but NOT sum(c_1c_2a):
- Not everything that overlaps a also overlaps c_1 or c_2 : c_3

Our counter model is ruled out by A_{sum}



- c and d underlap but NOT sum(cda):
- Not everything that overlaps a also overlaps c or d : e

Atomistic and atomless mereologies

Atoms

- An atom is an entity with no proper parts
- Definition: A x ≡¬(∃y) PP yx
- Questions
  - Are there atoms?
  - If yes is everything *entirely* made up of atoms?
  - Does *everything* comprise at *least of some* atoms?
  - Is *everything* made up of *atomless gunk*?

Axioms of different strength and character are added to the mereology at hand

Mereology is neutral

- All options are logically compatible with mereology developed so far
- Principles regarding atomism can be added to mereology at any level:
  M, MM, EM, CM, CEM, GEM
- Principles of atomicity and atomlessness themselves are *mutually incompatible*
- Need to be added *in separation* to mereology

Atomlessness

- There are no atoms
- Everything made up of atomless gunk
- ¬A x
- Adding (¬A x) to **M** yields A**M**
- Adding (¬A x) to EM yields AEM
- In general adding (¬A x) to X yields AX

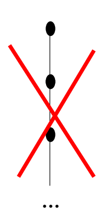
AGEM

- GEM = M+SSP+A_{sum}
- AGEM = GEM + ¬A x
- Example model:
  - Regular open sets of the Euclidian plane with 'P' interpreted as set-inclusion (Tarski 1935)
  - Proves that AGEM is consistent

Atomicity

Weak atomicity (AT0)

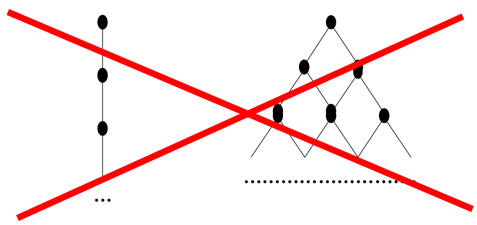
- There are atoms through not everything needs to have a complete atomic decomposition
- AT0 (∃x) Ax
- AT0 ensures that there is at least one atom



Atomicity (AT1)

- every entity has an atom as part
- AT1: (∃y)(Ay & P yx)
- Any finite model of M (EM,…,GEM) is atomistic, i.e., any finite model of M satisfies AT1
- Consistency of X+AT1
  - Trivial one-element model with 'P' interpreted as as identity
  - Models of AGEM: Boolean algebra with the bottom element removed

Models ruled out by AT1: (∃y)(Ay & P yx)



Atomic essentialism (AT2)

- Comes in two equivalent versions
  - AT2(a): ¬P xy ⇒(∃z)(A z & P zx & ¬P zy)
    (Atomic version of SSP)
    ¬P xy ⇒(∃z)(P zx & ¬O zy) (SSP)
  - AT2(b): (z)(A z ⇒ (P zx ⇒P zy)) ⇒ P xy
- Assignment:
  - prove the equivalence of AT2(a) and AT2(b)
  - prove that AT2 implies SSP, I.e., M+AT2 |- SSP

Equivalence of AT2(a) and AT2(b)

- Use the following logical equivalences:
  - Trans
  - QN
  - DN
  - DeM
  - Impl

Atomic essentialism (2)

We then can prove that

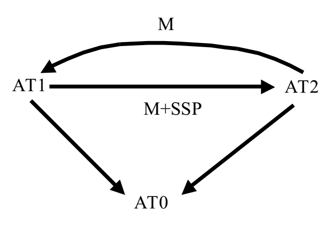
- Two things are identical iff they have the same atoms as parts
- x=y ⇔ (z)(A z ⇒ P zx ⇔ P zy)
- Very strong:
  - For identity it is sufficient to look at the atoms.
  - Other parts do not matter

Assignment

- Prove that
  x=y ⇔ (z)(A z ⇒ P zx ⇔ P zy)
  follows from AT2
- i.e.,
  M+ AT2 |-- x=y ⇔ (z)(A z ⇒ P zx ⇔ P zy)
- Hint:
  - it is easier to use AT2(b)
  - Use (P ⇒ (Q & R)) ⇒ ((P ⇒ Q)&(P ⇒ R))

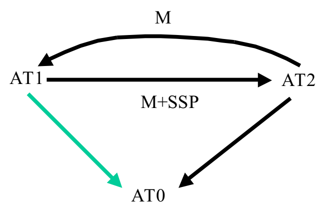
Relations between AT0,AT1,AT2

Masolo & Vieu 01



Relations between AT0,AT1,AT2

Masolo & Vieu 01



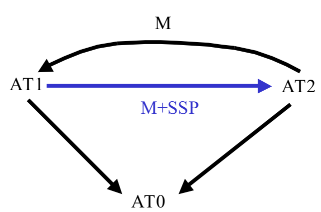
AT1 |-- AT0

AT1 |-- AT0

1.	(∃z)(A z & P zx)	AT1
2.	A z & P zx	
3.	A z	2 simp
4.	(∃z) A z	3 EG

Relations between AT0,AT1,AT2

Masolo & Vieu 01



M+SSP+AT1 |-- AT2

¬P xy ⇒(∃z)(A z & P zx & ¬P zy)

0.	P xy & ¬O yz ⇒ ¬O xz	assumed theorem
1.	¬P xy	ass
2.	(∃z)(P zx & ¬O zy)	1, SSP MP
3.	P zx & ¬O zy	
4.	(∃u)(A u & P uz)	AT1 UI
5.	A u & P uz	
6.	P uz & P zx	(5 simp), (3 simp) conj
7.	P ux	6, M3 MP
8.	P uz & ¬O zy	(5 simp), (3 simp) conj
9.	¬ O uy	8, 0 MP
10.	A u & P ux & ¬ O uy	(5 simp), 7, 9 conj
11.	(∃u)(A u & P ux & ¬ O uy)	10 EG
12.	¬P xy ⇒ (∃u)(A u & P ux & ¬ O uy)	1-11 CP

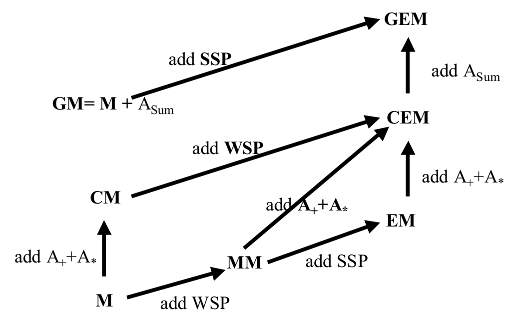
Weak atomlessness

- There is atomless gunk but not everything needs to be gunky
- (∃x)(∀y)(Pyx ⇒ ¬Ay)

Nihilism

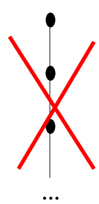
- Everything is an atom
- Ax

Summary



Weak atomicity (AT0)

- There are atoms through not everything needs to have a complete atomic decomposition
- AT0 (∃x) Ax
- AT0 ensures that there is at least one atom



Atomicity (AT1)

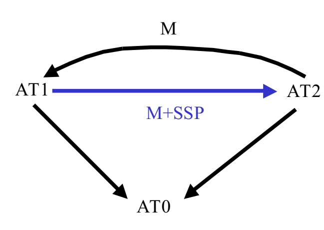
- every entity has an atom as part
- AT1: (∃y)(Ay & P yx)
- Any finite model of M (EM,…,GEM) is atomistic, i.e., any finite model of M satisfies AT1

Atomic essentialism (AT2)

- Comes in two equivalent versions
  - AT2(a): ¬P xy ⇒(∃z)(A z & P zx & ¬P zy)
    (Atomic version of SSP)
    ¬P xy ⇒(∃z)(P zx & ¬O zy) (SSP)
  - AT2(b): (z)(A z ⇒ (P zx ⇒P zy)) ⇒ P xy
- We then can prove that
  - Two things are identical iff they have the same atoms as parts
  - x=y ⇔ (z)(A z ⇒ P zx ⇔ P zy)

Relations between AT0,AT1,AT2

Masolo & Vieu 01



M+SSP+AT1 |-- AT2

AT1 |-- AT0

Assignments

- prove the equivalence of AT2(a) and AT2(b) (check hints above)
- prove that AT2 implies SSP, i.e., M+AT2 |- SSP
- Prove that $x=y \Leftrightarrow (z)(A\ z \Rightarrow P\ zx \Leftrightarrow P\ zy)$ follows from AT2 (check the hints above)