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LOGIC, FORM AND MATTER

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1. *Preamble.* What follows is an exercise in the philosophy of scientific or theoretical language. Language of this sort will be taken to be distinguished from other sorts of language at least in this: that it seeks to describe some segment of an independently existing reality. It will therefore be a condition on the philosophical understanding of theoretical language that it grapple with the idea of a segment of reality. Theoretical language is of course also distinguished by its characteristic internal (semantic and syntactic) organisation, and it has been this which has principally occupied the attentions of analytic philosophers interested in the structures of scientific theory. These attentions have in large effect been directed not to the theoretical object-domains associated with systems of theoretical sentences, but to certain abstract (set-theoretical) surrogates constructed exclusively for the purposes of laying bare this internal organisation. Ontology is thus usurped by semantics.

Bodies of theory, propositions and their parts, are however radically heterogeneous to the object-domains which they describe. Their respective sets of structural properties may therefore be expected to exhibit at least some degree of independence. This suspicion is confirmed even in relation to the simplest of structural properties of theoretical languages, those associated with the propositional logical constants. An ontological analogue of the logical '—and . . .' can be discerned in the notion of ontological conjunction or sum, and of the logical '—entails . . .' in the concept of ontological inclusion (that *the king is happy and drunk* entails *the king is happy* is mirrored, ontologically, in the fact that the states of affairs which make the consequent true are included amongst the states of affairs which make the antecedent true). Yet there are no corresponding ontological analogues of negation and disjunction. A directly depicting language will contain no negative or disjunctive propositions (cf. [1] and §4 of [9]).

The parallelism between logical and ontological conjunction

is deep-rooted. The formal operations that allow us to move from ' $\Gamma \rightarrow A \& B$ ' read: ' $A \& B$ is deducible from the sequence of sentences Γ ') to ' $\Gamma \rightarrow A$ ' and ' $\Gamma \rightarrow B$ ' as conclusions, and from premises ' $\Gamma \rightarrow A$ ' and ' $\Theta \rightarrow B$ ' to the conclusion ' $\Gamma, \Theta \rightarrow A \& B$ ', have almost exact analogues in the formal operations that take us from " $a + b$ " (read: 'the sum of a and b exists') to " a " (read: ' a exists') and " b ", and from premises " a " and " b " taken conjointly to " $a + b$ ". Moreover, there is a parallelism in the fact that the meanings of '&' and '+' are each characterised exhaustively by the purely formal operations that govern their use. (" $a + b$ " cannot for example signify that a and b are spatially or temporally proximate. In general we should argue that no spatial or temporal notion—and therefore also not the notion of identity through time—can be characterised exhaustively by purely formal operations.)

The constants '&' and '+' are however importantly distinct. Logical conjunction relates exclusively to propositions; ontological conjunction relates to all nameables (including propositions, though the result of conjoining propositions A_i ontologically is not their logical conjunction, a new, typically more complex proposition; the sum of the A_i is an n -fold propositional heap). This suggests a distinction between logical and ontological constants, and between corresponding structural properties of, respectively, bodies of theory and segments of the world. The logical constants are comparatively well-understood. Further light on the constants of formal ontology will be shed below.

Logical and ontological constants are discriminated by the fact that the meanings they express can be determined exhaustively by purely formal operations. We shall therefore refer to them as formal constants, leaving open the question whether all formal constants are either logical or ontological. The pressing issue is, of course, that of determining precisely what is to count as a formal operation. Fortunately the work of logicians of the past 100 years has contributed at least the skeleton of an answer. The theory of formal operations underlying Gentzen's concept of the sequent calculus serves to delineate one entity which comes near to discriminating the formal structural (or, as we shall now simply say: formal) properties of theoretical language. In a penetrating discussion of Gentzen's work [2], Ian Hacking advances a view of the logical constants as syntactic

items characterised by rules of inference "like Gentzen's" (p. 291). The problem, then, becomes one of determining what shall count as "like" Gentzen's rules. Hacking's own preferred solution to this problem, which has motives skew to those which will predominate here, stays close to the letter of Gentzen's work. In this it is too narrow. For if Gentzen's ideas are to yield up for us an account of 'formal operation', then Gentzen-like rules must select out also the formal-ontological constants, and not all of these can find a natural home in formal logic as this is conceived by either Gentzen or Hacking.

It is the formal logical notion of deducibility that forms the central core of Gentzen's work. The rules of the calculi tell us what is to count as an admissible transformation between statements of deducibility relations between sequences of sentences. (From ' $A \& B$ is deducible from Γ ', for example, we can infer ' A is deducible from Γ '.) Transformation rules are divided into two sorts: structural and operational. The structural rules embody basic facts about deducibility; operational rules serve to characterise the meanings of the formal logical constants. Of the conditions imposed by Hacking upon Gentzen-like operational rules, only one will be of immediate relevance to us here. This is the requirement that such rules be conservative: the formal constants, when added to an object language, should confound neither the basic facts about deducibility manifested in the structural rules of the calculus, nor any pre-logical deducibility relations (from ' a is red' infer ' a is coloured') latent in the given language (pp. 296, 304). The characteristic internal properties of theoretical language (or of theoretical reasoning) are captured so well in the notion of deductive closure that we should exhibit no surprise at the fact that Gentzen's rules, erected specifically to capture the concept of formal deducibility, should come so near to providing a convincing delineation of the province of logic. But what, in formal ontology (the science of the formal properties of things, including thing-manifolds, events, actions, states, processes, and the like) should serve to anchor the transformation rules which characterise the meanings of the formal ontological constants in the way that deducibility anchors the rules of formal logic? We can first of all point out, echoing Frege, for whom 'is a fact' constitutes the common predicate of all propositions (*Begriffsschrift*, §3), that there is a sense in which

'is deducible from' serves as a common (2-place) predicate of all of Gentzen's sequents. In much the same way, 'exists' will be seen to form the common predicate of all propositional pictures ("a", "a+b", etc.) of formal ontology, and it is this concept which will serve as our guide in building up the formal ontological calculus.

This calculus will have as its counterpart of Gentzen's structural rules a single rule [D] (after its inventor, Wolfgang Degen):

from any "t" move to any "s", where all well-formed parts of 's' are parts of 't'.

The force of this rule will become clear shortly.

We should now require, in echo of Hacking/Gentzen, that the operational rules of the calculus be conservative; they should not muck up the basic facts about 'exists', either those captured by [D] or those presupposed by the object-language(s) onto which the formal ontological constants are grafted.

What, then, are the basic facts about 'exists' which are presupposed by the theoretical languages with which we are familiar? Taking different types of scientific theory as their model, different formal ontologists, from Aristotle to the author of the *Tractatus*, have offered different lists of basic facts. The theory which will underlie what follows, taken from Husserl's *Logische Untersuchungen* (specifically from Investigation III, "On the Theory of Wholes and Parts"), would claim to be the most general possible account. Husserl's instincts as to the absolute generality of his theory—only one small part of which will be presented here—are supported by the fact that it has proved possible, by relatively simple side-constraints on the notion of well-formedness in [D], to generate within it close analogues of its principal competitors (cf. [10], [11]).

The key to Husserlian formal ontology is the concept of part. Material things (properties, events, processes, material relations, acts, bodies, syntactic items . . .) have (material) parts, which stand in certain (non-material) relations to each other. The existence of such non-material relations is demonstrated by a familiar argument. That material parts m_1 constitute an integral whole entails that the m_1 are bound to each other by certain relations (otherwise their sum is not a single whole, but an n -fold heap of disconnected parts). Not all such relations can be material relations, can be, that is to say, amongst the m_1 . For that the

material m_2 binds m_1 and m_3 does not yet tell us what it is which binds m_1 to m_2 and m_2 to m_3 . On pain of infinite regress, not all relations between the m_1 can be catered for by relations amongst the m_1 .

Husserl called the non-material ties between parts of an integral whole *foundation relations*. If a part a is such that of its nature it cannot exist except in association with a further part b , we say that a is founded upon b (LU III §14). A specific instance of a disease is in this sense founded upon its bearer. An act of thought is founded on a thinker. Repair of object a is founded upon prior damage, both repair and damage being in turn founded upon the object a itself. A syncategorematic term *qua* element in a meaningful utterance is founded on the categoremata with which it is associated.

The concept of foundation relation so defined is a formal concept; it can be characterised exhaustively by purely formal operations on corresponding expressions. We can now (somewhat tentatively—cf. the problems discussed in [8]) define an integral whole as a whole all of whose parts are connected to each other mediately or immediately by relations of foundation and none of whose parts are founded on any item discrete from the whole. This (formal) concept of integral whole approximates to the concepts of substance expounded by Aristotle and Brentano and it has affinities with Frege's concept of saturation. An integral whole is symbolised (pictured) by an expression like: \boxed{t} . Here the solid frame symbolises the (proximate) form of the entity in question; 't' is a name (picture) of the (proximate) matter. By [D], which allows us to strip away both form and matter from a propositional picture, we can infer from " \boxed{t} " (read: 'the integral whole t exists') either "t" ('the matter, t , of \boxed{t} exists) or " \square " ('an integral whole exists'). That \boxed{t} is a part of \boxed{s} we might symbolise by: $\boxed{\boxed{t}_s}$.

Thus polygonal frames share some syntactic and semantic properties with the ovoids of the calculus of Venn diagrams.

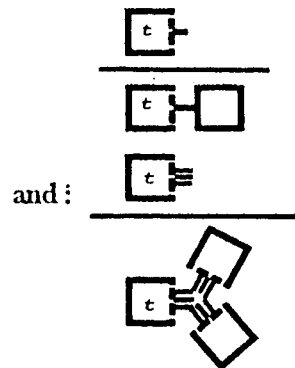
Not all nameables are integral wholes. (Hence in general we cannot proceed from "t" to " \boxed{t} ".) The derivability of this rule of inference is characteristic of the extensional theories of part-whole relations promulgated by Lesniewski and Goodman. It has the effect of annihilating the distinction between integrity

and non-integrity.) Let 't' stand for, say, the charge in a given conductor, or Anthony Blunt's knighthood. The latter is (or was, when it existed) founded upon an object (Blunt) which is discrete from itself. Aristotle called such non-object nameables accidents. Husserl however recognised that Aristotelian accidents fall within a much wider class of what he called dependent parts or *moments* (*LU III passim*). A knighthood or electric charge is said to be one-sidedly dependent on its bearer. Whilst the knighthood cannot exist except as part of a larger whole which includes its bearer, the converse is not true: Blunt can gain or lose his knighthood without thereby ceasing to exist. Not all non-object nameables are one-sidedly dependent in this sense. Colour-accidents and visual extents are two-sidedly or mutually dependent: a colour cannot exist without visual extension; visual extension cannot exist without colour. On the other hand, there are examples of relational accidents—sword-fights, matrimonial and chemical bonds, performances of trio sonatas—which are one-sidedly but multiply dependent on a manifold of bearers.

Dependent parts might therefore be symbolised as follows:



signifying, respectively, 1-, 2- and 3-fold one-sided dependence and 2-, 3- and 4-sided mutual dependence. The meanings of the formal constants symbolised by the broken frames are given by operational rules such as:



From: 'the dependent part *t* exists' infer: 'there is an integral whole with which *t* is configurated' or, equivalently, 'there is a configuration of *t* with some integral whole'.

From: 'the dependent part *t* exists' infer: 'there exist two further three-sidedly dependent parts with which *t* is configurated'.

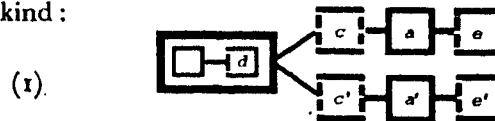
Examples of three-sided mutual dependence are provided by the hue, saturation and brightness of a colour, or the pitch, timbre and loudness of a tone. A colour-hue cannot exist except in a

larger whole which relates it foundationally to a saturation and a brightness.

If *t* and *s* are mediately or immediately linked by relations of dependence, then we say that they are foundationally related, and write "*t*[^]*s*". Trivially, by Degen's form-shedding principle, we may infer from "*t*[^]*s*" that "*ts*" (where left-right concatenation now does service for the '+' of ontological conjunction). From " \boxed{trs} " we may infer "*t*[^]*s*". Conversely, from "*t*[^]*s*" we may infer that there is a (possibly empty) term-complex '*r*', such that "*t*[^]*r*", "*r*[^]*s*", and " \boxed{trs} ". A whole *t* which is foundationally related to a second whole *s* is foundationally related to all superordinate wholes *r*. (This is a partial converse of [D]; compare theorems 2 and 4 of Husserl's formal ontology of wholes and parts in §14 of *LU III*).

[^] is an equivalence relation. It partitions the object-domain of a language into which it is embedded into maximally connected wholes. The ontology of absolute idealism may be simulated by the addition of a principle of upward closure (from "*ts*" infer "*t*[^]*s*"), which has the effect of guaranteeing the existence of a single maximally connected whole.

We shall provisionally restrict the lexicon of terms '*t*', '*s*', '*r*', . . . of the formal ontological calculus to the closure, under operations of left-right concatenation, [^]-concatenation and superposition, of the set of formal constants (frames) and material constants (simple names: '*a*', '*b*', '*c*', . . .). This allows us to build up terms (or, correlatively, propositional pictures) of the following kind:




(say: the bearer of disease *d* is undergoing courses of treatment *c* and *c'* from *a* and *a'* who are themselves suffering from diseases *e* and *e'*). From (1) we can infer that *a* exists; that the sum *ccc'aa'ee'* exists; that three objects exist (" $\square\square\square$ "), and so on.


Molecular terms such as (1) are pictures of states of affairs. The totality of existing states of affairs is the world.


2. *Logic Mattered*. That the elements of a picture are related to one another in a determinate way represents that things are related to one another in the same way. The language of the

formal ontological calculus is, we want to claim, a directly depicting language in exactly Wittgenstein's sense. It is as if the individual frames, with their matters, served as windows through which, if we strain our eyes, we can distinguish the correspondingly configured objects, events, properties and relations underneath.

A complex such as (1) gives us a view of the underlying state of affairs which is more articulated than that given by, say, the conjunction "dca'ae'e". "[a]" tells us that the object *a* (the integral whole picked out by the empty marker 'a' exists; "[]" tells us simply that an object exists. Degen has pointed out that there is a level of articulation intermediate between "[]" and "[a]" which is captured in language by the use of common nouns and common noun phrases ('a dog exists', 'a dog is suffering from an instance of influenza', 'an electric charge exists'). We shall therefore augment the catalogue of basic terms of the formal ontological calculus by material constants ' α ', ' β ', ' γ ', . . . which are to stand in for the simple common nouns of an associated object-language. For "[a]" read: 'a man exists'; for "[v]" read: 'a pox (an instance of the pox) exists', and so on. Additional expressive power is yielded by the device of superposing proper and common nouns:

(2) : 'the three men, Alfredo, Dick and Harry, exist';

(3)  'the Eiffel Tower, qua instance of the species *conductor of electricity*, carries an electric charge'.

 is something like a bifocal lens.

Simple common nouns or species markers are empty of form, but they are not, like simple proper names, empty of intelligible matter. In particular, they stand to each other in determinable-determinate relations, and the totality of common nouns is thereby partitioned into trees whose topmost determinable or highest material genus demarcates a region of materially cognate objects. It is such a region of objects, with its associated region of species, which forms, according to Husserl, the subject-matter of a properly constructed scientific theory.

How are determinable-determinate relations to be represented syntactically? We might experience an initial temptation to extend the formal ontological calculus by adding simple rewrite rules of the form:


$$[R] \frac{\text{---}\sigma\text{---}}{\text{---}\tau\text{---}}$$

Wherever ' σ ' occurs in a propositional picture, replace it by ' τ ' (any species marker further up the tree).

Here purely material articulation of a propositional picture is held to have no effect upon its formal structure. Unfortunately, form and matter are not in this respect independent; general names, unlike proper names, are not referentially transparent. Thus consider the propositional picture:

(4) 

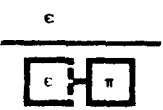
expressing the fact that husband and wife Hans and Erna are mutually founded on each other (a husband cannot exist except in association with a wife). 'Man' is a determinable of 'husband', yet

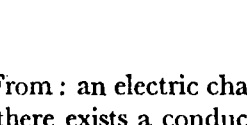
(5) 

cannot validly be inferred from (4). Qua man, Hans is not dependent for his existence upon the existence of his wife. This tells us that a distinction has to be made between relative and absolute (in)dependence. Relative to any whole in which he functions qua member of the species *man*, Hans is independent; relative to a whole in which he functions in his capacity as husband he is dependent. Relative to the chemical whole which is the manifold of molecules in my body, any single molecule is an independent object. Relative to me, each molecule is mediately or immediately founded on the remaining molecules (cf. *LU III* §§13,16ff).

Even refinements of [R] could however at most capture straightforward up-down relations amongst determinates and their determinables or logical parts. Of considerably greater scientific importance are the sideways leaps amongst determin-

ates which capture relationships amongst *disjoint parts* (LU III §1). Consider, for example :

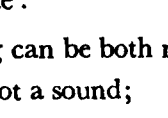
(6)  From : an electric charge exists' infer :
'there exists a conductor of electricity
in which the charge inheres'.

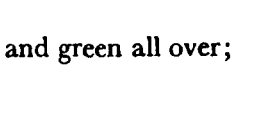
(7)  From : 'a hue exists' infer : 'there
exist mutually configured bright-
ness and saturation'.

The significance of such rules can best be demonstrated by translating all of the above from the ontological into the logical mode. Each propositional picture of the ontological calculus corresponds—with some creakage—to a system of one or more merely logically articulated propositions. (3), for example, corresponds to a system of propositions which includes :

e is a conductor of electricity,
 e carries an electric charge,
 $\exists x(x$ is a conductor of electricity),
etc.

Correlatively, to each valid ontological transformation there corresponds a family of logical inferences.

(8)  might correspond to, say :
 $\Gamma \rightarrow a$ is red
 $\Gamma \rightarrow \exists x(x$ is red)

(9)  to :
 $\Gamma \rightarrow \exists x(x$ is scarlet)
 $\Gamma \rightarrow \exists x(x$ is red)
 $\Gamma \rightarrow \exists x(x$ is coloured)

Moreover, in the presence of material rules such as (6)-(9), we can demonstrate for the extended ontological calculus certain metalinguistic rules on admissibility of propositional pictures

which will correspond, logically, to propositions of necessary exclusion like :

nothing can be both red and green all over ;
red is not a sound ;
a rose cannot bear a son ;
justice is not heavy ;
a murder cannot be committed by neglect ;

and so on.

The inference rules in (8) are formally valid. They correspond to purely analytic laws, that is, to laws true purely in virtue of the *forms* of the (simple and defined) terms which they contain (LU III §11 and cf. also §148 of Bolzano's *Wissenschaftslehre*). But as we can see by examining (9), not all valid transformation rules are analytic in this sense, and this remains true even when only a priori transformation rules—i.e. rules whose admissibility is not merely inductively established—are taken into account. Analytic philosophers have strained long and hard to maintain the view of all a priori transformations as analytic. Thus they have been constrained to show that certain overtly simple material terms are formally complex (cf. *Tractatus* 6.3751). Whilst this project has met with some success with respect to certain restricted classes of determinates ('bachelor', 'vixen', 'biped'), similar successes can by now surely be excluded in the case of core terms like 'colour', 'pitch' and 'hue' ([5]) and, more generally, in relation to all core terms appearing in rules which, like (7), express a priori relations between disjoint parts. The meanings of such terms are formally unarticulated. They cannot be grasped by means of definitions, but only by a process of lifting up the page and straining one's eyes to see the matters underneath.

We can now begin to see why it has mattered so much to analytic philosophers to maintain the view that all a priori propositions can be exhibited as disguised formal laws. Should it once come to be accepted that there exist a priori propositions not formally resolvable, then it would follow that there are inference rules, both logical and ontological, enjoying an irreducibly material validity. The science of valid inferences would therefore comprehend not only formal logic, but also certain material a

priori disciplines wholly alien to the analytic conception of philosophy.

3. *Synthetic a priori Structures.* The theory of logic and formal ontology sketched above rests on ideas developed by Husserl in order to find a place, within the realm of science, for the synthetic a priori propositions of his own new discipline of phenomenology. 'α', 'β', 'γ', . . . function, from Husserl's point of view, as markers of species, essences or natures, and a synthetic a priori propositional picture such as the consequent of (7) he would conceive as a picture of an a priori configuration of corresponding species-instances. All propositions of descriptive phenomenology express synthetic a priori relations amongst species in this sense.

Such propositions are established by a method, developed by Husserl from hints in the works of Bolzano, Twardowski and Stumpf, which he calls *eidetic variation*. Beginning with an instance *a*, real or imagined, of some not simply inductively delineated species *α*, *a* and its surroundings are allowed to vary in imagination along all possible dimensions. Some types of variation will lead to the annihilation of *a* qua member of the species *α*. Thus as soon as we attempt to imagine an act of memory relating to an event in the future we see that such an act is impossible. (The phenomenological method, as Wittgenstein saw, establishes not regularities, but mere possibilities: *Philosophische Bemerkungen*, p. 51f, *Wittgenstein und der Wiener Kreis*, p. 63.) It is a constant part of the structure of the species *act of memory*, that each of its instances relates to a prior event. A speck in the visual field, though it need not be red, must have some colour. That is, it is a constant part of the structure of the species *datum of visual extent*, that its instances be associated with instances of the species *colour datum*. Precisely which determinate colours are so associated (which determinate hues and which determinate degrees of saturation and of brightness) is something which is left variable. Each axis of independent variation in a species corresponds to a distinct dimension of disjoint parts in the structure of its instances.

Husserl himself applied this method exclusively to the subject-matters of descriptive psychology and, by extension, to cognitive formations such as scientific theories and the structures of lan-

guage. His ideas were however applied by the early phenomenologists also in other spheres. Adolf Reinach, for example, in his *The A priori Foundations of Civil Law* [7], uses the method to develop an elaborate theory of the species of performative utterances in terms of the different types of dependence relations between speech-acts and the obligations, claims and other material actions and conditions with which they are associated. Thus it is a synthetic a priori truth, according to Reinach, that an instance of the species *act of promising* results in mutually correlated obligation and claim on the part of promiser and promisee. And it is a synthetic a priori truth that a speech-event is an instance of the species *command* only if its content is issued in such a way that it is capable of being received and understood by the party to whom it is addressed.

Not all synthetic a priori propositions are propositions expressing foundation relations amongst instances of species. Consider:

'time preference is not negative';

'orange lies between red and yellow in the order of similarity';

'given three distinct tones, one lies intermediate between the other two';

'if *a* is warmer (taller, heavier, temporally more remote) than *b*, and *b* is warmer (. . .) than *c*, then *a* is warmer (. . .) than *c*';

'if *a* is preferable to (more probable than, more guilty than) *b*, then *b* is not preferable to (. . .) *a*'.

The determinates of determinables such as 'mental act', 'performative', 'mammal', do not designate species capable of being organised according to differences of intensity or degree, of more and less. Nor do they designate species whose instances are capable of being pieced (demarcated extensionally into contiguous mutually independent parts: *LU III §§17, 25*) as are instances of the species *spatio-temporal extent*. A priori propositions in the theory of measurement, and propositions expressing a priori relations of phenomenal *Steigerung*, particularly those rooted in the multi-dimensionality of colour- and tone-space and in linear orderings such as cold-warm-hot- . . . occupied Wittgenstein in very many of his writings. But each of his

attempts to exhibit the "logical structure" of such propositions shipwrecked on the banks of the materially specific structures in virtue of which they are true. Husserl's work on phenomenal qualities, on the other hand, which is concerned precisely to disclose the given material structures, is descriptively much more successful (see [14]).

Corresponding to the set-theoretical prejudice in general ontology, the ontology of measurement has been beset by the prejudice of cardinalism. Cardinal metrics would seem however to be directly associable only with what Husserl calls extensive manifolds, manifolds involving spatio-temporal extendedness and therefore capable of being correspondingly pieced (LU, *loc. cit.*). Linear manifolds involving degrees of intensity, those generated by, for example, sensations of loudness or heat, or by 'logical weight' (fn. to 1 of the *Tractatus*) are not extensive manifolds in this sense. Their structure is characteristically an ordinal, not a cardinal structure. (Though here 'ordinal' may have to be understood widely enough to comprehend mixed order-types such as that exhibited by the numbering system of the *Tractatus*. That this is not a decimal system, as Wittgenstein himself thought, is seen by considering the problem he would have faced had he felt the necessity to make ten or more comments of equal logical weight on a single proposition. Wittgenstein seems rather to have stumbled upon something like a system of ordinal fractions. A more general system is obtained if we add the facility to introduce, without upsetting the original numbering, a comment on $n.m$ which is intermediate in logical weight between it and $n.m.1$. (This problem of ordinal sports has its analogue in the theory of diplomatic protocol: where, at a dinner party constrained by rigidly stratified seating arrangements, is one to put the visiting Estonian archbishop?)

The a priori theory of intensive magnitudes was developed by theoretical psychologists such as Meinong, Stumpf and Kreibitz at the end of the 19th century. Their ideas found application in probability theory, in the theory of subjective preference orderings, and also in actuarial and legal science. Meinong's work is still partially alive in Russell's *Principles of Mathematics*, but problems in the theory of non-cardinal magnitudes are unfortunately seldom discussed by analytic philosophers of the present day.

4. *Mathematics*. The identification of 'form' with 'logical form' and of 'structure' with 'logical structure' on the part of analytic philosophers has not been accidental. This narrow conception has proved itself extraordinarily felicitous in the representation of the structures of mathematics. Analytic philosophers labouring under the conception have indeed produced their best results almost exclusively within the philosophy of mathematics, and this has consolidated the prejudice that it is considerations derived from mathematics which must serve as the test of an account of form. How well, then, does Husserl's theory stand up to this test?

Here we can do no more than scratch the surface of his views, but his account of the relation between mathematics and logic is sufficiently closely related to mainstream logicism to mean that much of what we have learned from the failures and successes of the latter can be applied also to his work. The principal difference between Husserlian and analytic philosophy of mathematics turns on the fact that the latter, having evolved no clear separation of logical and ontological form, has skirted the question of the relation between mathematics and ontology. Thus consider the two sets of rules:

$$\begin{array}{l}
 [Q1] \quad \frac{\neg \exists xFx}{OxFx} \\
 [N1] \quad \frac{\boxed{\alpha}}{(1 \boxed{\alpha})}
 \end{array}
 \quad
 \begin{array}{l}
 [Q2] \quad \frac{nx(Fx \ \& \ x \neq a) \ \& \ Fa}{S_nxFx} \\
 [N2] \quad \frac{\boxed{\alpha} \ (n \boxed{\alpha})}{(n \boxed{\alpha})}
 \end{array}$$

Both pairs of rules generate sequences of natural numbers ($O, SO, SSO, \dots; 1, 11, 111, \dots$) as indices of formal operations. But where the numbers defined by [Q1, 2] have properly been recognised by analytic philosophers as formal *logical* constants (properties of concepts), these same philosophers have failed to recognise that numbers generated by rules such as [N1, 2] are formal *ontological* (are, in this case, formal ontological properties of object-manifolds). What is remarkable is that Husserl is responsible not only for the earliest clear separation of formal logic and formal ontology, but also for the first account of numbers as

formal constants, constants whose meanings are characterised exhaustively by the rules which govern their use as 'indices [*Etalons*] in a field of definite operations'.*

Husserl's wider philosophy of mathematics falls, like Gentzen's, within the tradition of Leibniz and Bolzano. Both logic and mathematics he conceived as purely formal disciplines but, because they have different goals, their respective treatments of form are crucially distinct. Logic matters because it is the science of the formal properties of scientific or theoretical language; it is the "science of the essential parts of genuine science, as genuine" ([4], §5). Thus not every systematic treatment of a philosophical topic involving the replacement of commonly occurring words by Greek letters or fancy symbols is logic in Husserl's sense. Since theoretical reasoning is, for example, essentially finitary, Husserlian logic can involve only formal constants defined by transformation rules of finite complexity. Mathematics arises when this and other restrictions on formal operations are lifted, when the theorist allows himself absolute freedom of variation of the concept *formal operation*. The pure *mathesis universalis* thereby generated falls into two parts: the formal theory of theories (or of syntactic structures in general, including infinitary, modal and other deviant calculi), and the formal theory of manifolds (including discrete and continuous manifolds, finite and infinite sets and orderings, geometrical and topological structures, probability spaces, etc.). This yields a theory of the applicability of mathematics: certain formal manifolds are instantiated, materially, by manifolds of objects in the world. And it generates an account of the mathematisability of logic (where it is normally left unexplained why the deducibility relations which obtain between theoretical sentences should admit of mathematical treatment in a way that, for example, the grammars of such sentences do not). But these issues are formidable. Several thousands of pages of Husserl's writings are devoted to logic and the philosophy of mathematics. We plead only that philosophers show some slight readiness once more to admit his ideas into the corpus of their thoughts.

* [3], p. 475, cf. *Tractatus* 6.021. See also *op. cit.* p. 485: 'It belongs to the concept of operation that the result of an operation can become the substrate of a further operation of the same type. Results of operations are species of the same genus as their bases . . .' and compare *Tractatus* 5.251ff.

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 I am most heavily indebted to [6], almost the only serious commentary on the formal ontology of Husserl's *Logical Investigations*. I have also profited greatly from the logical writings of Peter Simons and Wolfgang Degen.