Quantum Mereotopology

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Abstract

Abstract: Mereotopology faces problems when its methods are extended to deal with time and change. We offer a new solution to these problems, based on a theory of partitions of reality which allows us to simulate (and also to generalize) aspects of set theory within a mereotopological framework. This theory is extended to a theory of coarse- and fine-grained histories (or finite sequences of partitions evolving over time), drawing on machinery developed within the framework of the so-called 'consistent histories' interpretation of quantum mechanics.

Keywords: mereotopology, granularity, ontology, presentism, partitions, histories, interpretation of quantum mechanics

Time and Mereology

It will be useful to formulate our problem against the background of recent work on spatial reasoning by Casati, Cohn, Egenhofer, Galton, Stell, Varzi, Worboys and others. These authors have shown that it is possible to conceive spatial reasoning in terms of the manipulation of corresponding spatial objects within a framework of mereology supplemented by topological notions. It has proved difficult, however, to extend this mereotopological framwork to comprehend not only spatial but also temporal features of the objects in question. Our goal in what follows is to rectify this problem by providing the basis for adding time and change into mereotopology. We shall not provide a full theory of temporal granularity (on this, see Bettini et al, 1998); rather we shall sketch only those features of such a theory which are needed for our meretopological purposes.

To put the matter very simply, once objects are allowed to exist at different times and to survive the gain or loss of parts, then central axioms of mereology—for example the axioms of extensionality and of transitivity of parthood—are no longer valid.

Philosophical ontologists have offered three different sorts of solution to this problem:

(1) Four-dimensionalism, which imposes a framework according to which it is not three-dimensional objects in space, such as Hamburg or your brother, which should constitute the domain of the theory, but rather four-dimensional spatiotemporal worms. (Quine 1960)

(2) Phase-theories, which impose a slicing of normal objects into their instantaneous temporal sections; normal objects themselves are then re-conceived as logical constructions—effectively, as dense sequences of such instantaneous temporal sections (as *entia successiva*). (Chisholm 1973)

(3) Presentism, which imposes a view according to which 'existence' and 'present existence' are to be taken as synonymous. (Prior 1968) We can still refer to past and future objects, on the presentist perspective, but only as objects which did or will exist. Presentism in this general sense is consistent with both four-dimensionalist and phase ontologies. (Brogaard 2000) It can also, however—and this is what is important for our purposes here—be combined with an ontology which takes normal objects seriously as these are conceived in our everyday processes of reasoning. Since such objects exist only at a single time (namely: now, in the present), the standard difficulties facing cross-temporal mereology can thereby be avoided.

(1) yields an ontology within which time is treated, in effect, as an additional spatial dimension. One problem with this ontology is that it is no longer possible to formulate in coherent fashion the familiar distinctions between things and events (or in other words between continuants and occurrents)—a distinction which many four-dimensionalists would in fact reject, but which seems central to our reasoning about spatiotemporal objects. An even more pressing problem for the four-dimensionalist turns on the fact that change and becoming are strictly speaking not capable of being represented within this ontology: that an object becomes warmer or cooler is, rather, analogous to static variation of the sort that is instantiated by a banner that is red at one end and blue at the other. Analogous difficulties are faced also by (2), which replaces ordinary names with time-indexed expressions of the form 'Lemberg at noon on October 25, 1998'. Here, too, available systems fall short of providing an ontology within which our reasoning about ordinary spatial objects (things and events) can be represented in a natural way.

(3), on the other hand, is more promising. It takes over ordinary names for ordinary objects and operates not with time-indices but rather with tenses, or in other words with

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just the standard sorts of modifications of verbs that are used in ordinary reasoning. Presentism retains also the fundamental ontological distinction between objects and events (or between continuants and occurrents). This is another distinction crucial to ordinary reasoning that is undermined on alternatives (1) and (2).

The problems with (3) arise when we wish to represent processes of reasoning which relate to objects not existing in the present. Where (1) and (2) solve the problems of temporal mereology by embracing a temporally extended universe but reconceiving the objects in this universe in such a way that standard mereology can be applied, (3) achieves this same effect by holding on to things and events as normally understood, but reconceives the universe itself as being, at any given time, temporally unextended. How, then, is the presentist to represent time and change? If she allows within her ontology not only objects which exist now but also two families of objects which did and which will exist, then she will resurrect the very problems the presentist ontology was designed to solve. In addition the presentist faces new problems which arise when she seeks to do justice to those types of reasoning about past or future objects which involve the simultaneous manipulation of objects existing at different times or the adoption of different temporal perspectives on the part of the reasoner. Consider: 'It was during the cleaning up after the flood that I remembered that I would later need to go on to the circus'.

The framework defended in what follows is a generalization of presentism as applied to ordinary objects, which resolves the mentioned problems by allowing the manipulation not only of objects existing within the present but also of objects existing at various selected times in the past or future. Simply put, it allows not just one but (finitely) many time-indexed presents (instantaneous snapshots through time) within the framework of a single ontology. It draws in this respect on the ontology of fine- and coarse-grained histories proposed by the physicist Robert Griffiths and used by Griffiths himself, and also by Gell-Mann, Hartle, Omnès, and others, as the basis for an interpretation of quantum mechanics. (Omnès 1994) Our title alludes to this quantum-mechanical background, and more specifically to the fact that the approach here advanced induces a certain sort of quantization or granularization on objects in space and time.

Grids and Partitions

When you think of John cooking his dinner in the kitchen, then you do not think of all the parts of John or of his surroundings. For you set John into relief in a highly specific way in relation to the rest of the world. You do not think of the follicles in his arm or the freckles on his cheek. You do not think of the fly next to his ear or the neutrinos that pass through his body. Rather, you impose what we shall call a partition upon reality which induces a fiat separation between what is focused upon and what is ignored. When we focus our attention on France, then similarly we set France into relief in relation to the rest of the world; and we effect similar partitions, though in more complex ways, when we focus on a map of France depicting its 91 *départements* or its 311 *arrondissements*.

Partitions as here conceived may be of coarser or of finer granularity, but they must have cells of finite size. Hence, they cannot be dense. The division of the line into real or rational numbers does not define a partition, and neither does the (whole) system of lines of latitude and longitude on the surface of the globe. A partition is, intuitively, the result of applying some sort of grid to a certain portion of reality. For a partition to do its work, its cells need to be large enough to contain the objects (observables) that are of interest in the portion of reality which concerns us. At the same time these cells must be not too large, in order that they may allow us to factor out the details which do not concern us. A partition is thus an instrument for focusing upon and also for ignoring things—for placing certain parts and moments of reality into the foreground of our attentions, in such a way that other parts and moments are traced over in the background of our attentions.

A grid is a way of dividing up the world, or some portion of the world, into cells. A partition is the result of such division. The verb 'to partition' is thus to be understood in what follows as a success verb. The grid of a partition is in each case laid like a net over the relevant object-domain in such way that, like a net, its cells are transparent: they allow the objects in the domain over which it is laid to show through in undistorted fashion. The notion of a partition is in this respect a generalization of the notion of set. Where, however, the elements exist within a set without order or location—they can be permuted at will and the set remains identical—a partition comes with a specific order and location of its constituent cells. A partition brings with it an address system—of the sort that is found, for example, in models of the human genome, or in the ROM-BIOS memory of a computer's central processing unit, or in a map of the monasteries of France. This means that a partition, in contrast to a set, may include empty cells.

Partitions are distinguished from sets also in this: where an object can be an element of a set (or singleton) in only one way, an object can be in a cell within a partition in any number of ways. For there is no requirement that an object must fit its cell exactly. Compare an object in a cell to a bacterium in a petri dish, or to a guest in a hotel room.

A set is an abstract structure; its members are (in the cases relevant to our deliberations here) parts of concrete reality. Partitions, similarly, belong to the realm of abstracta (the realm of our theoretical representations), over against the concrete realm of represented things and events. We can think of the boundaries of each cell in a partition as fiat boundaries. (Smith 1995) These boundaries are then not physical discontinuities in the underlying domain of objects, but are rather the products of our acts of demarcation (analogous, once again, to the results of drawing lines on a map).

Each partition can then itself be thought of as a sum total of fiat boundaries comprehending and at the same time parceling out in determinate fashion certain concrete portion of the world. The cells of a partition may be purely spatial, as in a map which effects a two-dimensional partition of a certain portion of the surface of the globe. But partitions may be constructed also in such a way as to involve non-spatial demarcations into cells. Thus they may comprehend dimensions determined by various properties—of velocity, temperature, density, or what have you – associated with the objects to which the partition is applied. At the opposite end of the spectrum we have very simple partitions, for example the Spinoza partition which comprehends the whole universe in a single cell. Similarly we can define for each given object x what we might call the object (or foreground/background) partition for x. This has two cells, one of which contains, precisely, x; the second cell contains x's complement (the mereological sum of all the objects in the universe disjoint from x).

Objects and Cells

An object is a constituent part of the world. It is what and where it is independently of any acts of human fiat and independently of our efforts to understand it theoretically. It is governed by the classical mereotopology of the bona fide realm. A cell or complex of cells, by contrast, is an artefact of our theoretical activity: it reflects a possible way of dividing up the world into parts, and it exists only within the context of the partition to which it belongs and by which it is determined. It is governed by the non-classical mereotopology of the fiat realm. (Smith 1997, Smith and Varzi 2000) Granularity itself is properly at home only in the fiat realm: it pertains not to the objects themselves on the side of reality, but rather only to the ways we partition these objects in our theorizing.

Let the variables $z, z', \ldots$ range over cells and complexes of cells. Let '$z \leq_A z'$' be read as meaning: z is a sub-complex of the complex z' within the partition A. $\leq_A$ defines a partial order, by analogy with the usual set-theoretic subset relation, with *A* the maximal element.

A cell in a partition is, intuitively, a complex of cells which has no sub-complexes. We can define what it is for a complex to be minimal in this sense in the following way:

$$C_A(z) =: z \leq_A A \ \wedge \ \forall z' (z' \leq_A z \Rightarrow z' = z)$$

We can rule out infinite complexity of partitions by imposing the requirement that all descending chains in a partition-structure terminate in a minimal cell:

If $\ldots \leq_A z_1 \leq_A z_0 (z_i \leq_A A;\ i \in N)$, then there is some $m \in N$ such that $z_m = z_{m+1=} \ldots$

As complexes of cells are in some respects like sets, so cells are in some respects like singletons. Thus we can draw here on David Lewis's conception of sets as mereological fusions of singletons. (Lewis 1991) Partitions satisfy the standard set-construction principles of union and intersection. If two complexes belong to the same partition, then their union is also a complex in that partition:

$$z_1, z_2 \leq_A A \Rightarrow z_1 \cup z_2 \leq_A A$$

The associated principle for the intersection of complexes can also be accepted:

$$z_{1,} z_2 \leq_A A \ \wedge \ z_1 \mathrm{o}_A z_2 \Rightarrow z_1 \cap z_2 \leq_A A$$

If two individuals are overlapping complexes in a partition, then their intersection is also a complex in that partition, which is however a trivial consequence of the definition:

$$z_1 \mathrm{o}_A z_2 := \exists z (z \leq_A A \wedge z \leq_A z_1 \wedge z \leq_A z_2).$$

For complements we have:

$$z <_A A \Rightarrow -z <_A A$$

If z is a proper constituent complex in a partition, then the complement of z is also a constituent complex in that partition.

Formally, the span of a partition *A* is defined as the mereological sum of all the cells in the partition. The span of a partition itself is a partition in which the interior fiat boundaries have, as it were, been smeared away. It is a partition with a single cell.

We shall say that a partition is extended by another partition if all of the cells in the former are also cells in the latter. We write $A \preceq A'$ to signify: A is extended by A'. We can then define extension as follows:

$$A \preceq A' := \forall z \ (z \leq_A A \Rightarrow z \leq_{A'} A')$$

A partition may be extended either by enlargement or by refinement. If a partition is enlarged, then more cells are added at its outer border. If a partition is refined, then more cells are included in its interior while the span is kept constant. This can occur either via imposition of a finer grain in the existing dimensions of the partition, or through combination (multiplication) with another partition in a way which amounts to the construction, within the mereotopological framework, of an analogue of the standard set-theoretic notion of Cartesian product.

Objects and Partitions

Consider a partition *A* relating to plants of given types within a given area. We partition the space into cells along two spatial dimensions and one dimension determined by plant types. If *x* is a plant within a given cell *z* in this partition, then we write:

$$\mathrm{L}_A(x, z)$$

$\mathrm{L}_A(x, z)$, which may be read as meaning 'x is located at z in A', is a primitive concept. (Casati and Varzi 1999) Location is to be understood in such a way that cells have objects located in them, and complexes may have mereological sums of objects located in them.

We define *x is recognized by A*, as follows:

$$x \in A := \exists z (z \leq_A A \wedge \mathrm{L}_A(x, z))$$

We define exact location in terms of simple location as follows :

$$\mathrm{L}_A{*}(x,z) := \mathrm{L}_A\,(x,z) \wedge \forall x'(\mathrm{L}_A\,(x',z) \Rightarrow x' \leq x)$$

If x is exactly located in z, then x is a maximal occupant of z. Intuitively, all boundaries of x then coincide with those of z. Compare the relation between a concrete parcel of land and the corresponding cell in a cadastre.

In a given partition, if an individual is exactly located both at the complex z, and at the complex z', then z and z' are identical.

$$\mathrm{L}{*}_A\,(x,z) \wedge \mathrm{L}{*}_A\,(x,z') \Rightarrow z = z'$$

We also have:

$$\mathrm{L}{*}_A\,(x,z) \wedge \mathrm{L}{*}_A\,(x',z) \Rightarrow x = x'$$

Since objects compose to form more composite objects, the objects located in a given cell or complex of cells satisfy the following Principle of Closure for sums:

$$\mathrm{L}_A\,(x,z) \wedge \mathrm{L}_A(y,z) \Rightarrow \mathrm{L}_A(x+y,z)$$

If two objects are located at two different cells, then the sum of these objects is located at the sum of these cells:

$$\mathrm{L}_A(x,z) \wedge \mathrm{L}_A(x',z') \Rightarrow \mathrm{L}_A(x+x',z+z')$$

Crucially, an object is never in two cells which do not overlap:

$$\mathrm{L}_A\,(x,z) \wedge \mathrm{L}_A\,(x,z') \Rightarrow z \text{ o } z'$$

We might call this the Principle of Classical Realism.

If an object x is located at a complex z in the partition A, and if y, a part of this object, is recognized by A, then y is located in z:

$$\mathrm{L}_A\,(x,z) \wedge y \leq x \wedge y \in A \Rightarrow \mathrm{L}_A(y,z)$$

We then define 'minimal object' relative to a partition A in the obvious way as follows:

$$\mathrm{M}_A\,(x) =: x \in A \wedge \sim \exists y(y < x \wedge y \in A)$$

For some partitions, which we can call *distributive*, if object x is a part of object y, where y is located at a complex z, then x is also located at that complex:

$$\text{dist}(A) := \forall x \forall y \forall z(x \leq y \wedge \mathrm{L}_A(y,z) \Rightarrow \mathrm{L}_A(x,z))$$

A set is a simple example of a non-distributive partition.

Partitions and Extensions

If, given a partition A and a certain portion of reality w, we write A_w to designate the result of restricting A to w, we can then define a second notion of extension, taking account not merely of the partition as a system of cells, but also of what is located in those cells, as follows:

$$A_w \preceq A'_{w\prime} := \forall x \forall z(\mathrm{L}_{Aw}(x,z) \Rightarrow \mathrm{L}_{A'w\prime}(x,z))$$

A_w is extended by $A'_{w'}$ if and only if all object-cell relations true in A_w are also true in $A'_{w\prime}$. Once again, extension can arise through either enlargement, for instance when two partitions are glued together topologically, or through refinement, when the cell-density or number of dimensions of a partition is increased while the domain is kept fixed.

Some partitions may cut through reality in ways that are skew to each other. One partition may divide a state into its separate counties, and a second partition divide it according to its soil types or population density. The two resulting partitions will then contain no cells in common, though they do in some sense share a common space of objects. We can accordingly create a single partition which includes them both, effectively by taking the Cartesian product of the two partitions with which we begin. This larger partition then stands to our initial partitions in the relation of refinement.

We can define "consistency" of partitions in these terms as follows:

$$A_w \vartriangle A'_{w\prime} := \exists A''_{w''}\ (A_w \preceq A''_{w''} \wedge A'_{w'} \preceq A''_{w''})$$

Two partitions are consistent when there is some third partition which extends them both.

Histories

We can conceive of a chess game in terms of the theory of partitions as follows. The game determines a partition having 64 minimal cells, at most 32 of which have objects located within them. Minimal objects are then the 32 separate pieces. Now, however, we need to take into account not just one partition but rather a coarse-grained temporal sequence of partitions, corresponding to the successive positions in the game. We shall call such a sequence of partitions a history. A partition stands to a history as an instantaneous snapshot stands to the sequence of successive frames which constitutes a film. A history corresponds to a sequence of successive observations, for example as these are made in the course of a physical experiment.

A history can be described by means of a conjunction of sentences of the form: The individual x is located at time i in the cell z:

$$\mathrm{L}_i\,(x,z)$$

where x is an object, z a cell in the partition, and i is an index for the successive reference times on the basis of which the given history is constructed.

A history may be more or less coarse-grained according to the number of reference-times and of cells which we employ in its construction. Consider the history which picks out John's location at three successive times. The rest of the world at the three times is ignored, as are all matters pertaining to the world at other times. Suppose John's locations (cells) at these three times are successively: Kennedy, De Gaulle, and Abu Dhabi airports. We can then describe John's movements in terms of a three-cell partition and three reference-times. We are not concerned with the people in the airport, the stewardesses in the successive planes or the food John is eating in the airports. These things, whatever they are, could have varied without affecting any detail of the given history.

We can, however, create a finer-grained history by constructing partitions that contain either more details about John and the places at which he is located, or more reference times. We use 'H' as a variable ranging over histories (finite sequences of partitions) and we write $A \in H$ for: A is a partition in history H. A history H is extended by another history H' if and only if all partitions in H are extended by partitions in H':

$$H \preceq H' := \forall A(A \in H \Rightarrow \exists A'(A' \in H' \ \wedge \ A \preceq A'))$$

Whatever holds (eventuates) in a history H holds in all extensions of H.

We can define the domain DH of a history in the obvious way as the ordered sequence of the domains of the corresponding partitions. A history is then refined through another history H' just in case H is extended by H' and H and H' have the same domain.

Suppose your entire knowledge of John's trip to Abu Dhabi is encapsulated by a given course-grained history. There are then many finer-grained histories all of which are consistent with your knowledge (though of course not all of these need correspond to what in fact eventuates). Each coarse-grained history can be identified with a certain class of fine-grained histories, namely the class of fine-grained histories that vary in respect of the details ignored in the given coarse-grained history. We shall say that two fine-grained histories H' and H'' are equivalent with respect to a coarse-grained history H if they satisfy:

$$H' \approx_H H'' := H \preceq H' \ \wedge \ H \preceq H''.$$

Libraries

There are alternatives to any given coarse-grained history H. John might fly to Abu Dhabi via London instead of via Paris. The coin, which landed on its head, might have landed on its tail. A coarse-grained history H' that is an alternative to H employs the same reference-times,but the objects are distributed differently across the underlying cells. The location predicate is then not an instantiation or occupation predicate simpliciter, but rather an occupation predicate with respect to a given history H in a family of alternative histories.

Alternative coarse-grained histories are in some respect analogous to alternative possible worlds. The consistency of a coarse-grained history can be understood in terms of the consistency of the sentences of the form $\mathrm{L}_i(x, z)$ by which it is described, in a way which can be used to generate maximal families of alternative histories. The family of histories over John's behavior at the given sequence of times is an exhaustive totality of mutually exclusive, exhaustive coarse-grained histories over his behavior at those times. We shall call such a maximal class of consistent coarse-grained histories a library. A library is analogous to a truth-table (Omnès 1994 calls a library a 'logic'): it specifies all possible ways in which a given system may behave. We can then assign probabilities to the different consistent histories in a given library. The probability that John goes to Abu Dhabi via Paris might be 75%, while the probability that he goes via London is 10%. The probabilities assigned to the histories in a given library must sum to 1. Hence, the probability that John goes neither via Paris nor via London is 15%. The library over John's behavior at the given reference times tells us the chance distribution over alternative histories of a given granularity.

The coarse-grained history in which John goes via Orly, and the alternative history in which he goes via Heathrow are mutually exclusive. That is, there is no larger, consistent history that contains them both.

We write '$H \in L$' for: H is a history in library L. We can then define an equivalence relation on fine-grained histories, relative to a given library of coarse-grained histories, as follows:

$$H' \approx_L H'' := \exists H \in L(H \preceq H' \ \wedge \ H \preceq H'')$$

$\approx$ partitions fine-grained histories into equivalence classes in the obvious way.

Consistent Histories and Quantum Mechanics

A library is maximal relative to a given granularity of cells and reference-times and relative to a given domain of constituent partitions. However, a library can be extended by increasing the number of reference times, or by using a finer partition for cells.

Two libraries L and $L'$are then called mutually consistent when there is a larger library of consistent histories containing them both:

$$L \vartriangle L' := \exists L''(L \preceq L'' \ \wedge \ L' \preceq L'')$$

Two libraries L and L' are called 'complementary' when there is no such larger library.

The theory of consistent histories and of probability assignments to histories within libraries was originally developed by Griffiths in (1984, 1993) and also by Gell-Mann, Hartle, and Omnès as the basis for a new interpretation of quantum mechanics. What distinguishes the quantum from the classical world, in addition to the pervasive and ineliminable role of probabilities in its description, is that to do justice to the evolution of physical systems within the quantum world we must employ not one but many libraries which are complementary (mutually incompatible) in the sense defined above. Experiments, from this perspective, are courses of events, like any other, to be apprehended within consistent histories (and thus within encompassing libraries) of appropriate type.

In the quantum world, it is sometimes possible for a particle to have contrary properties—a phenomenon called 'superposition'. For example, a photon can sometimes have two positions (be in two different places) at one and the same time. It can, in other words, contravene the Principle of Classical Realism as formulated in the above. To represent such a state of affairs in consistent fashion, the consistent historians hold, it is necessary for physicists to embrace different and mutually incompatible libraries in relation to one and the same physical system. All reasoning about that system must then take place exclusively within some one of these selected libraries. If reasoning takes place across libraries, then inconsistency will result.

Suppose physicists A and B have each made calculations with respect to the behavior of photons within some given apparatus involving, say, a photon source, a screen with right and left slits, and a detector. They each are allowed to set up experiments to measure the location of photons in order to test the accuracy of their calculations. A, working within one library and its associated repertoire of experiments, conceives the photon as a particle and constructs experiments designed to detect whether the photon goes through either the right or the left slit in the apparatus. B, working within a complementary library and repertoire of experiments, conceives the photon as a wave and constructs experiments designed to measure interference effects as the wave passes through both slits. Both libraries give rise to predictions of astonishing accuracy which are repeatedly confirmed in successive experiments. A's and B's predictions are, to be sure, inconsistent with each other. But such inconsistency can never be detected in relation to any given system of photons, since it is impossible for A and B to carry out the necessary experiments simultaneously.

Each experiment carried out by either A or B corresponds to a certain family of coarse-grained histories (libraries). Their respective libraries are inconsistent with each other. But they each give rise to equally good predictions, and no experiment can be designed which will establish a privileged status of one library over against another, complementary library.

Provided that a history is a member of a consistent family of histories, it can be assigned a probability (Griffith 1984, 1993), and within a given consistent family the probabilities function in the same way as do those of a classical stochastic theory: one and only one history occurs, just as, when we are tossing coins, one and only one succession of heads and tails in fact corresponds to reality. But histories can be assigned probabilities only if they are of sufficiently coarse grain. (Gell-Mann and Hartle 1991, 1993) This is for technical reasons, turning on the ways in which superposition effects can be said to 'decohere' (and thus become negligible) when we are dealing with physical systems of sufficient size and complexity. That the theory of consistent histories can be applied to the macroscopic phenomena (to the ordinary macroscopic objects) of our everyday reality might seem, in comparison, to be a trivial matter. That the theory can allow the extension of the mereotopological ontology to deal with change and becoming among such objects seems, however, to be of consequence nonetheless.

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